Design and Analysis of Algorithms I

Data Structures

Universal Hash Functions: Motivation

Hash Table: Supported Operations

Purpose : maintain a (possibly evolving) set of stuff.
(transactions, people + associated data, IP addresses, etc.)

Insert : add new record

Delete : delete existing record
easier/more common with chaining than open addressing

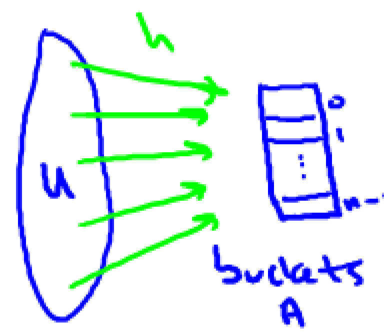
Lookup : check for a particular record
(a "dictionary")

Using a "key"

AMAZING
GUARANTEE
All operations in
O(1) time ! *

* 1. properly implemented 2. non-pathological data

Resolving Collisions

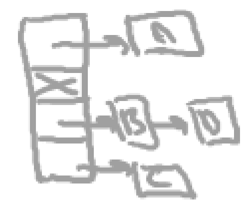


Collision : distinct x,y in U such that h(x) = h(y).

Solution#1: (separate) chaining.

-- keep linked list in each bucket

-- given a key/object x, perform Insert/Delete/Lookup in the list in A[h(x)]



bucket for x

linked list for x

Solution#2 : open addressing. (only one object per bucket)

-- hash function now specifies probe sequence h1(x), h2(x), …

(keep trying till find open slot)

-- examples : linear probing (look consecutively), double hashing

use 2 hash functions

The Load of a Hash Table

Definition : the load factor of a hash table is

$$\alpha := \frac{\text{\# of objects in hash table}}{\text{\# of buckets of hash table}}$$

Which hash table implementation strategy is feasible for load factors larger than 1?

- Both chaining and open addressing
- Neither chaining nor open addressing
- Only chaining
- Only open addressing

The Load of a Hash Table

Definition : the load factor of a hash table is

$$\alpha := \frac{\text{\# of objects in hash table}}{\text{\# of buckets of hash table}}$$

Note : 1.) $\alpha = O(1)$ is necessary condition for operations to run in constant time.

2.) with open addressing, need $\alpha << 1$.

Upshot#1 : good HT performance, need to control load.

Pathological Data Sets

Upshot#2 : for good HT performance, need a good hash function.

Ideal : user super-clever hash function guaranteed
to spread every data set out evenly.

Problem : DOES NOT EXIST! (for every hash function, there is a pathological data set)

Reason : fix a hash function h : U -> {0,1,2,…,n-1}

$\Rightarrow$a la Pigeonhole Principle, there exist bucket i such that at least |u|/n elements of U hash to I under h.

$\Rightarrow$ if data set drawn only from these, everything collides !

Pathological Data in the Real World

Preference : Crosby and Wallach, USENIX 2003.

Main Point : can paralyze several real-world systems (e.g., network intrusion detection) by exploiting badly designed hash functions.

-- open source

-- overly simplistic hash function

(easy to reverse engineer a pathological data set)

Solutions

1. Use a cryptographic hash function (e.g., SHA-2)

   -- infeasible to reverse engineer a pathological data set

2. Use randomization. ⟵In next 2 videos

   -- design a family H of hash functions such that for all data sets S, "almost all" functions $h \in H$ spread S out "pretty evenly".

   (compare to QuickSort guarantee)

Overview of Universal Hashing

Next : details on randomized solution (in 3 parts).

Part 1 : proposed definition of a "good random hash function".
("universal family of hash functions")

Part 3 : concrete example of simple + practical such functions

Part 4 : justifications of definition : "good functions" lead to "good performance"