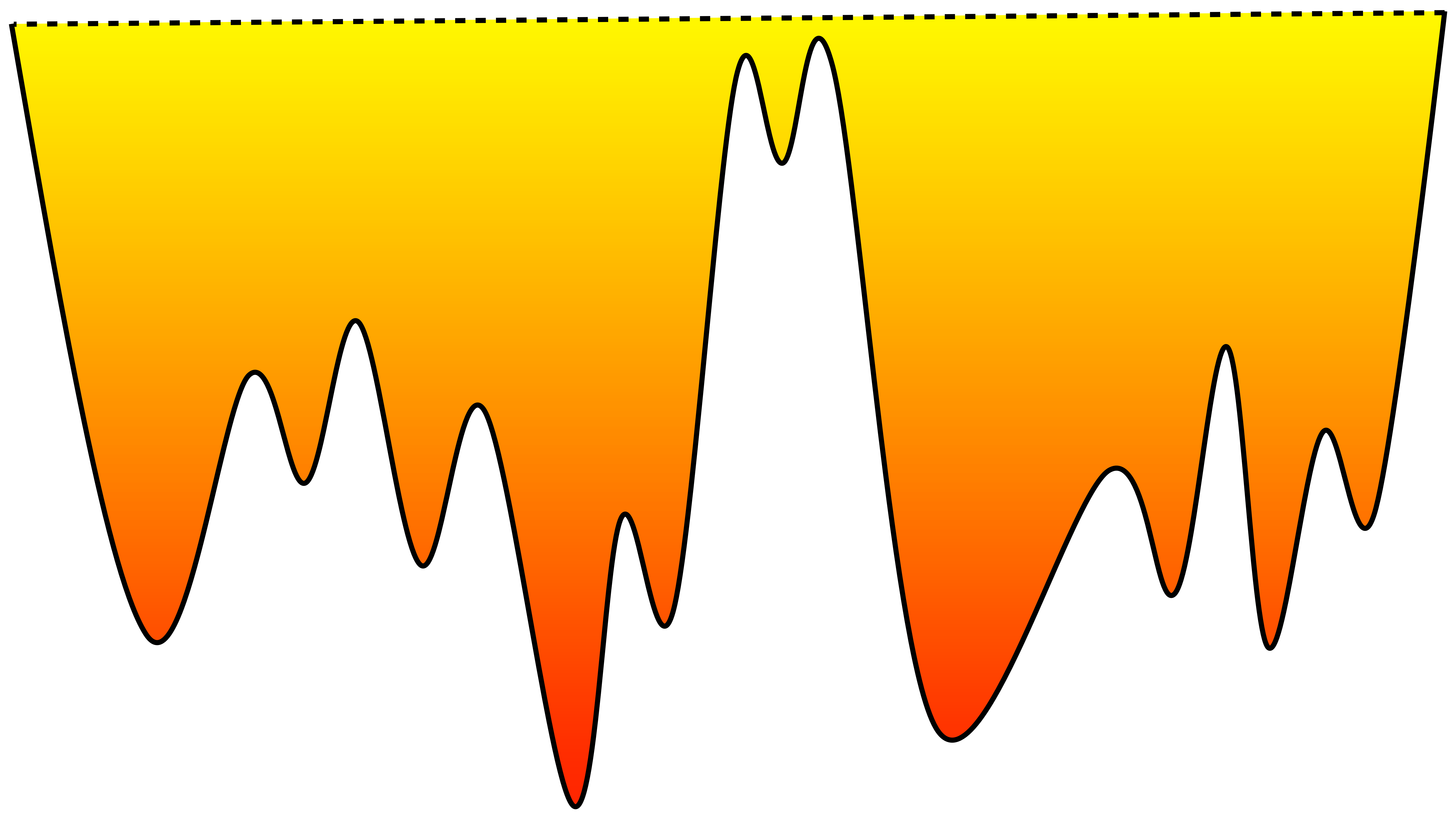
Discrete Optimization

Local Search: Part VIII

Goal of the Lecture

- Local search
  - meta-heuristics
  - multi-start search
  - simulated annealing
  - tabu search

Escaping Local Minima



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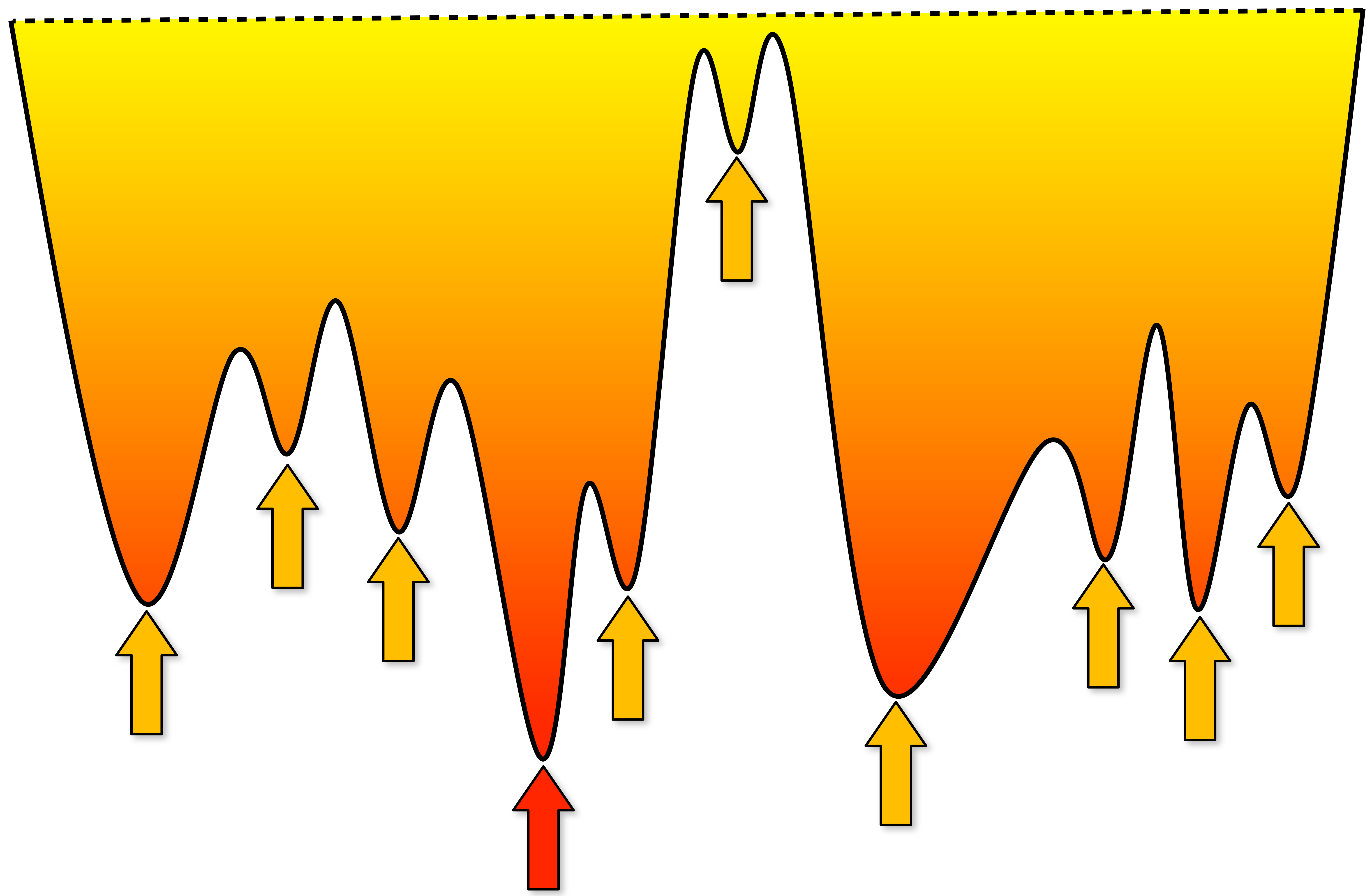
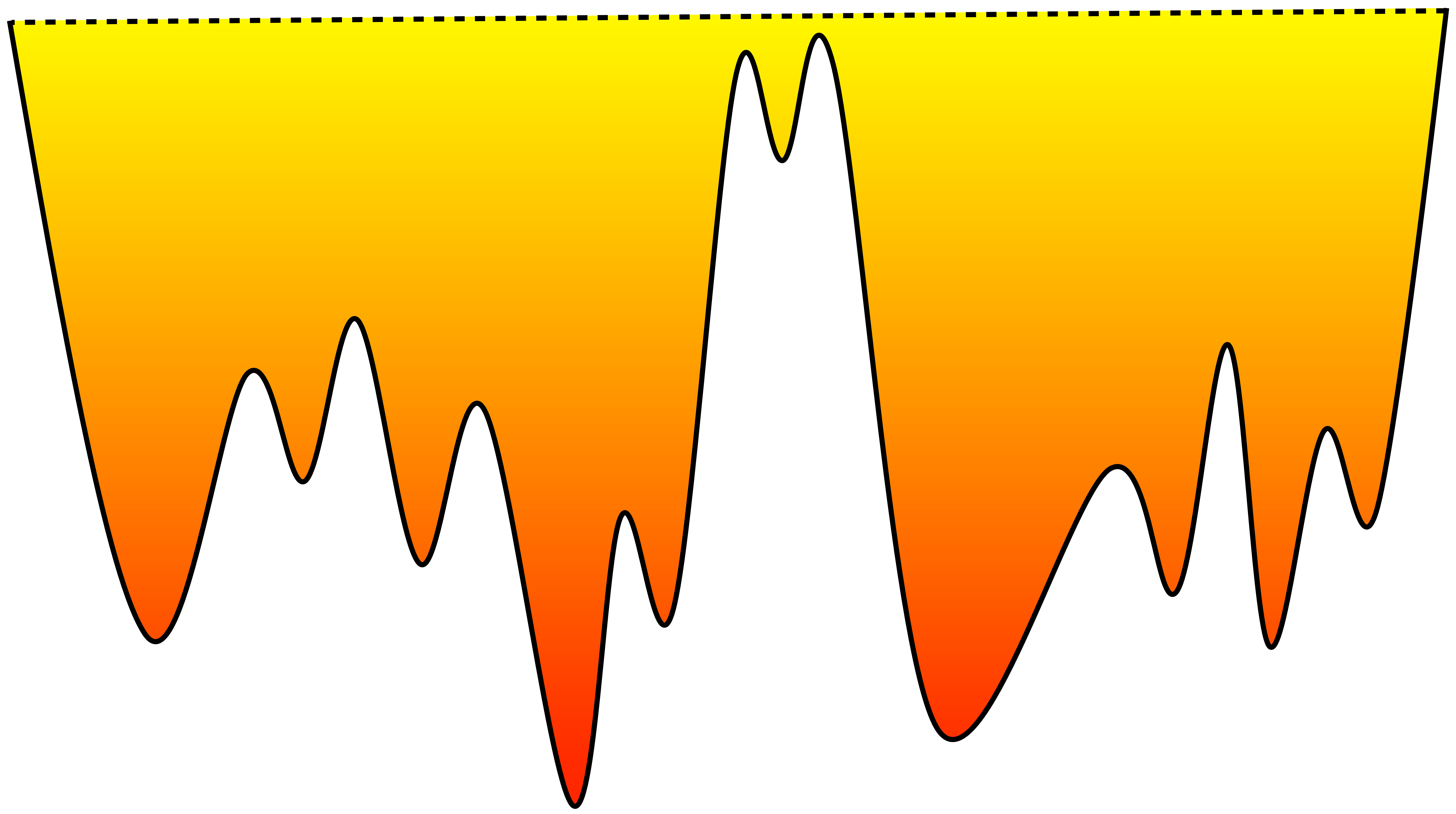


