Design and Analysis of Algorithms I

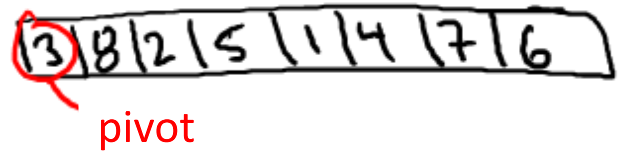
QuickSort

The Partition Subroutine

Partitioning Around a Pivot

Key Idea : partition array around a pivot element.

-Pick element of array



-Rearrange array so that

-Left of pivot => less than pivot

-Right of pivot => greater than pivot

Note : puts pivot in its "rightful position".

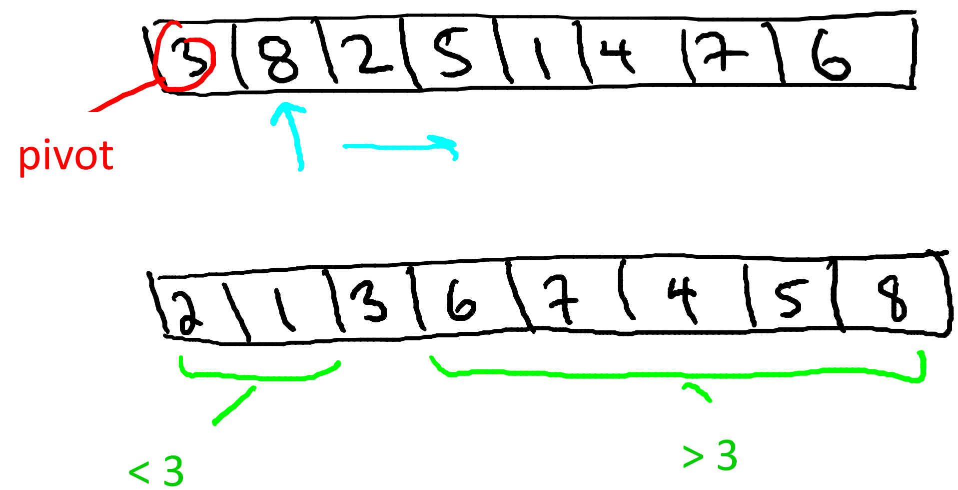
Two Cool Facts About Partition

1. Linear O(n) time, no extra memory
   [see next video]

2. Reduces problem size

The Easy Way Out

Note : Using O(n) extra memory, easy to partition around pivot in O(n) time.

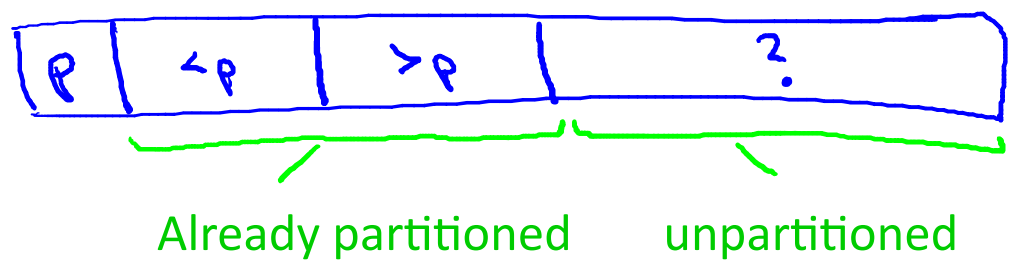


In-Place Implementation

<u>Assume</u> : pivot = 1^{st} element of array

[if not, swap pivot <--> 1^{st} element as preprocessing step]

