Dirichlet distribution

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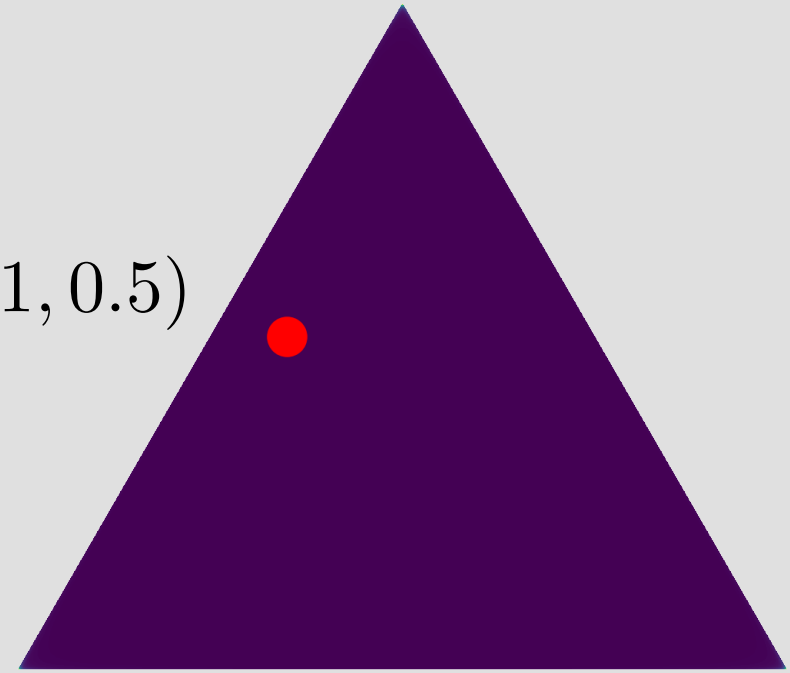
θ: $\sum_k \theta_k = 1$

$\theta_k \geq 0$



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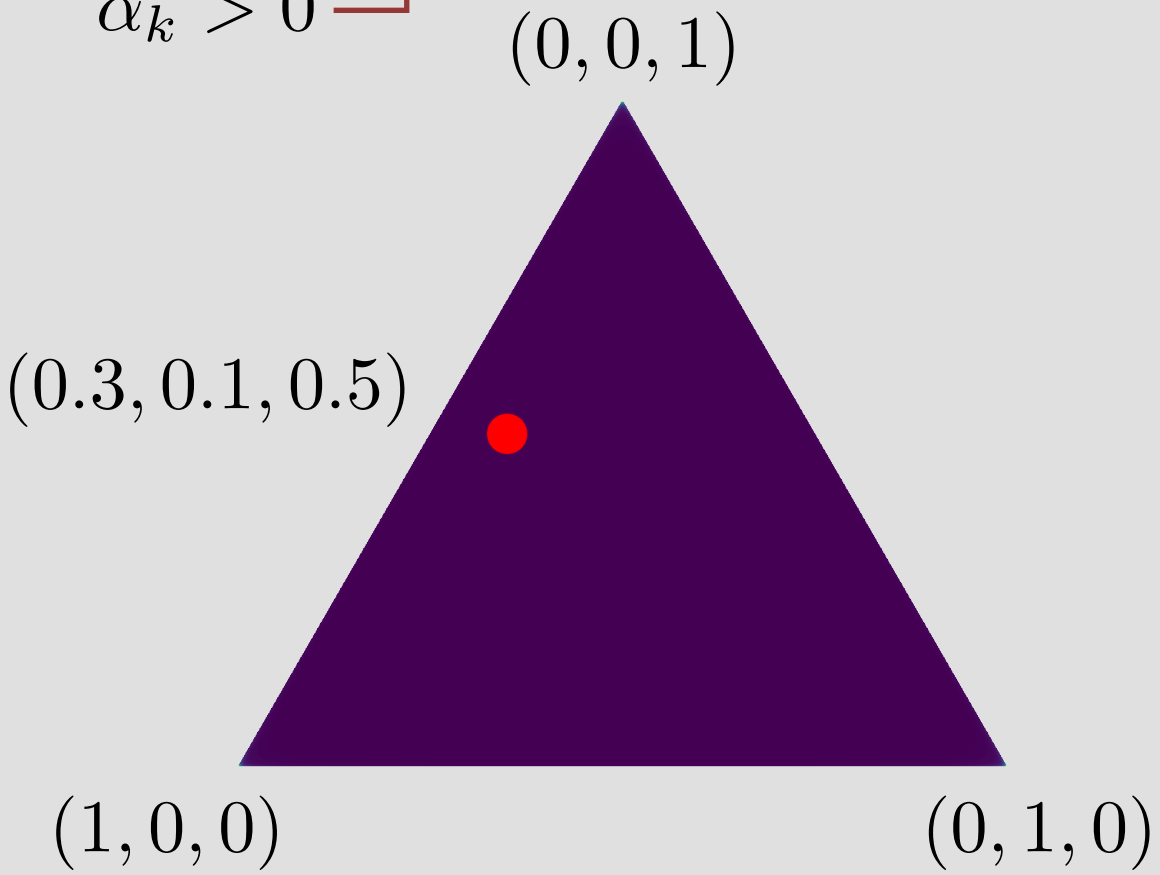
$\alpha_k > 0$