Iterated Local Search



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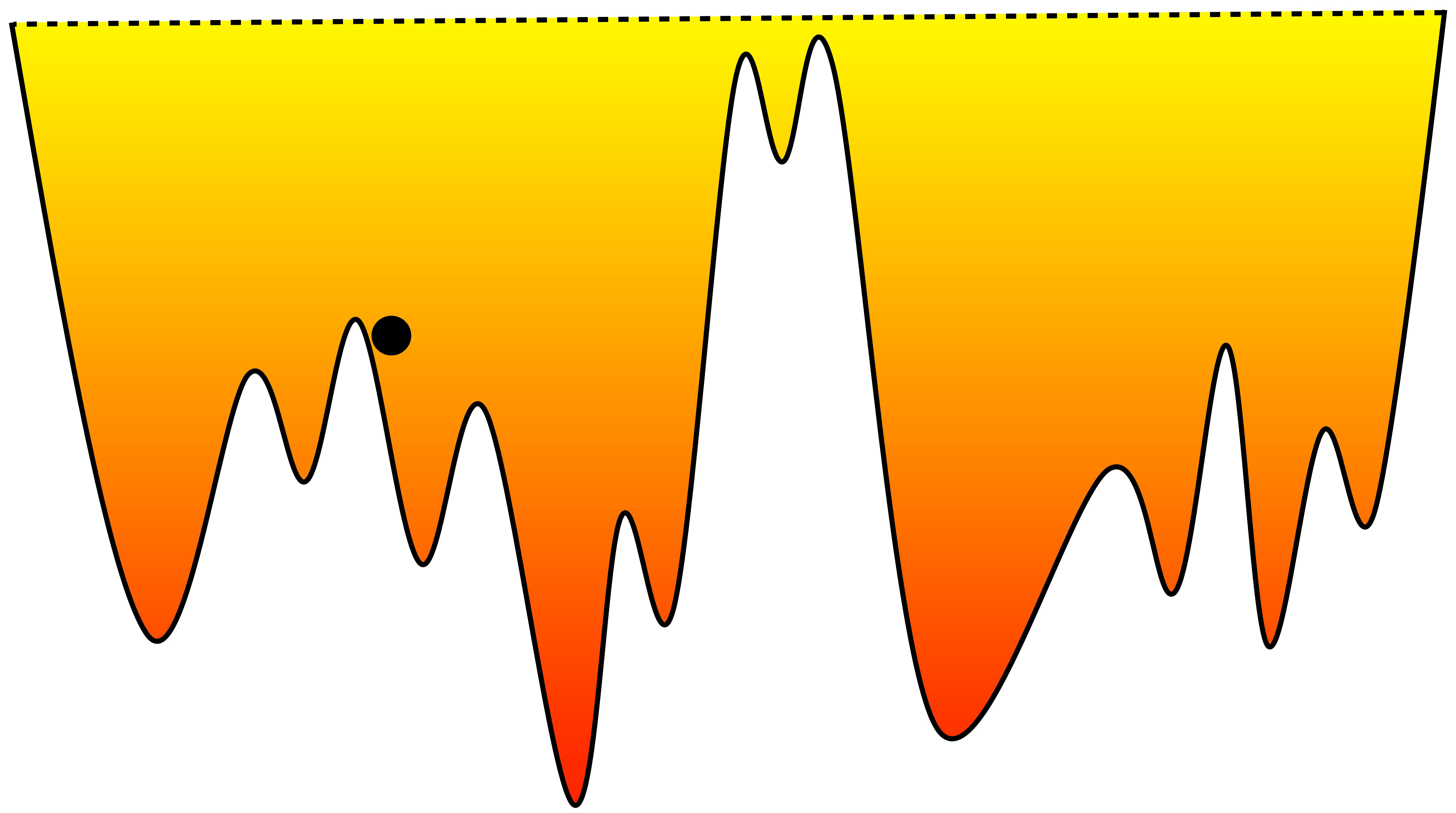

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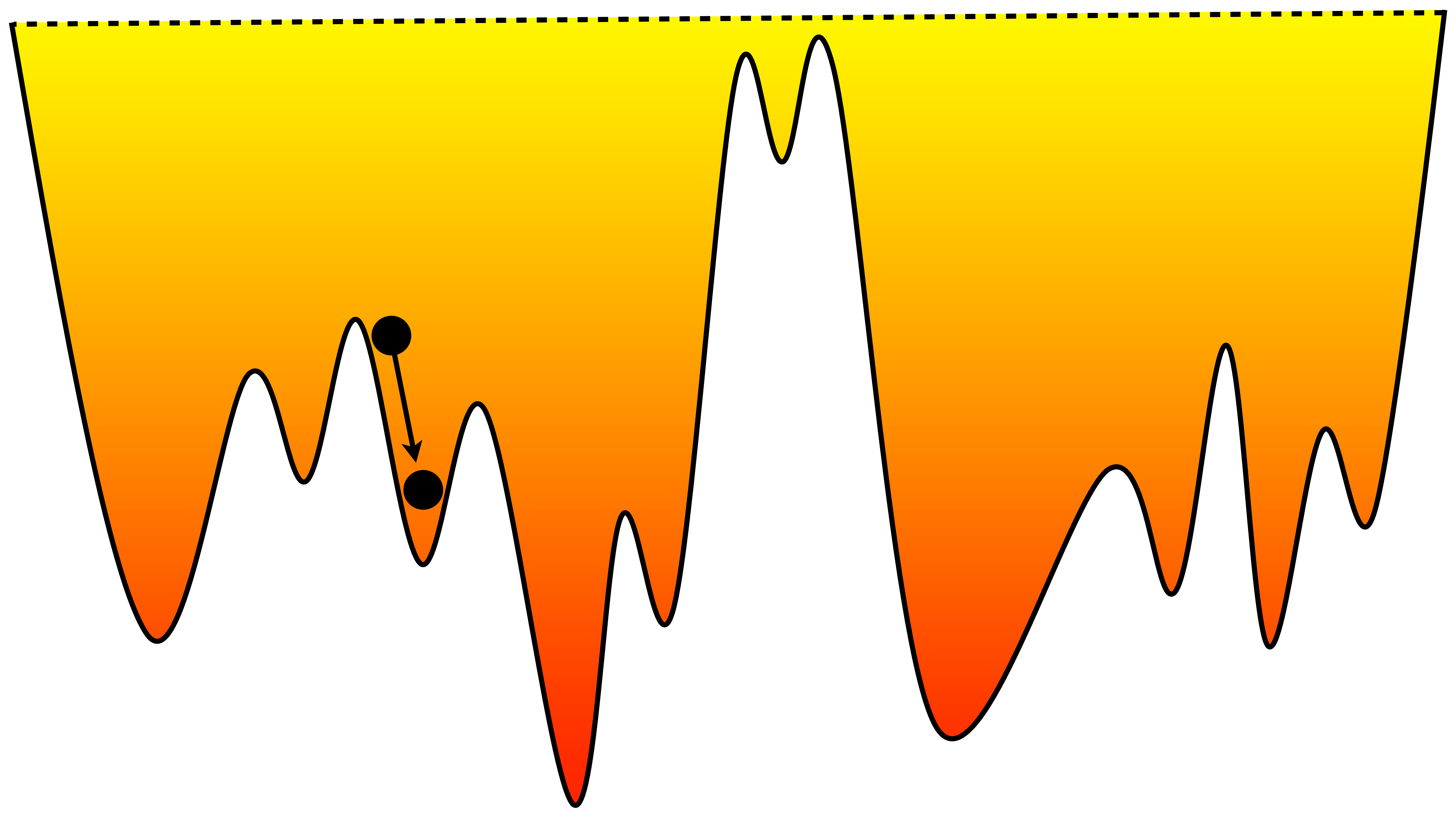