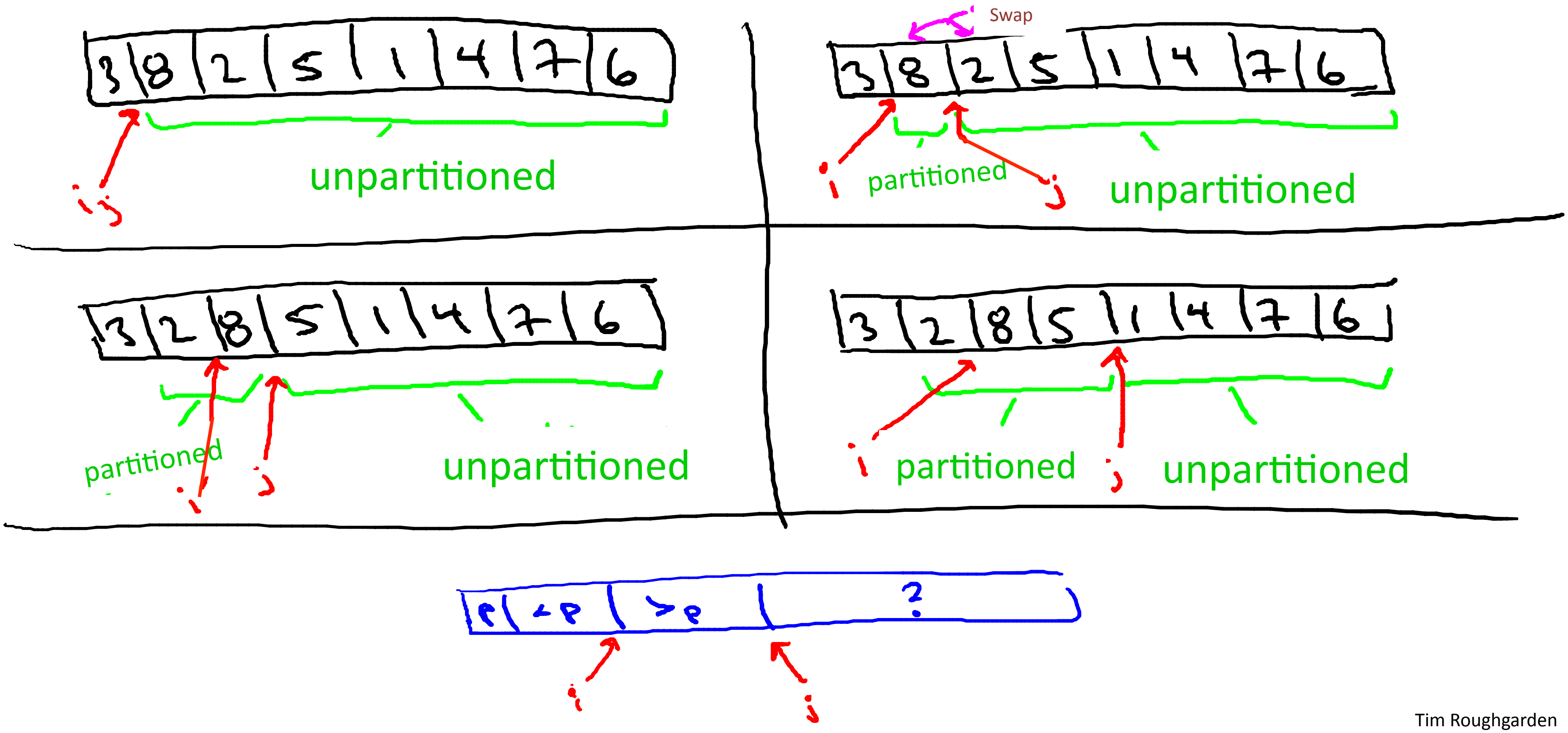
<u>High – Level Idea</u> :



