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2. Find best approximation $q(z)$ of $p^*(z)$:

$$\mathcal{KL}\left[q(z) \parallel p^*(z)\right] \to \min_{q \in Q}$$

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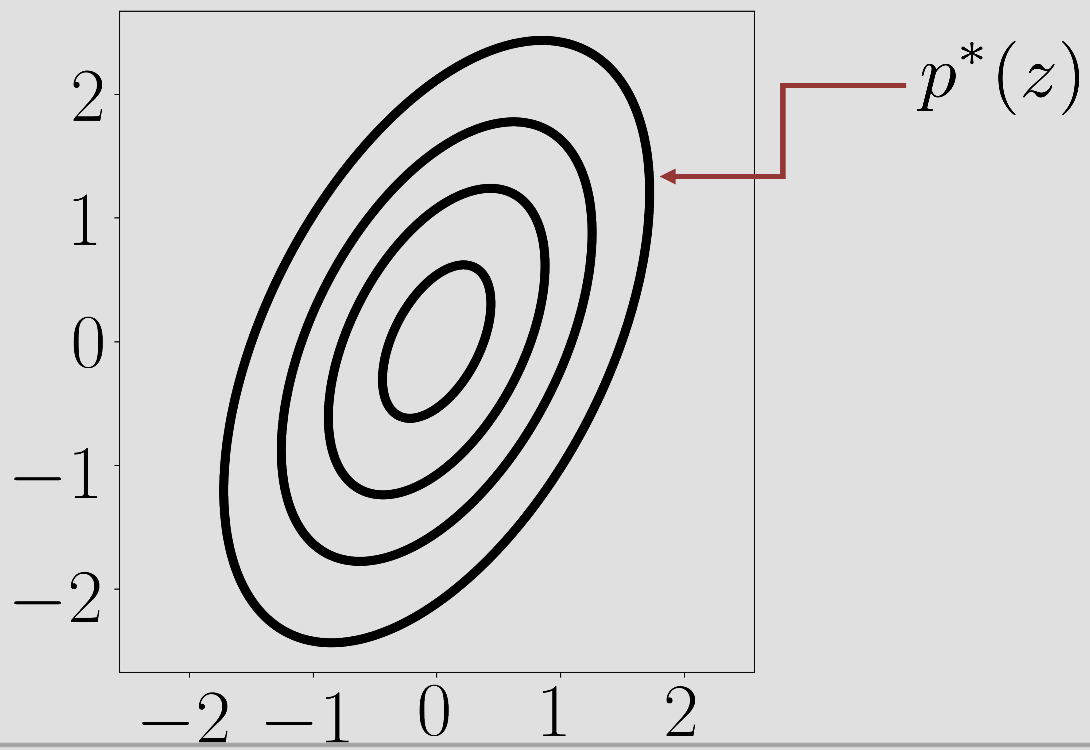

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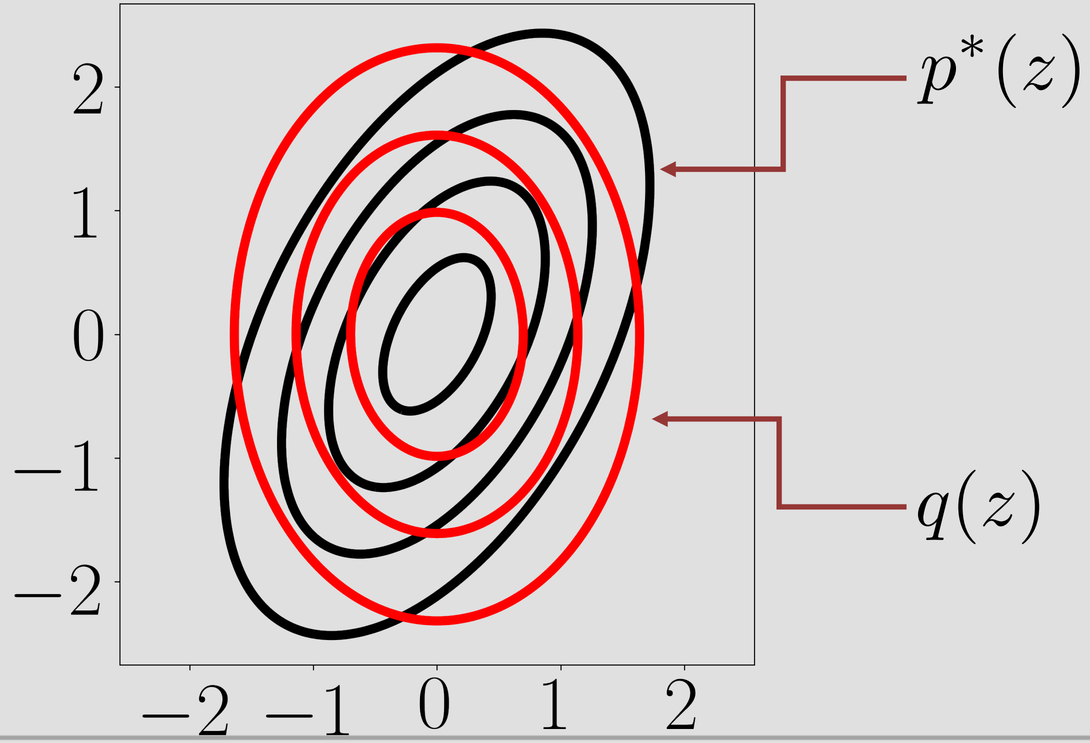

Optimization

$$\mathcal{KL}(q \,\|\, p^*) = \mathcal{KL}(\prod_{i=1}^{d} q_i \,\|\, p^*) \to \min_{q_1, q_2, \ldots, q_d}$$

Coordinate descend:

1. $\mathcal{KL}(q \,\|\, p^*) \to \min\limits_{q_1}$
2. $\mathcal{KL}(q \,\|\, p^*) \to \min\limits_{q_2}$
3. ...

Технический слайд (<= 12.5 min)

На доске вывод основной формулы + conditional conj.

$$\sum_X \prod_i q_i(x_i) \log \frac{\prod_i q_i(x_i)}{p(x)} = \sum_{x_j}\sum_{x_{-j}} q_j(x_j)\prod_{i\neq j} q_i(x_i)\left[\sum_k \log q_k(x_k) - \log p(x)\right] =$$

$$= \sum_{x_j} q_j(x_j)\sum_{x_{-j}}\prod_{i\neq j} q_i(x_i)\left[\sum_{k\neq j}\log q_k(x_k) + \log q_j(x_j)\right] - \sum_{x_j} q_j(x_j)\sum_{x_{-j}}\prod_{i\neq j} q_i(x_i)\log p(x) =$$

$$= \sum_{x_j} q_j(x_j)\sum_{x_{-j}}\prod_{i\neq j} q_i(x_i)\log q_j(x_j) - \sum_{x_j} q_j(x_j)\,\mathbb{E}_{x_{-j}}\log p(x) + const =$$

$$= \sum_{x_j} q_j(x_j)\left[\log q_j(x_j) - \mathbb{E}_{x_{-j}}\log p(x)\right] + const \longrightarrow \min_{q_j(x_j)}$$

$$\log q_j(x_j) = \mathbb{E}_{x_{-j}}\log p(x) + const.$$

$$q_j(x_j) = \frac{1}{Z}\cdot\exp\left(\mathbb{E}_{x_{-j}}\log p(x)\right)$$