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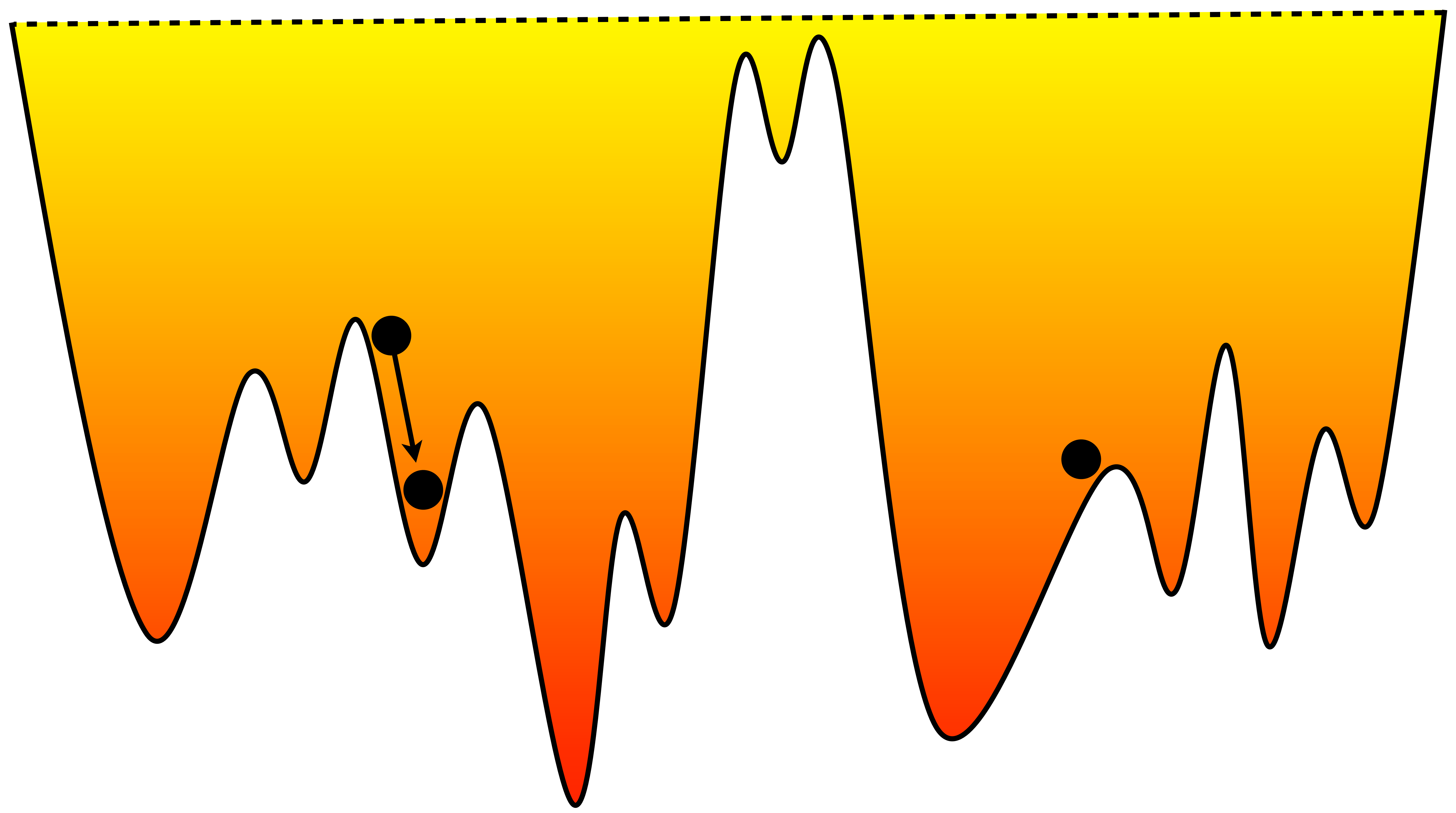

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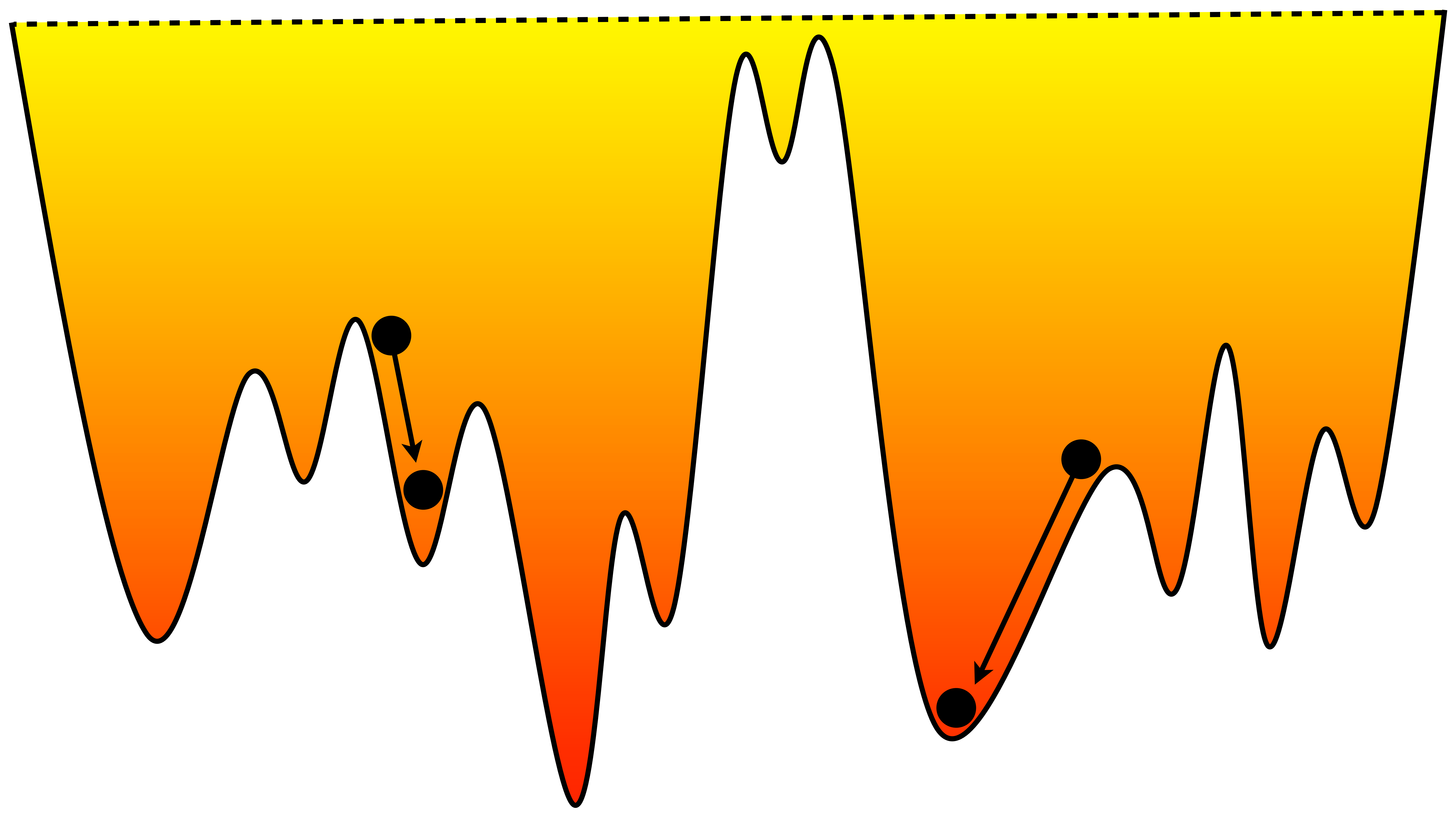

