Variational Inference

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Variational inference (ТЕХНИЧЕСКИЙ СЛАЙД)

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VARIATIONAL FAMILY

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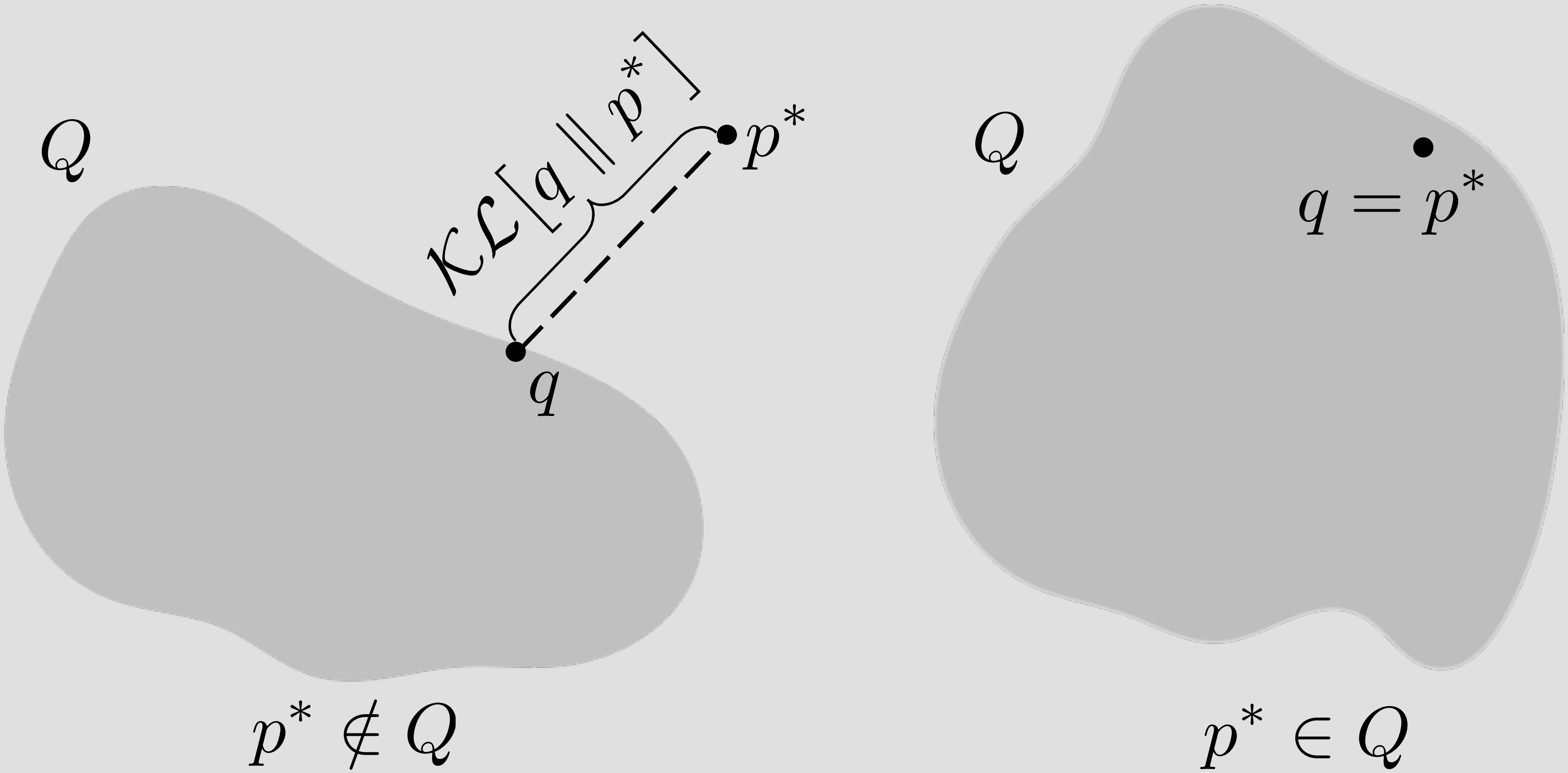
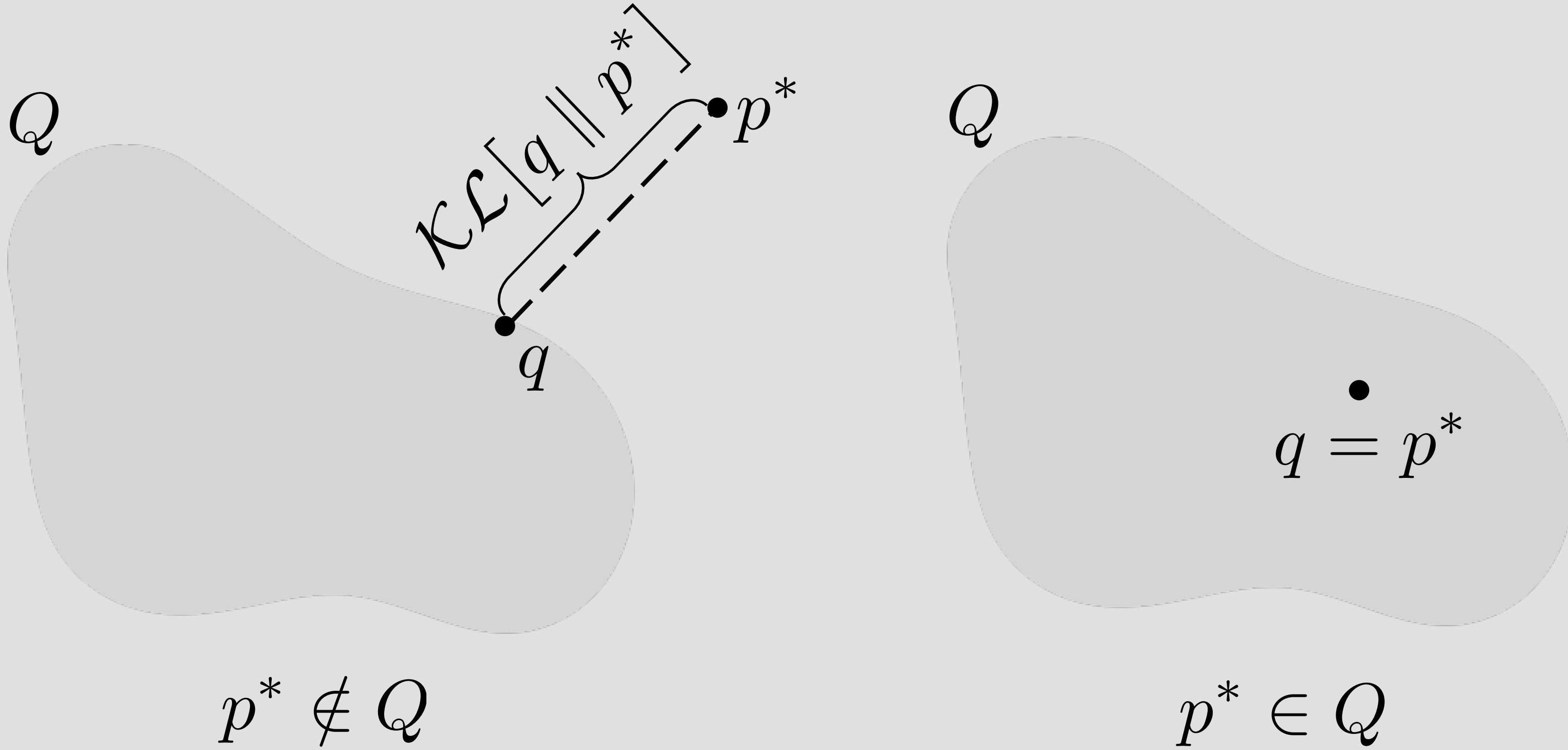
$$\mathcal{KL}\left[q(z) \parallel p^*(z)\right] \to \min_{q \in Q}$$

Choice of variational family



Choice of variational family (ТЕХНИЧЕСКИЙ СЛАЙД)

В стиле если Q не накроет p*, то будет расстояние какое-то.
А если накроет, то q и p* совпадут + немного темнее фигуру



LARGER Q => HARDER

Unnormalized distribution

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