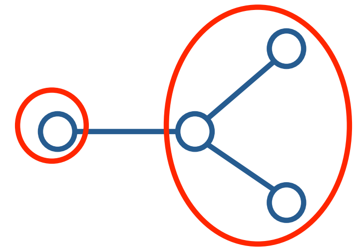
Design and Analysis
of Algorithms I

Contraction Algorithm

Counting Mininum Cuts

The Number of Minimum Cuts

NOTE: A graph can have multiple min cuts.
[e.g., a tree with n vertices has (n-1) minimum cuts]

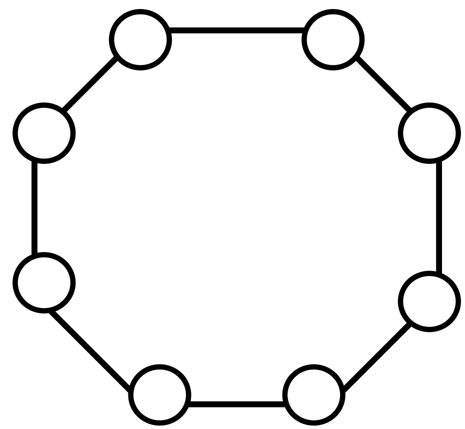


QUESTION: What's the largest number of min cuts that a graph with n vertices can have?

ANSWER: $\binom{n}{2} = \frac{n(n-1)}{2}$

The Lower Bound

Consider the n-cycle.



(n = 8)

NOTE: Each pair of the n edges defines a distinct minimum cut (with two crossing edges).

- ➢ has $\geq \binom{n}{2}$ min cuts

The Upper Bound

Let (A_1, B_1) , (A_2, B_2) , …, (A_t, B_t) be the min cuts of a graph with n vertices.

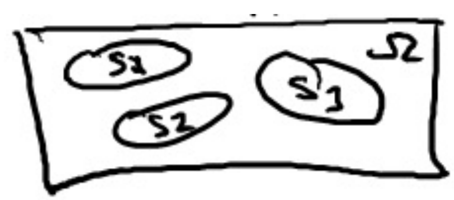
By the Contraction Algorithm analysis (without repeated trials):

$$\Pr[\underbrace{output = (A_i, B_i)}_{S_i}] \geq \frac{2}{n(n-1)} = \frac{1}{\binom{n}{2}} \quad \forall i = 1, 2, .., t$$

Note: S_i's are disjoint events (i.e., only one can happen)

➢ their probabilities sum to at most 1

Thus: $$\frac{t}{\binom{n}{2}} \leq 1 \Rightarrow t \leq \binom{n}{2}$$



QED !