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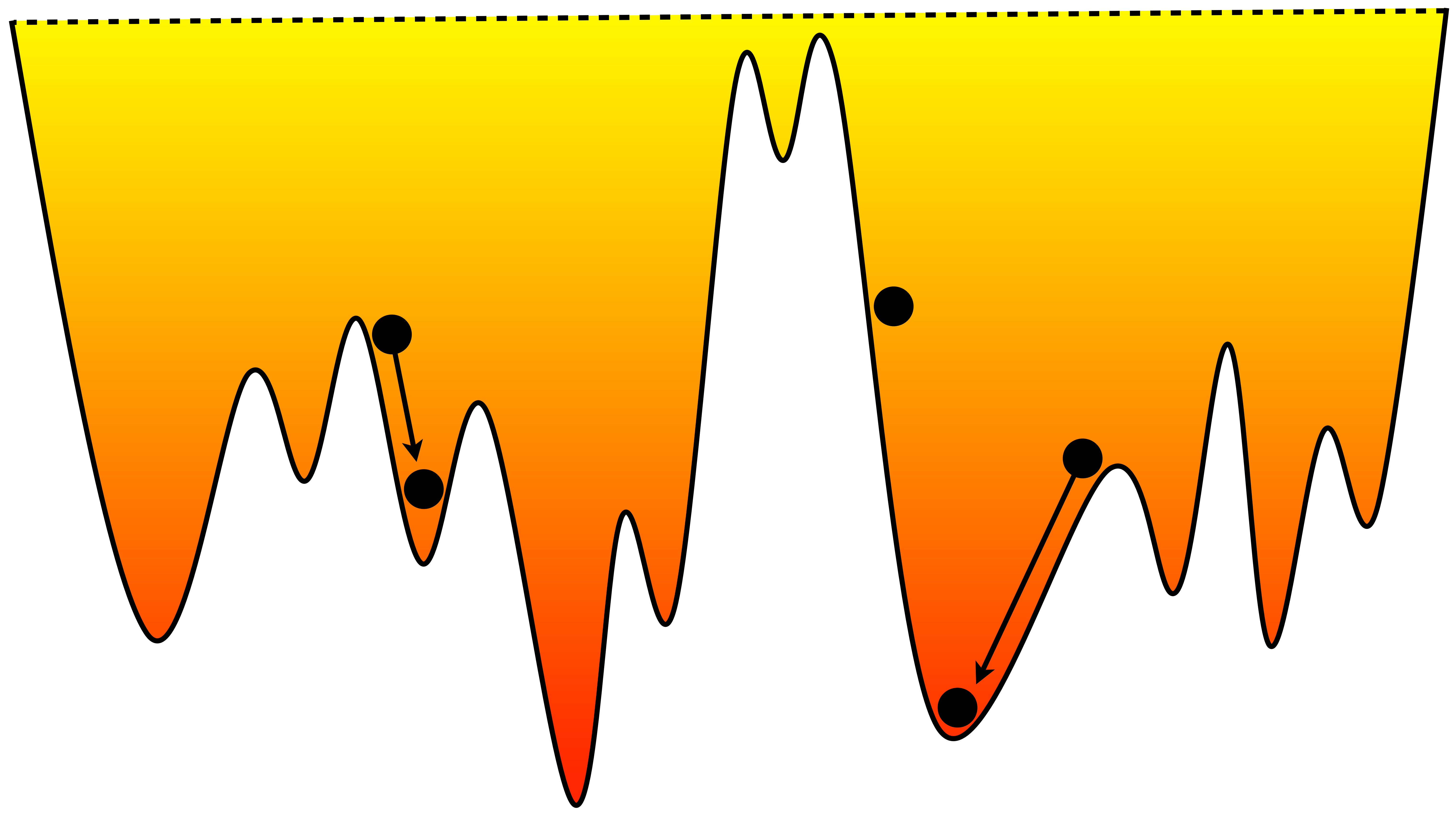

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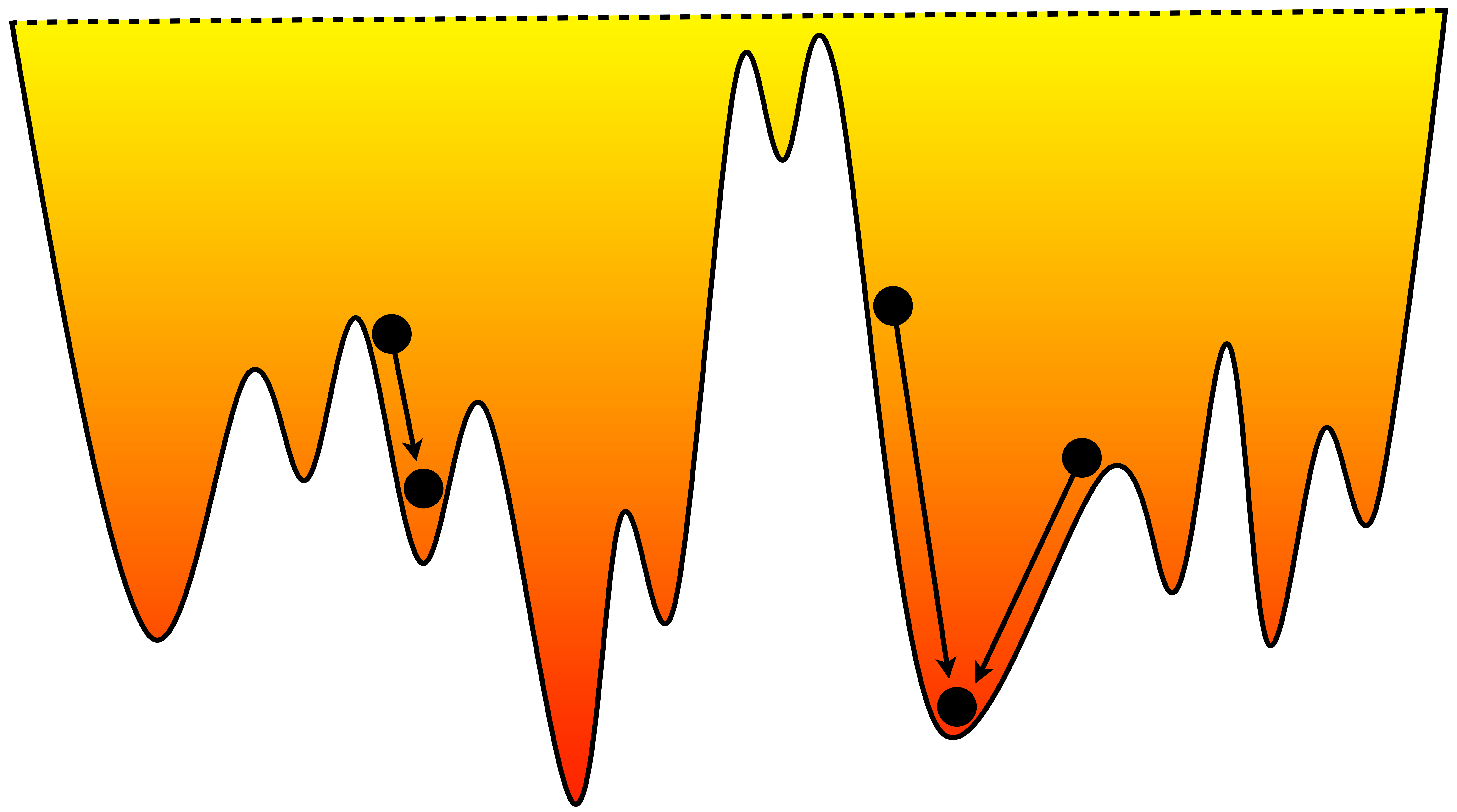

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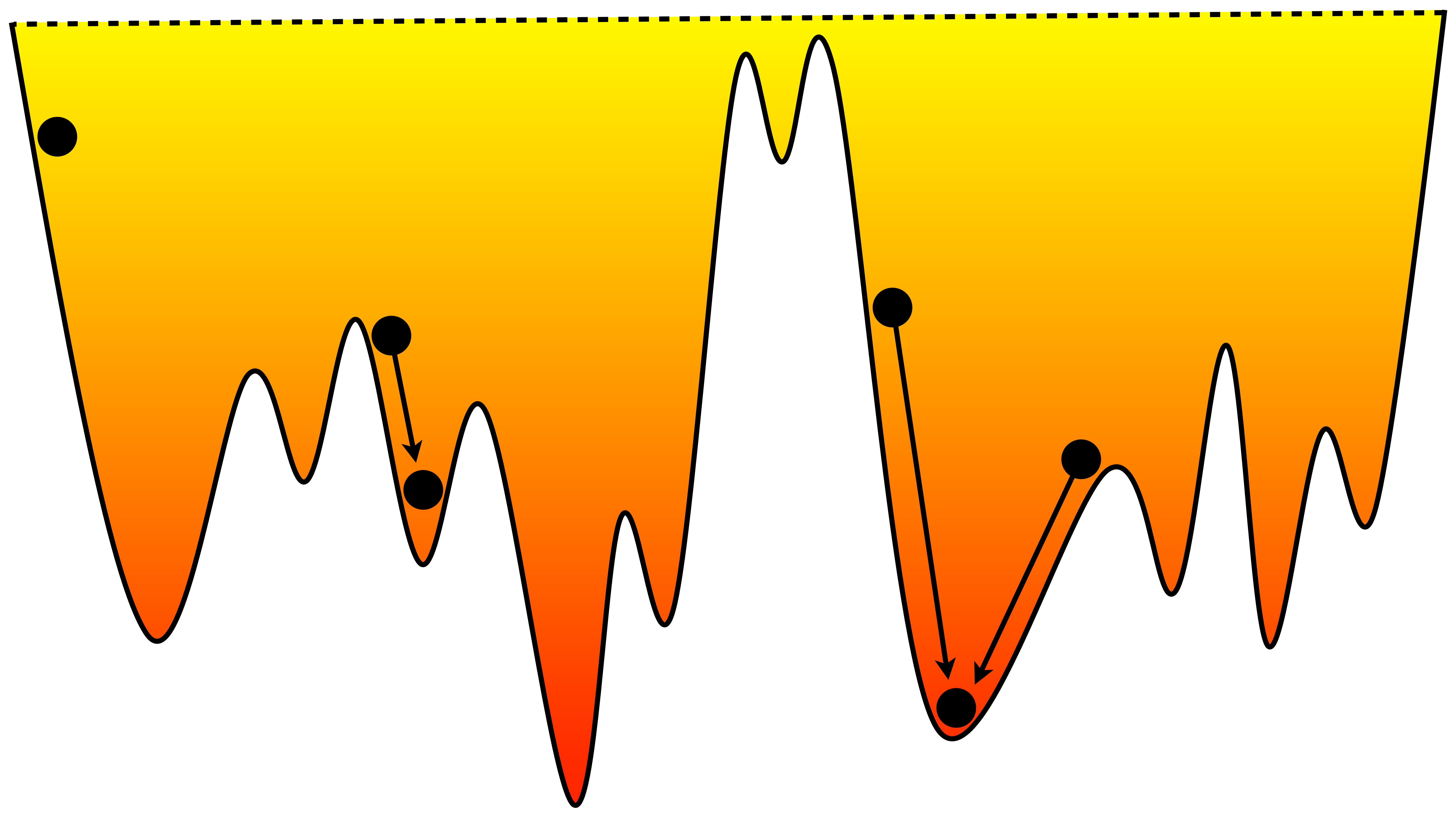

