Contraction Algorithm

Overview

Design and Analysis of Algorithms I

Goals for These Lectures

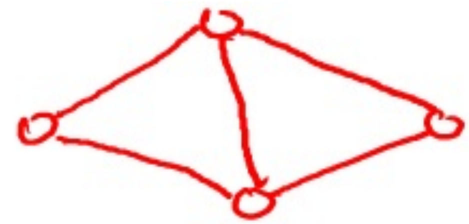
- Further practice with randomized algorithms
  - In a new application domain (graphs)
- Introduction to graphs and graph algorithms

Also: “only” 20 years ago!

Graphs

Two ingredients

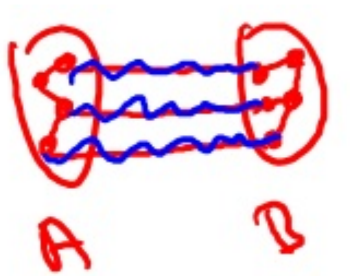
- Vertices aka nodes (V)
- Edges (E) = pairs of vertices
  - can be undirected [unordered pair] or directed [ordered pair] (aka arcs)



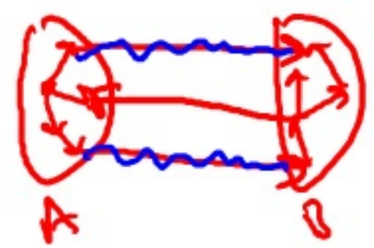
Examples: road networks, the Web, social networks, precedence constraints, etc.

Cuts of Graphs

Definition: a cut of a graph (V, E) is a partition of V into two non-empty sets A and B.



[undirected]



[directed]

Definition: the crossing edges of a cut (A, B) are those with:

- the one endpoint in each of (A, B) [undirected]
- tail in A, head in B [directed]

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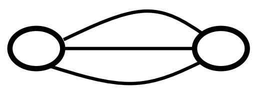
Roughly how many cuts does a graph with n vertices have?

- ○ n
- ○ n^2
- ○ 2^n
- ○ n^n

The Minimum Cut Problem

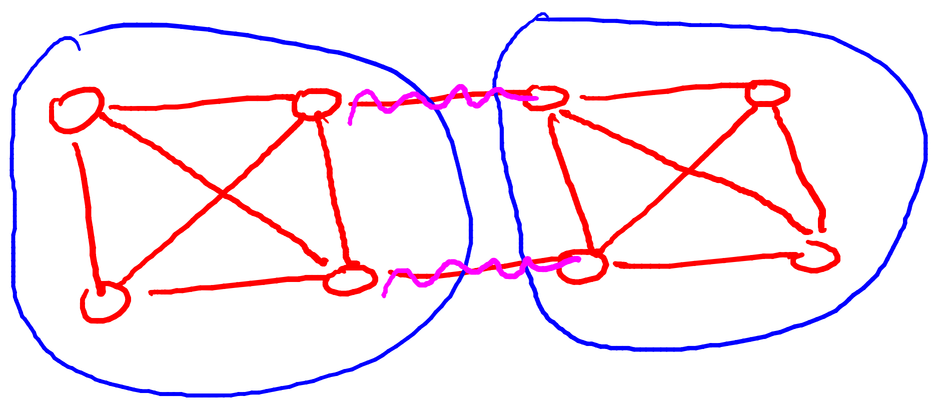
- INPUT: An undirected graph G = (V, E).

[Parallel  edges allowed]

[See other video for representation of the input]

- GOAL: Compute a cut with fewest number of crossing edges. (a min cut)

What is the number of edges crossing a minimum cut in the graph shown below?



- 1
- 2
- 3
- 4

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A Few Applications

- indentify network bottlenecks / weaknesses
- community detection in social networks
- image segmentation
    - input = graph of pixels
    - use edge weights

      [(u,v) has large weight ⇔ "expect" u,v to come from some object]

  hope: repeated min cuts identifies the primary objects in picture.