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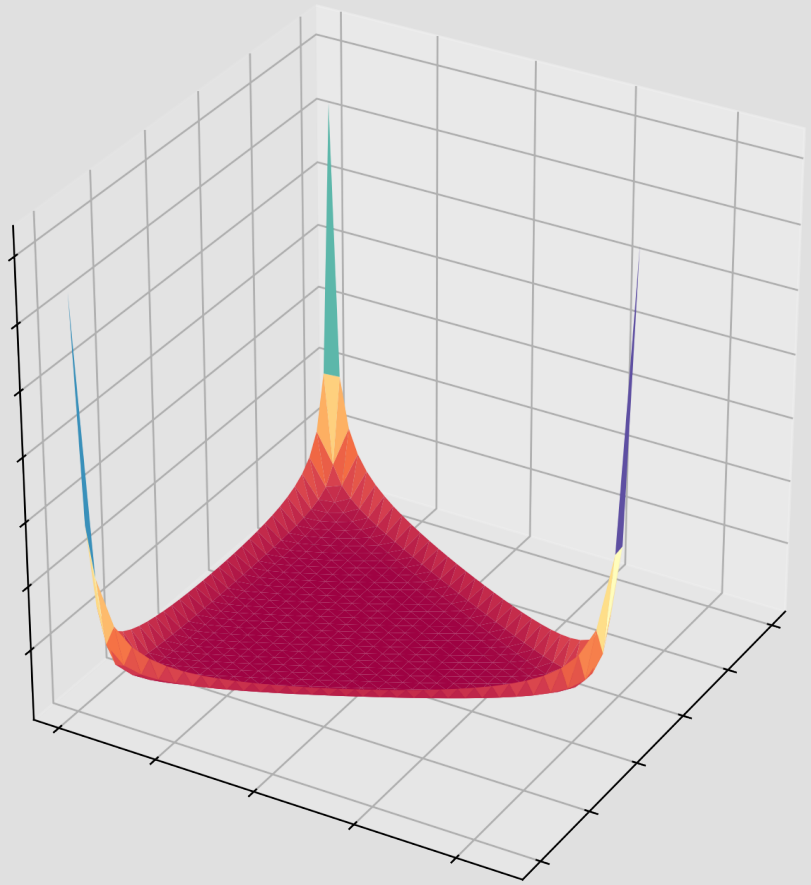
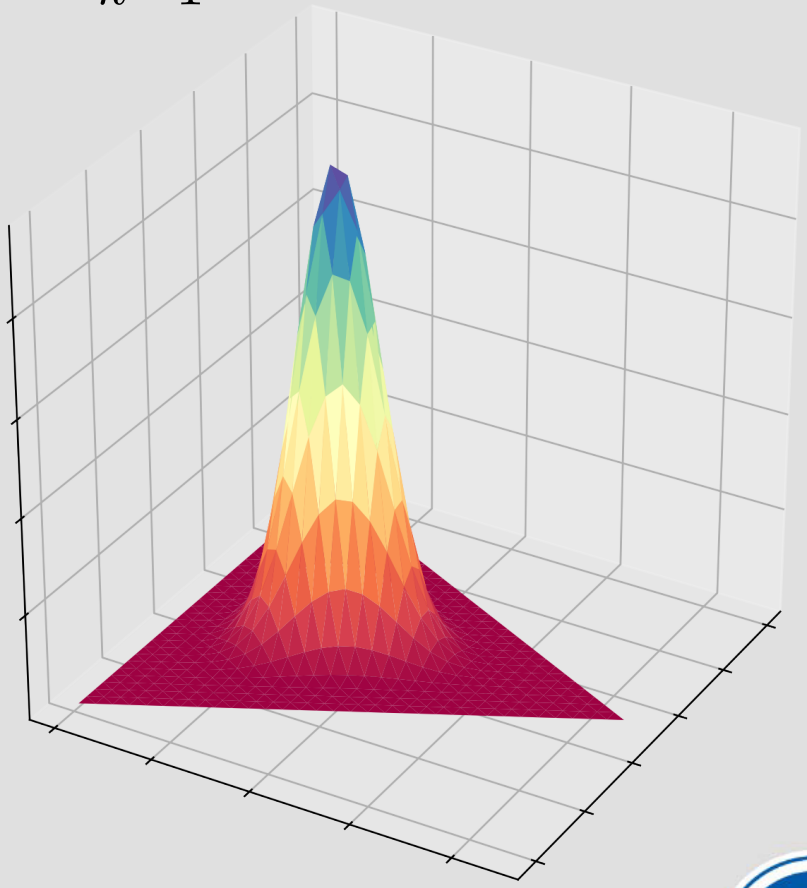