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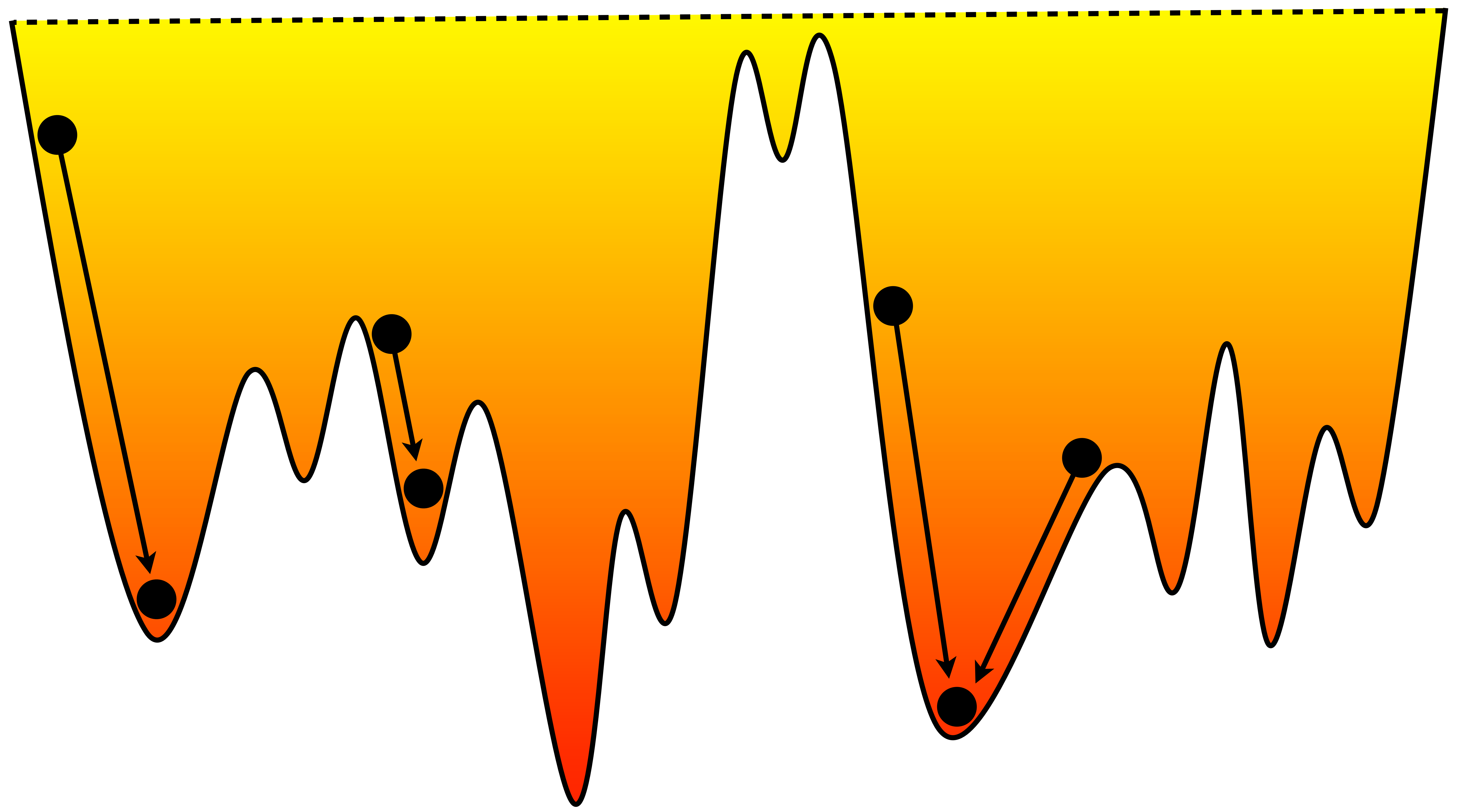

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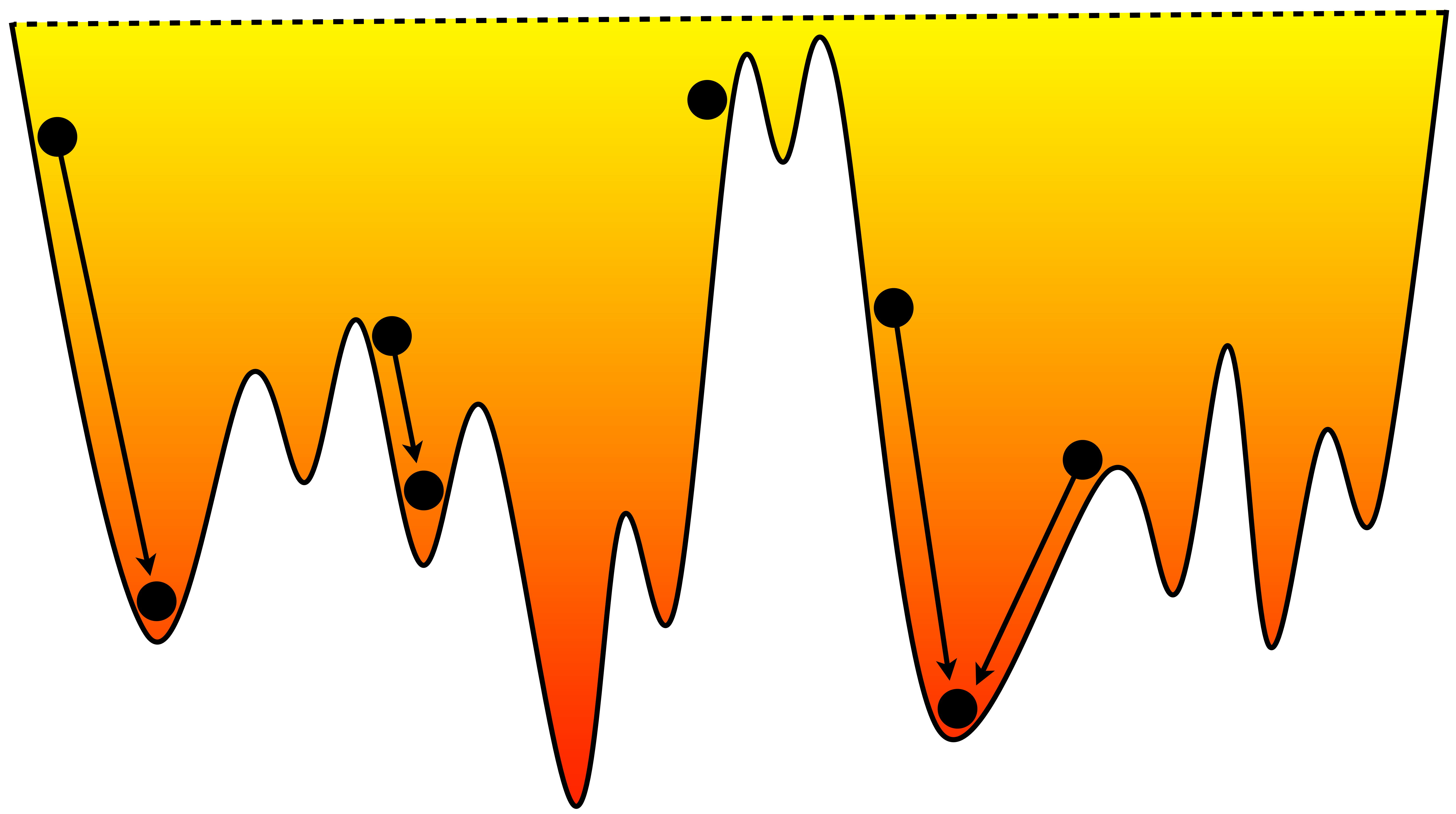

