Design and Analysis of Algorithms I

Divide and Conquer

Closest Pair II

Correctness Claim

$min\{d(p_1,q_1), d(p_2,q_2)\}$

Claim : Let $p \in Q, q \in R$ be a split pair with d(p,q) < δ

Then: (A) p and q are members of S_y

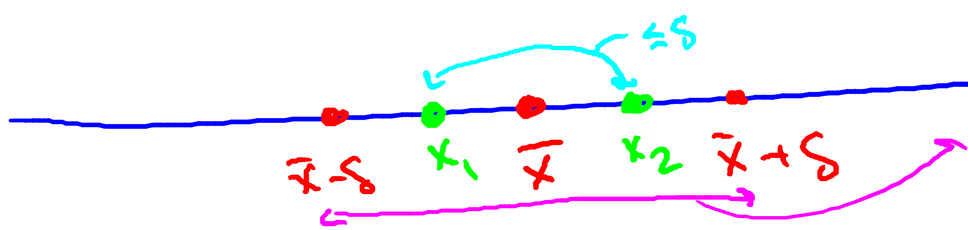
(B) p and q are at most 7 positions apart in S_y.

Proof of Correctness Claim (A)

Let $p = (x_1, y_1) \in Q,\ q = (x_2, y_2) \in R,\ d(p,q) \le \delta$

Note : Since $d(p,q) \le \delta,\ |x_1 - x_2| \le \delta\ \ and\ \ |y_1 - y_2| \le \delta$

Proof of (A) [p and q are members of S_y i.e. $x_1, x_2 \in [\bar{x} - \delta, \bar{x} + \delta]$]



Strip that defines S_y

Note : $p \in Q => x_1 \le \bar{x}\ \ and\ \ q \in R => x_2 \ge \bar{x}.$

=> $x_1, x_2 \in [\bar{x} - \delta, \bar{x} + \delta]$

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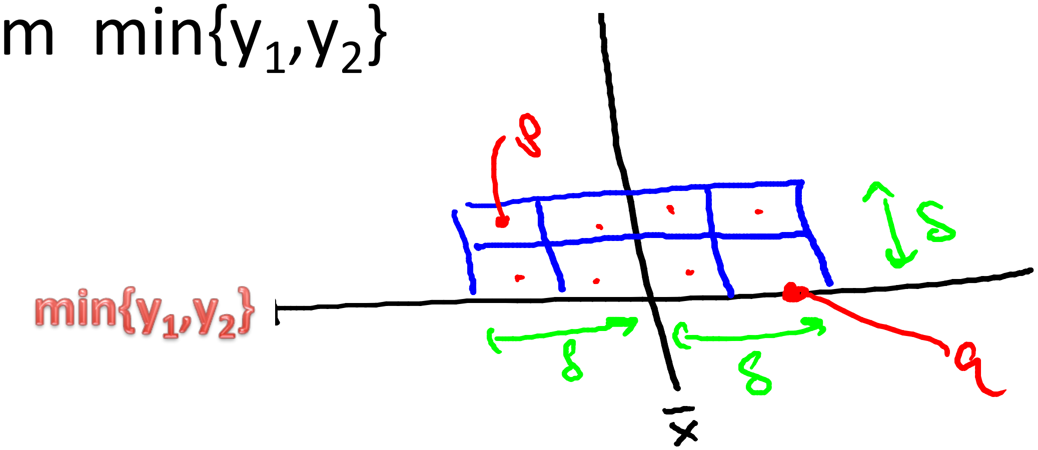
Proof of Correctness Claim (B)

(B) : p = (x_1,y_1) and q = (x_2,y_2) are at most 7 positions apart in S_y

<u>Key Picture</u> : draw $\delta/2 \times \delta/2$ boxes with center $\bar{x}$ and bottom $\min\{y_1,y_2\}$



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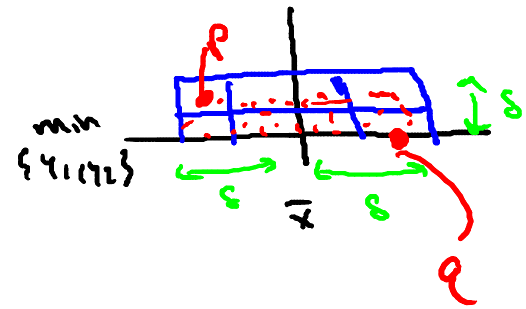
Proof of Correctness Claim (B)



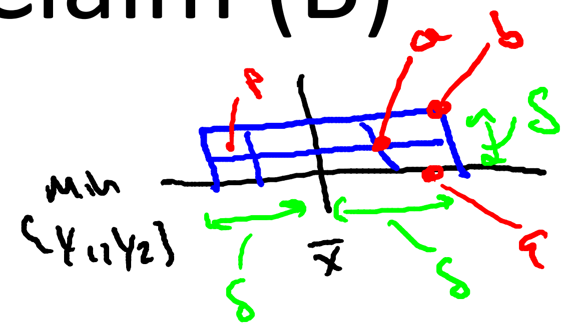
Lemma 1 : all points of S_y with y-coordinate between those of p and q, inclusive, lie in one of these 8 boxes.

Proof : First, recall y-coordinates of p,q differ by $< \delta$

Second, by definition of S_y, all have x-coordinates between $\bar{x} - \delta \ \ and \ \ \bar{x} + \delta$

Q.E.D

Proof of Correctness Claim (B)



Lemma 2 : At most one point of P in each box.

Proof : by contradiction

Suppose a,b lie in the same box. Then :

I. a,b are either both in Q or both in R

II. $d(a,b) \leq \frac{\delta}{2} \cdot \sqrt{2} \leq \delta$

But (i) and (ii) contradict the definition of δ (as smallest distance between pairs of points in Q or in R)

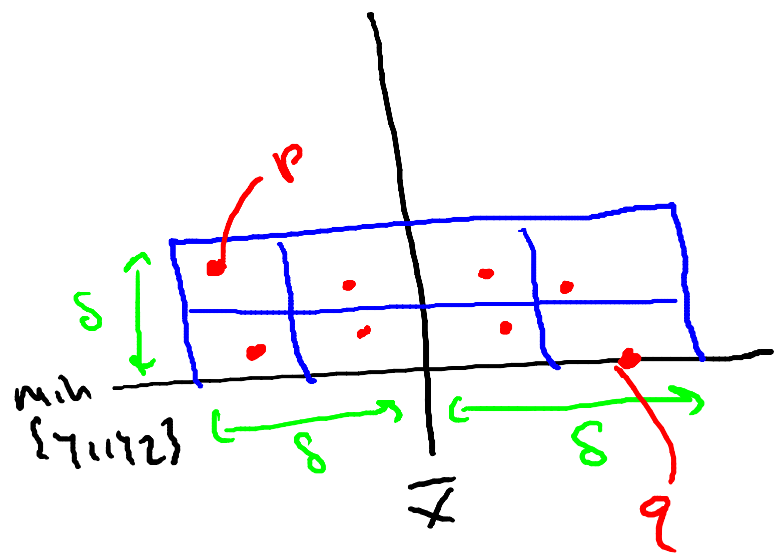
Q.E.D

Tim Roughgarden

Final Wrap-Up

Lemmas 1 and 2 => at most 8 points in this picture (including p and q)

=> Positions of p,q in S_y differ by at most 7



Q.E.D