$\alpha = (0.1, 0.1, 0.1)$



$\alpha = (10, 10, 10)$

Dirichlet distribution

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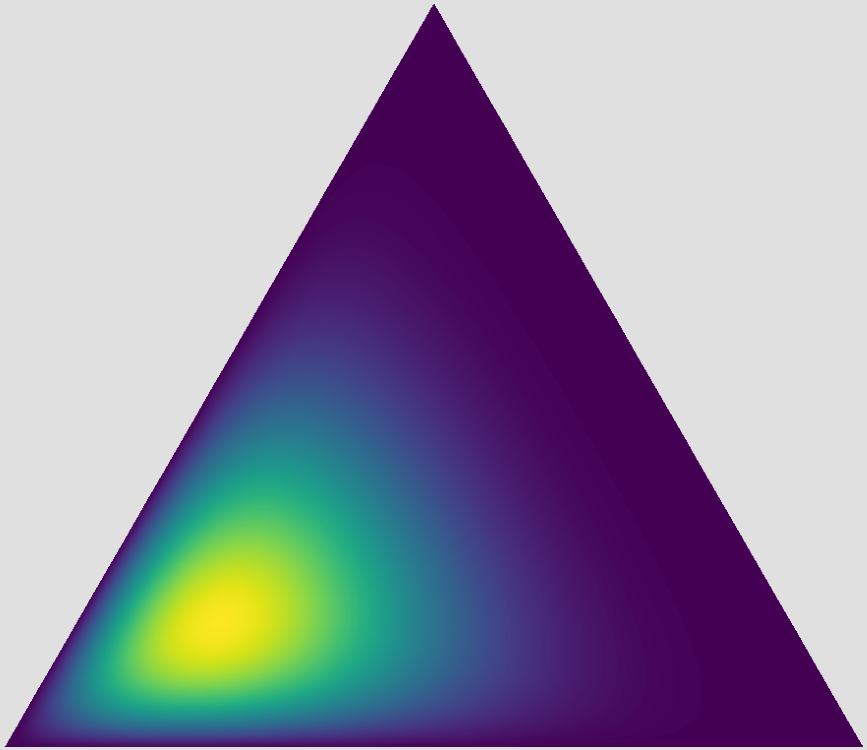
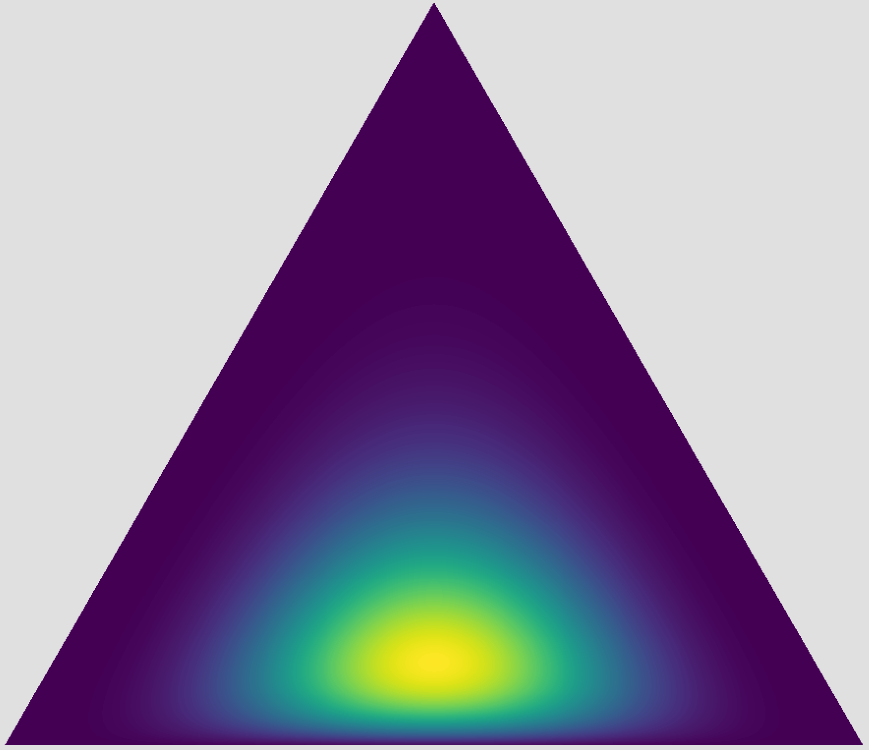


$\alpha = (5, 2, 2)$



$\alpha = (5, 5, 2)$

Statistics

$$\mathrm{Dir}(\theta|\alpha) = \frac{1}{B(\alpha)} \prod_{k=1}^{K} \theta_k^{\alpha_k - 1}$$