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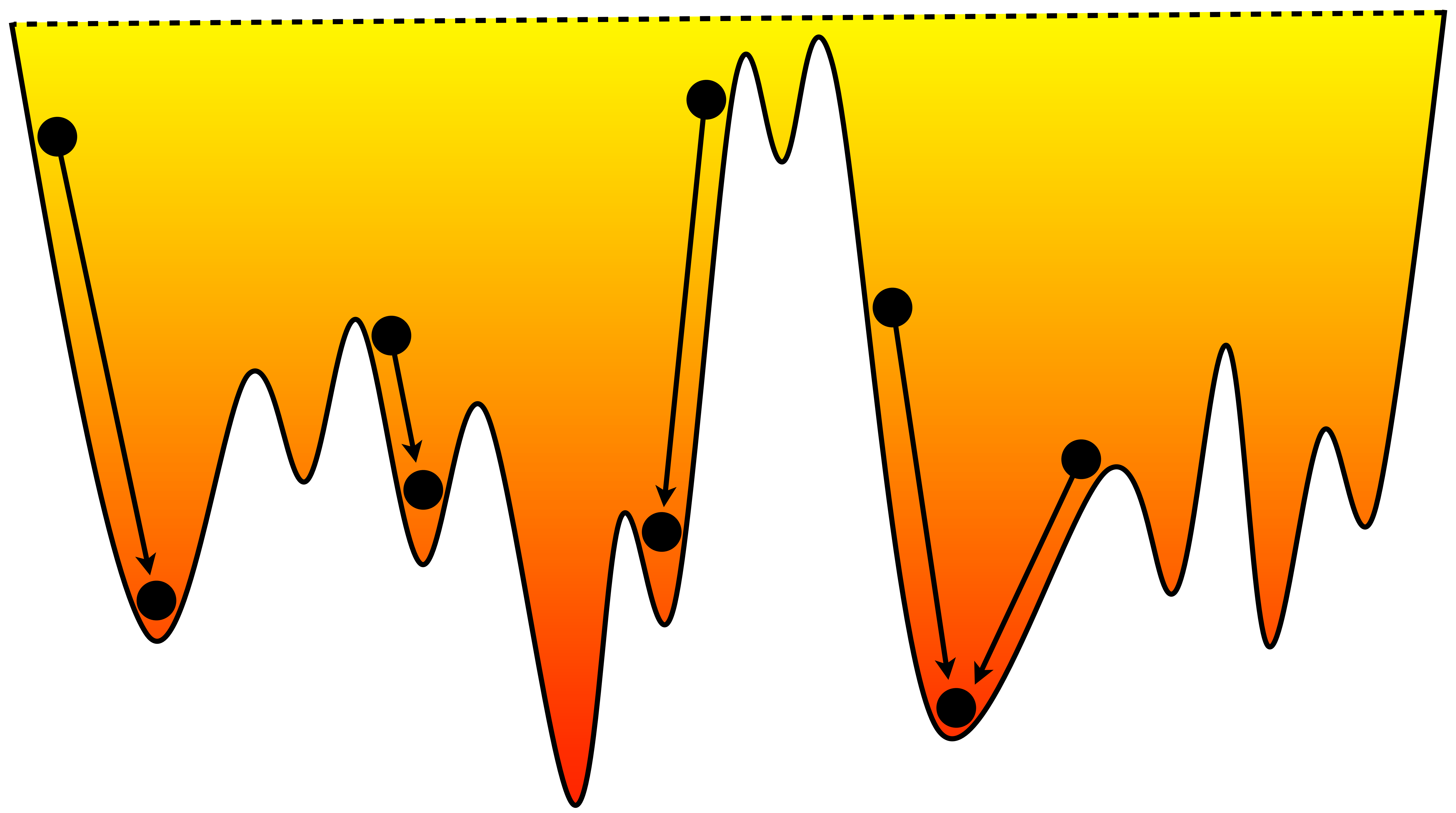

Iterated Local Search

- Execute multiple local search from different starting configuration
  - generic
    - can be combined with other metaheuristics
    - multistarts or restarts

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```
1.  function IteratedLocalSearch($f, N, L, S$) {
2.      $s :=$ generateInitialSolution();
3.      $s^* := s$;
4.      for $k := 1$ to $MaxSearches$ do
5.          $s :=$ LocalSearch($f, N, L, S, s$);
6.          if $f(s) < f(s^*)$ then
7.              $s^* := s$;
8.          $s :=$ generateNewSolution($s$);
9.      return $s^*$;
10. }
```

Metropolis Heuristic

- Basic idea
  - accept a move if it improves the objective value or, in case it does not, with some probability
  - the probability depends on how “bad” the move is
  - inspired by statistical physics

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- Basic idea
  - accept a move if it improves the objective value or, in case it does not, with some probability
  - the probability depends on how “bad” the move is
  - inspired by statistical physics
- How is the probability chosen?
  - t is a temperature
  - Δ is the difference f(n) - f(s)
  - a degrading move is accepted with probability,

$$\exp(\frac{-\Delta}{t})$$

Metropolis Heuristic

```
1.  function S-METROPOLIS[t](N,s)
2.      select $n \in N$ with probability $1/\#N$;
3.      if $f(n) \leq f(s)$ then
4.          return $n$;
5.      else with probability $exp(\frac{-(f(n)-f(s))}{t})$
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- What happens for a large $\Delta = f(n) - f(s)$?
  - the probability of accepting the move becomes very small

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- What happens for a large t ?
  - the probability of accepting a degrading move is large

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- Based on statistical physics
  - heating and cooling schedules to produce crystals with few defects
- Key idea
  - Use Metropolis algorithm but adjust the temperature dynamically
  - start with a high temperature
    - essentially a random walk
  - decrease the temperature progressively
  - when the temperature is low
    - essentially a local improvement search

Simulated Annealing

```
1.  function SimulatedAnnealing($f$, $N$) {
2.      $s$ := generateInitialSolution();
3.      $t_1$ := initTemperature(s);
4.      $s^*$ := $s$;
5.      for $k$ := 1 to $MaxSearches$ do
6.          $s$ := LocalSearch($f$,$N$,L-All,S-Metropolis[$t_k$],$s$);
7.          if $f(s) < f(s^*)$ then
8.              $s^*$ := $s$;
9.          $t_{k+1}$ := updateTemperature($s$,$t_k$);
10.     return $s^*$;
11. }
```

Simulated Annealing

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  - connected neighborhood
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    - slower than exhaustive search

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- guaranteed to converge to a global optimum
  - connected neighborhood
  - slow schedule
    - slower than exhaustive search
- In practice
  - some excellent results on some hard benchmarks
    - e.g., TTP, minimizing tardiness in scheduling
  - reasonably fast schedule

$$t_{k+1} = \alpha \; t_k$$

Simulated Annealing

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  - restarts
  - reheats
  - see also tabu search later

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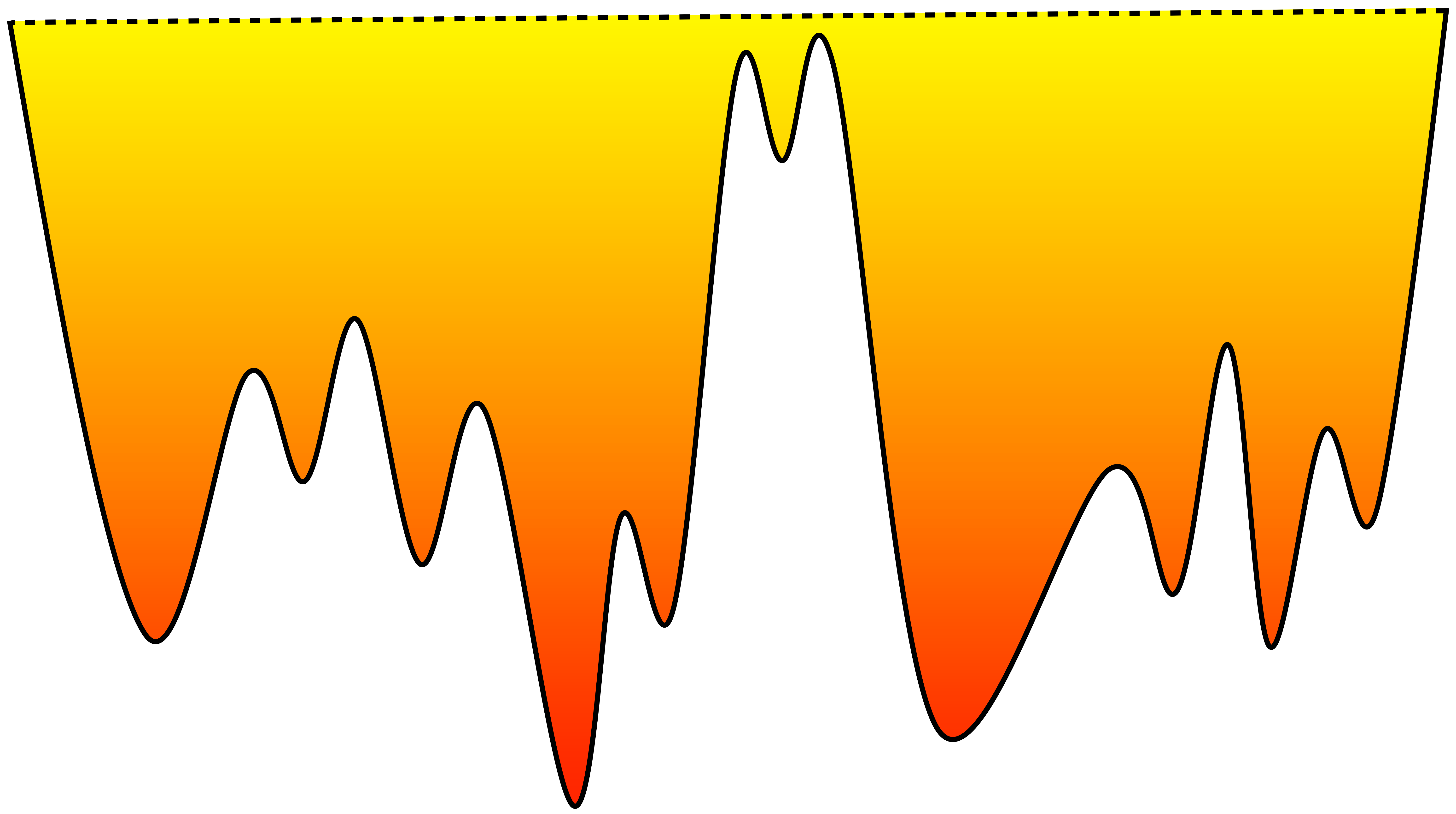
  - see also tabu search later
- Restarts
  - like in multi-start procedure
- Reheat
  - increase the temperature