-Single scan through array

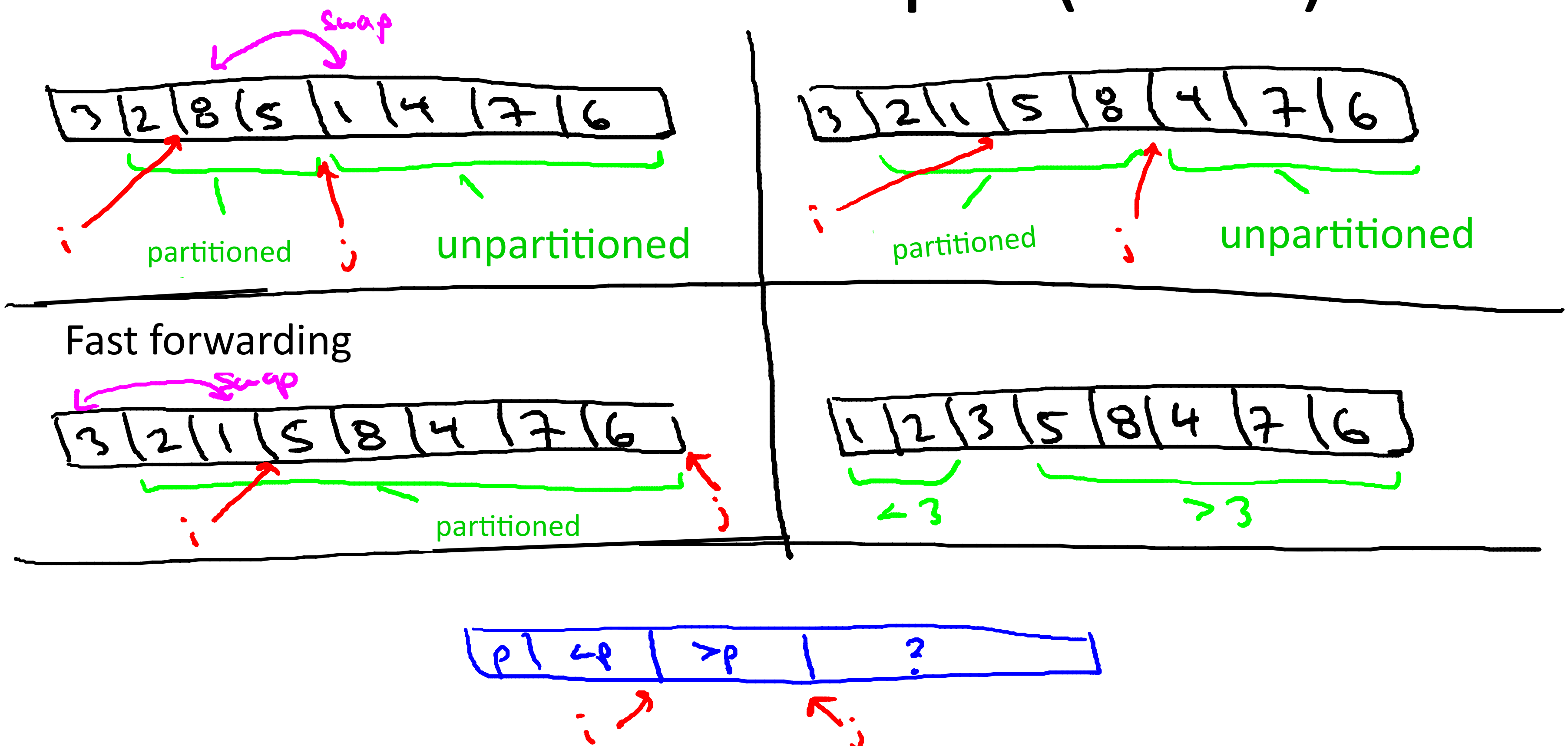
- invariant : everything looked at so far is partitioned

Partition Example



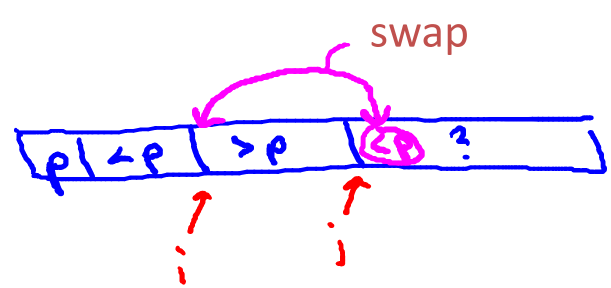
Tim Roughgarden

Partition Example (con'd)



Pseudocode for Partition

```
Partition (A,l,r)                    [ input corresponds to A[l…r]]
    - p:= A[l]
    - i:= l+1
    - for j=l+1 to r
        - if A[j] < p                [if A[j] > p, do nothing ]
          -swap A[j] and A[i]
          - i:= i+1
    - swap A[l] and A[i-1]
```



Running Time

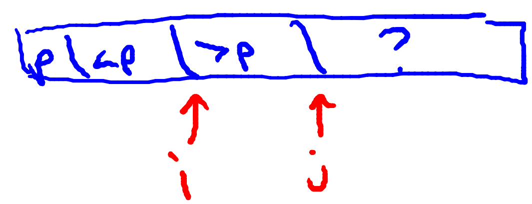
Running time = $O(n)$, where $n = r - l + 1$ is the length of the input (sub) array.

<u>Reason</u> : $O(1)$ work per array entry.

<u>Also</u> : clearly works in place (repeated swaps)

Correctness

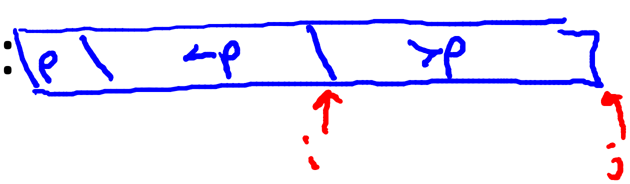
Claim : the for loop maintains the invariants :



1. A[l+1],..,A[i-1] are all less than the pivot
2. A[i],...,A[j-1] are all greater than pivot.

[Exercise : check this, by induction.]

Consequence : at end of for loop, have:



=> after final swap, array partitioned around pivot.

Q.E.D