$$\mathbb{E}\theta_i = \frac{\alpha_i}{\alpha_0}$$

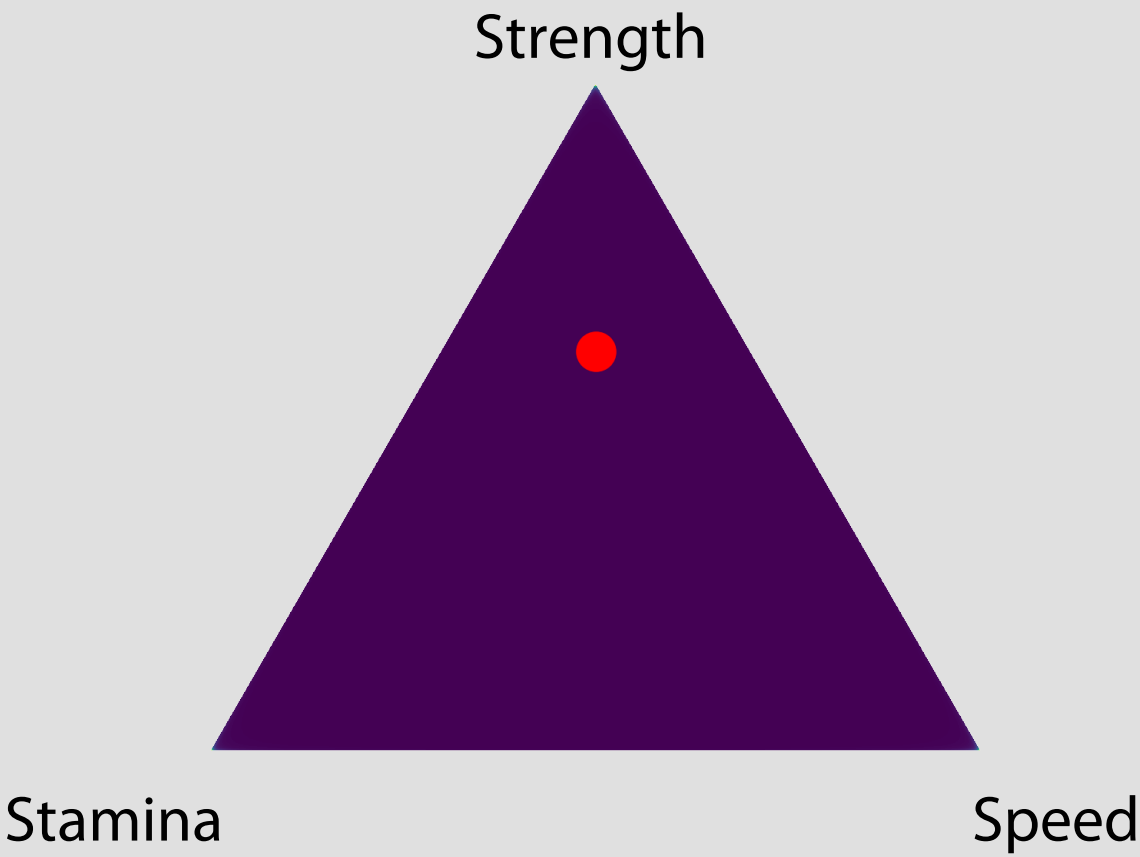
$$\mathrm{Cov}(\theta_i, \theta_j) = \frac{\alpha_i \alpha_0 [i=j] - \alpha_i \alpha_j}{\alpha_0^2(\alpha_0+1)}$$

$$\alpha_0 = \sum_{k=1}^{K} \alpha_k$$

Example

Massively multiplayer online role-playing game (**MMORPG**)