Tabu Search



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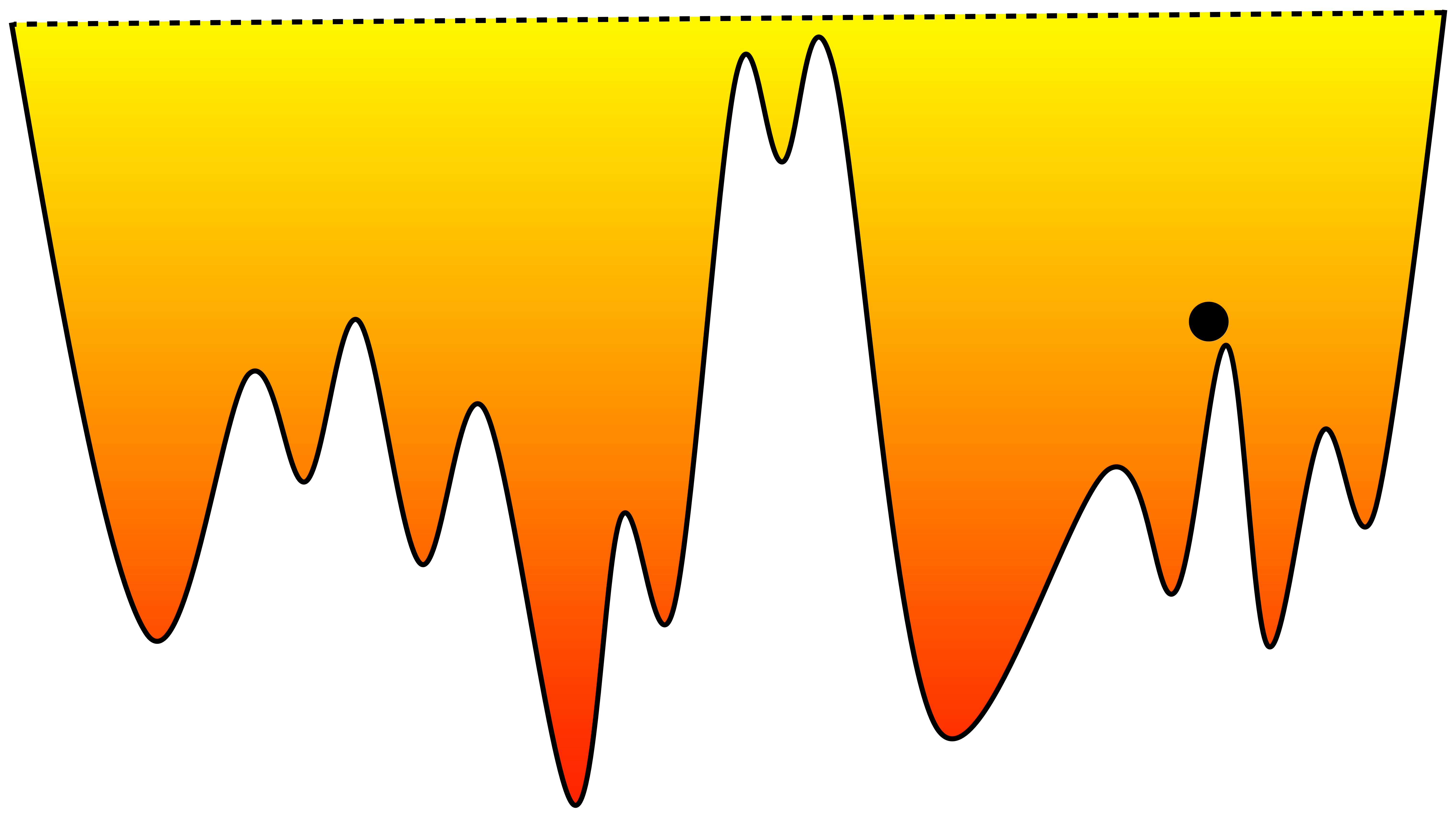

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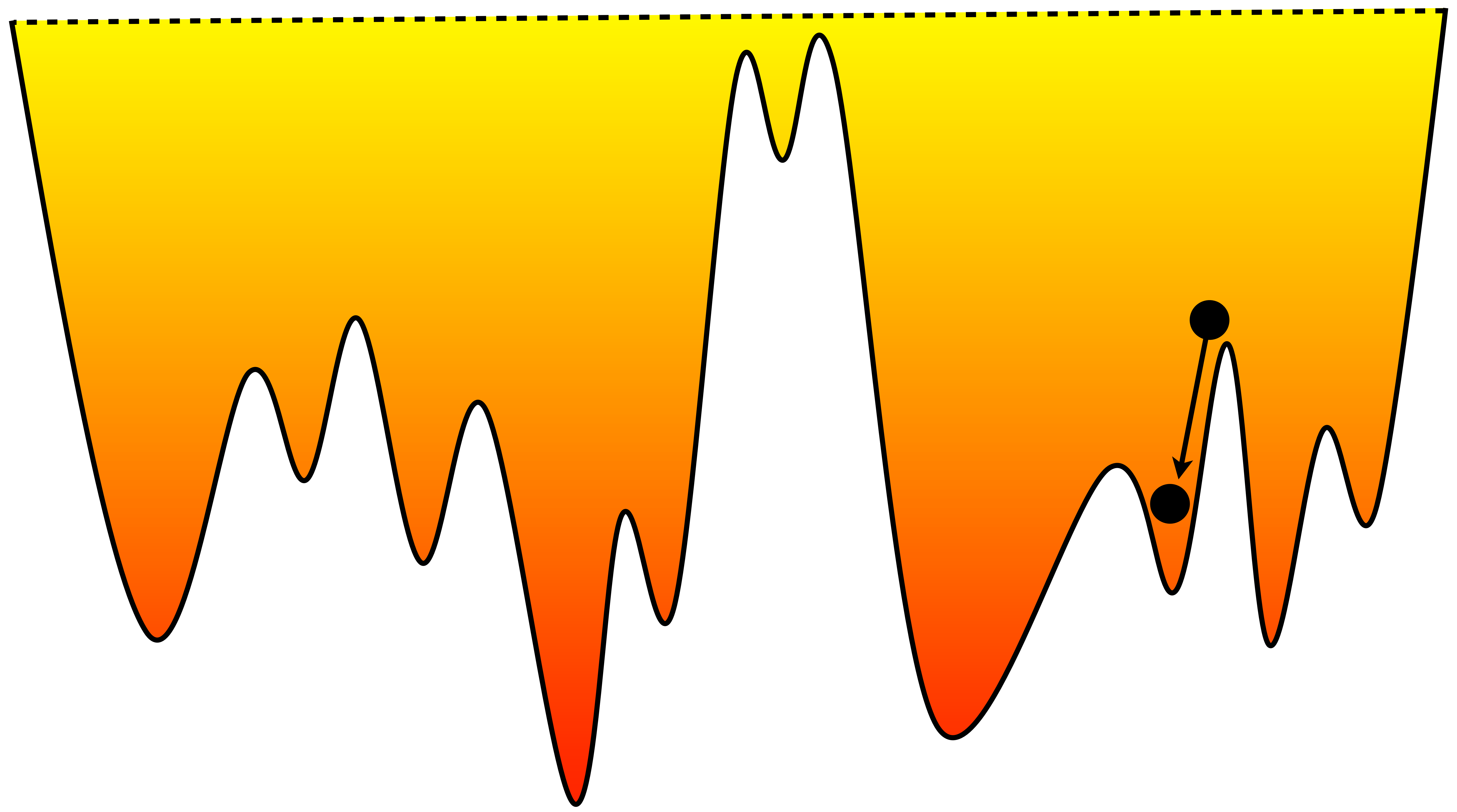

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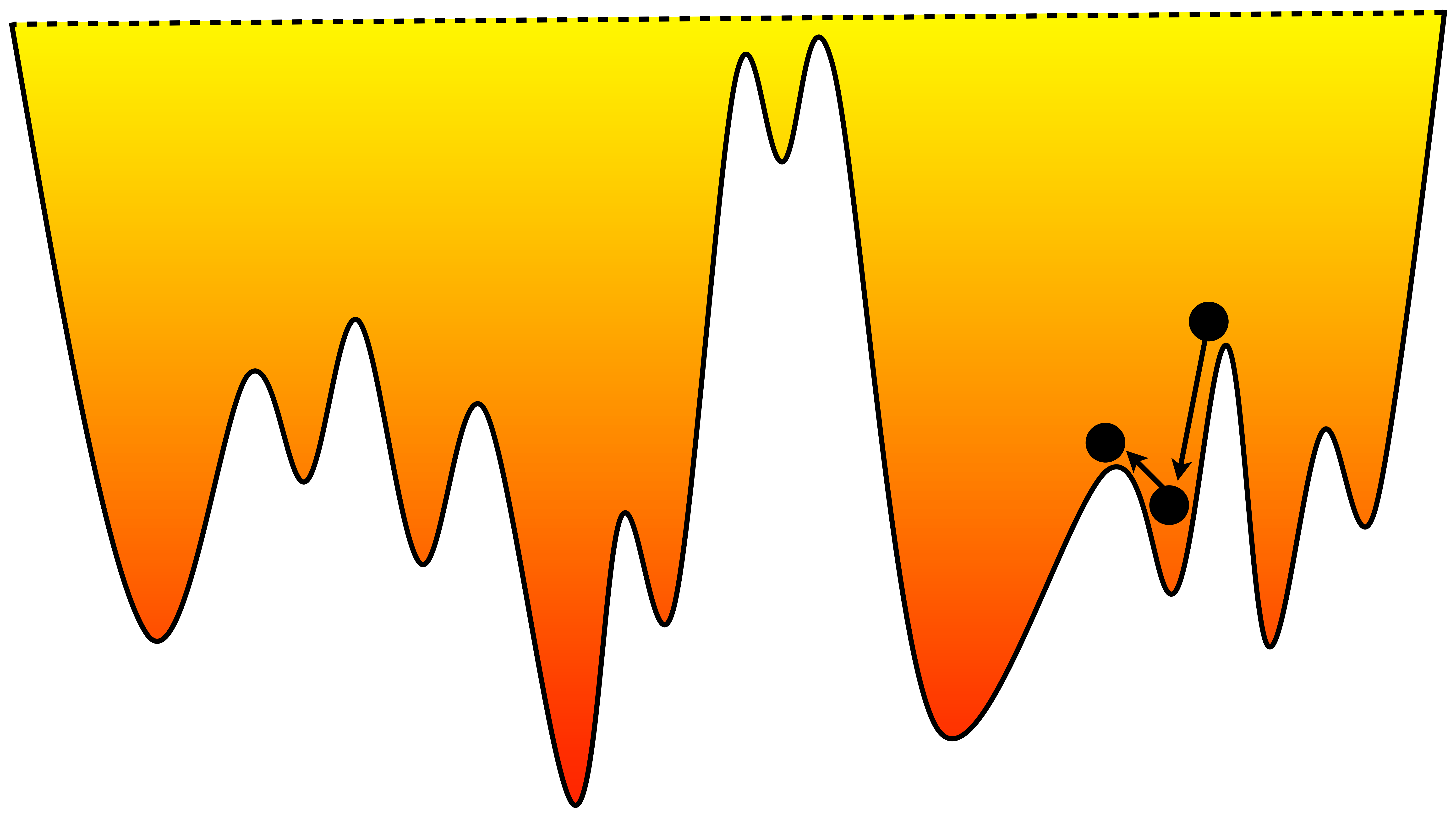