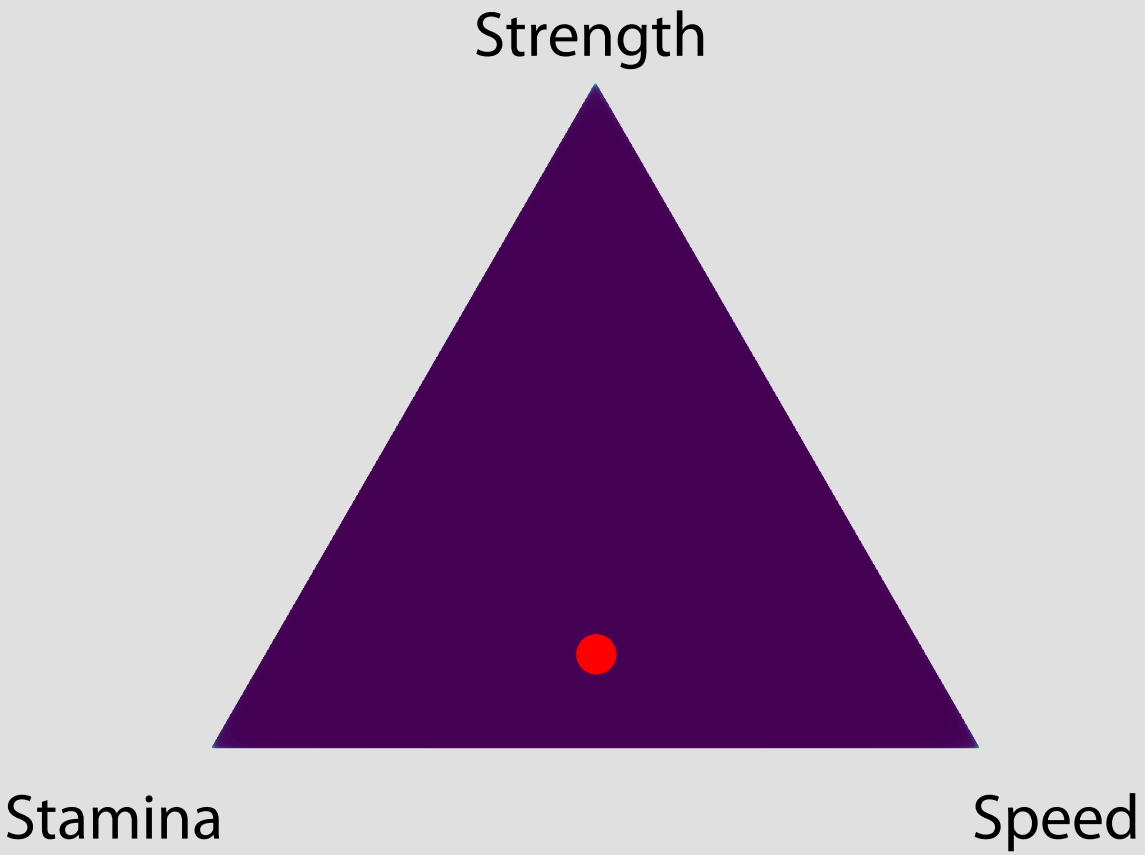
Player 1:



Example

Massively multiplayer online role-playing game (**MMORPG**)

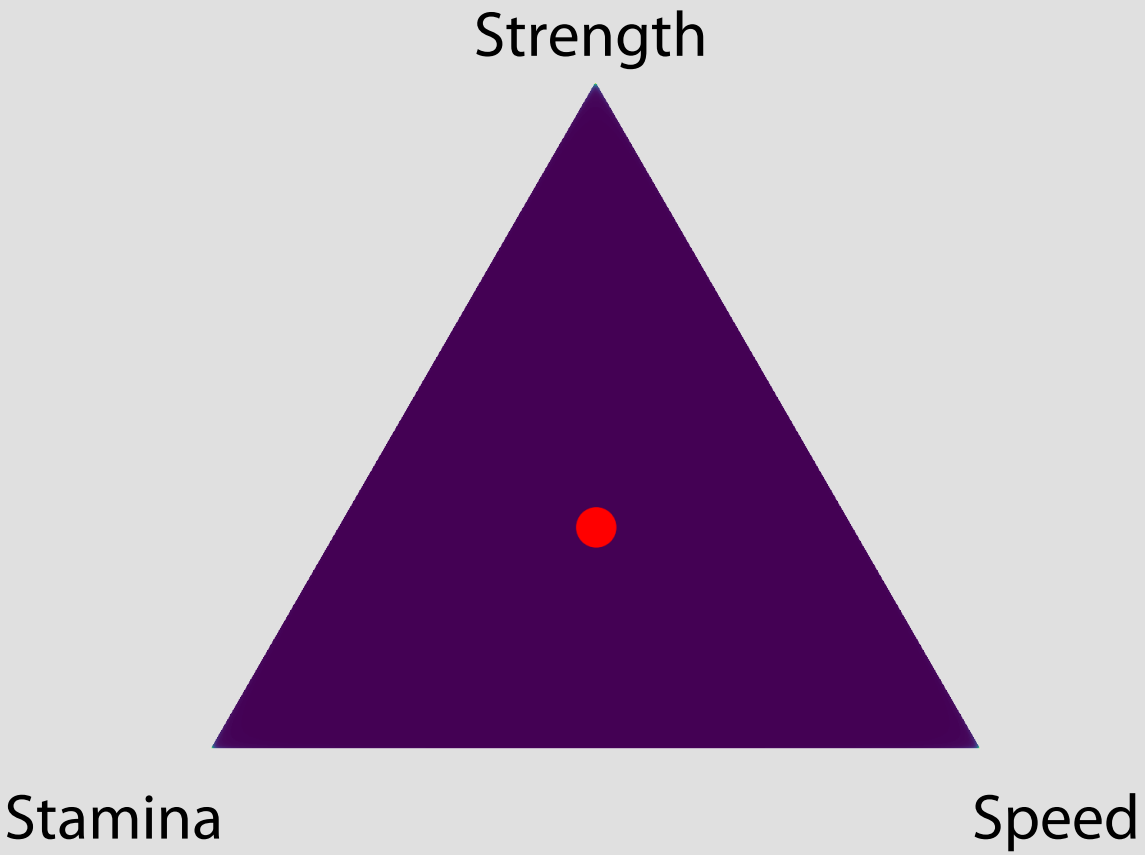
Player 2:



Example

Massively multiplayer online role-playing game (**MMORPG**)

Player 3:

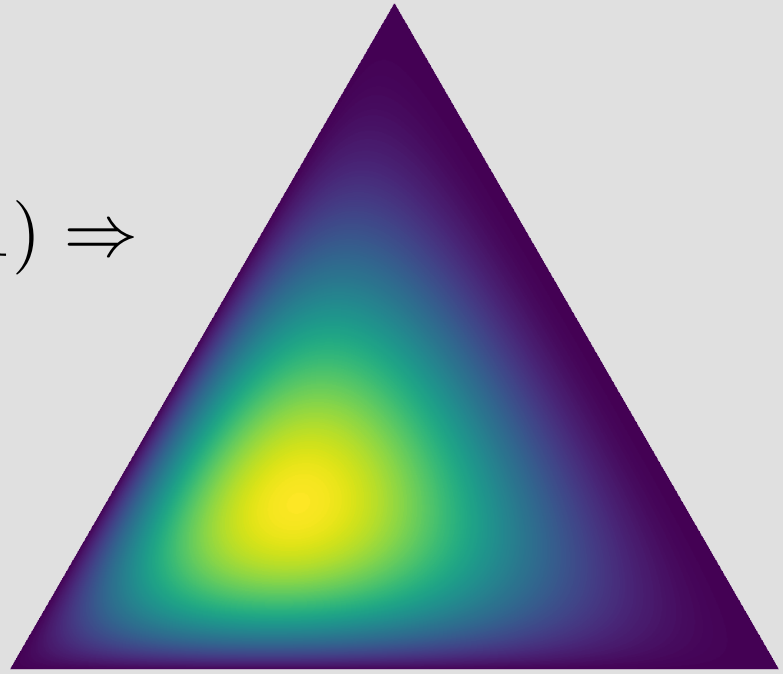


Example

Massively multiplayer online role-playing game (**MMORPG**)

Average over all players:

$\alpha = (3, 1, 1) \Rightarrow$



Conjugate prior

$P(\theta)$ is **conjugate** to $P(X|\theta)$:

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)}$$

$\mathcal{A}(v)$ — $\mathcal{A}(v)$

Multinomial likelihood

$$P(X|\theta) = \frac{n!}{x_1!\ldots x_K!}\theta_1^{x_1}\ldots\theta_K^{x_K}$$

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$$p(\theta|X) \propto \prod_{k=1}^{K}\theta_k^{\alpha_k+x_k-1}$$

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$$p(\theta|X) = \mathrm{Dir}(\theta|\left(\begin{array}{c}\cdots\\ \alpha_k+x_k\\ \cdots\end{array}\right))$$