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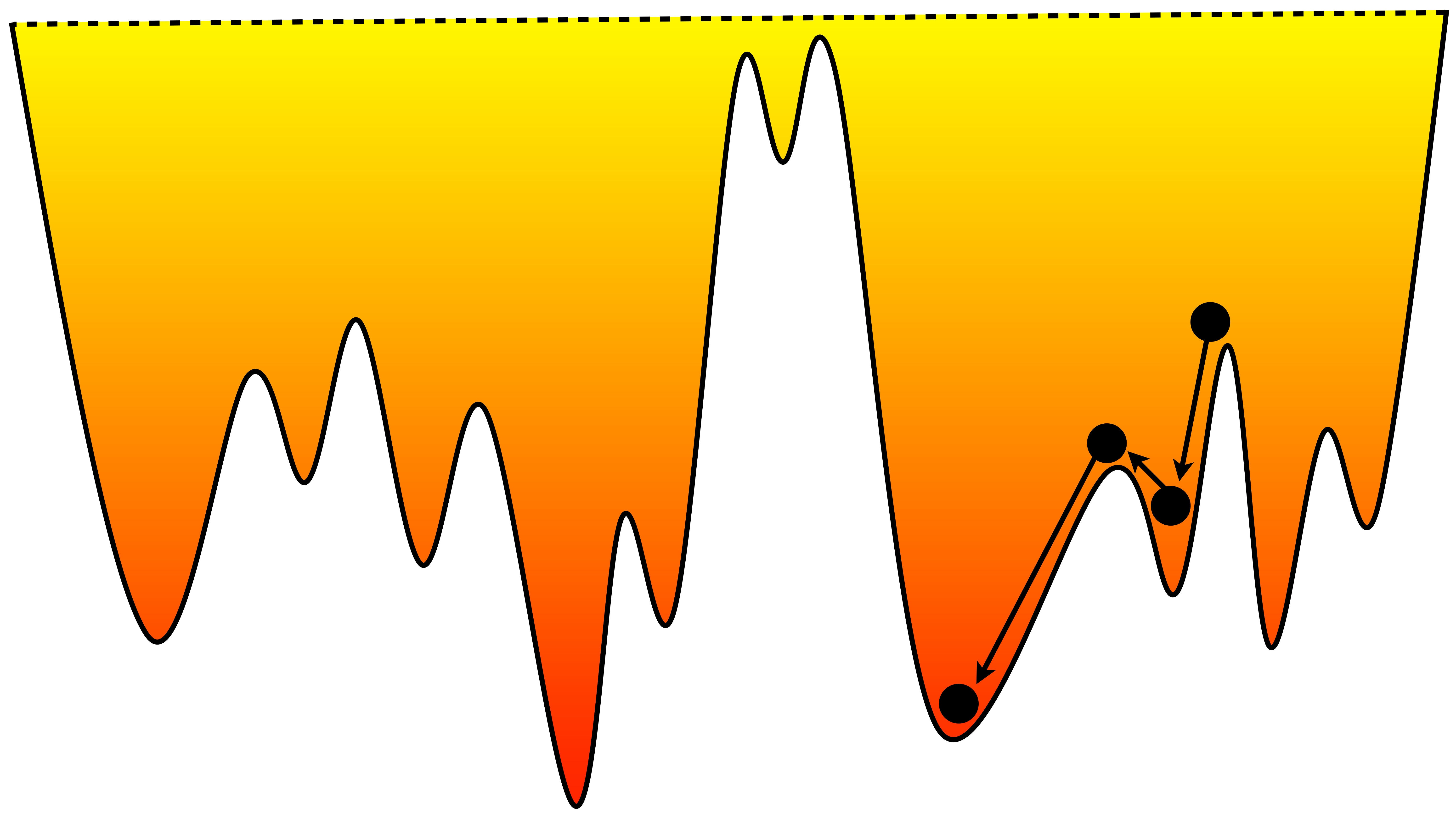

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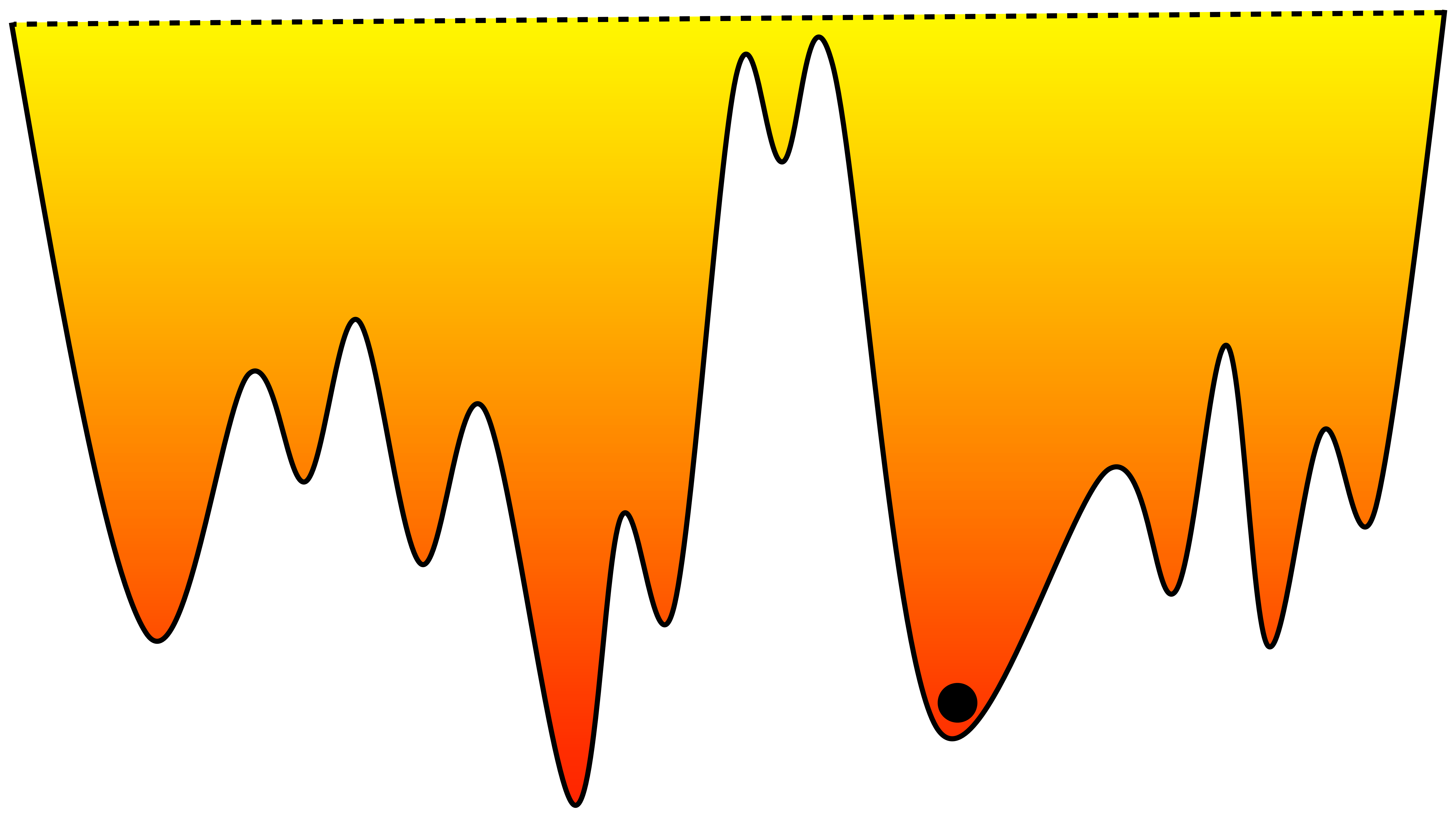

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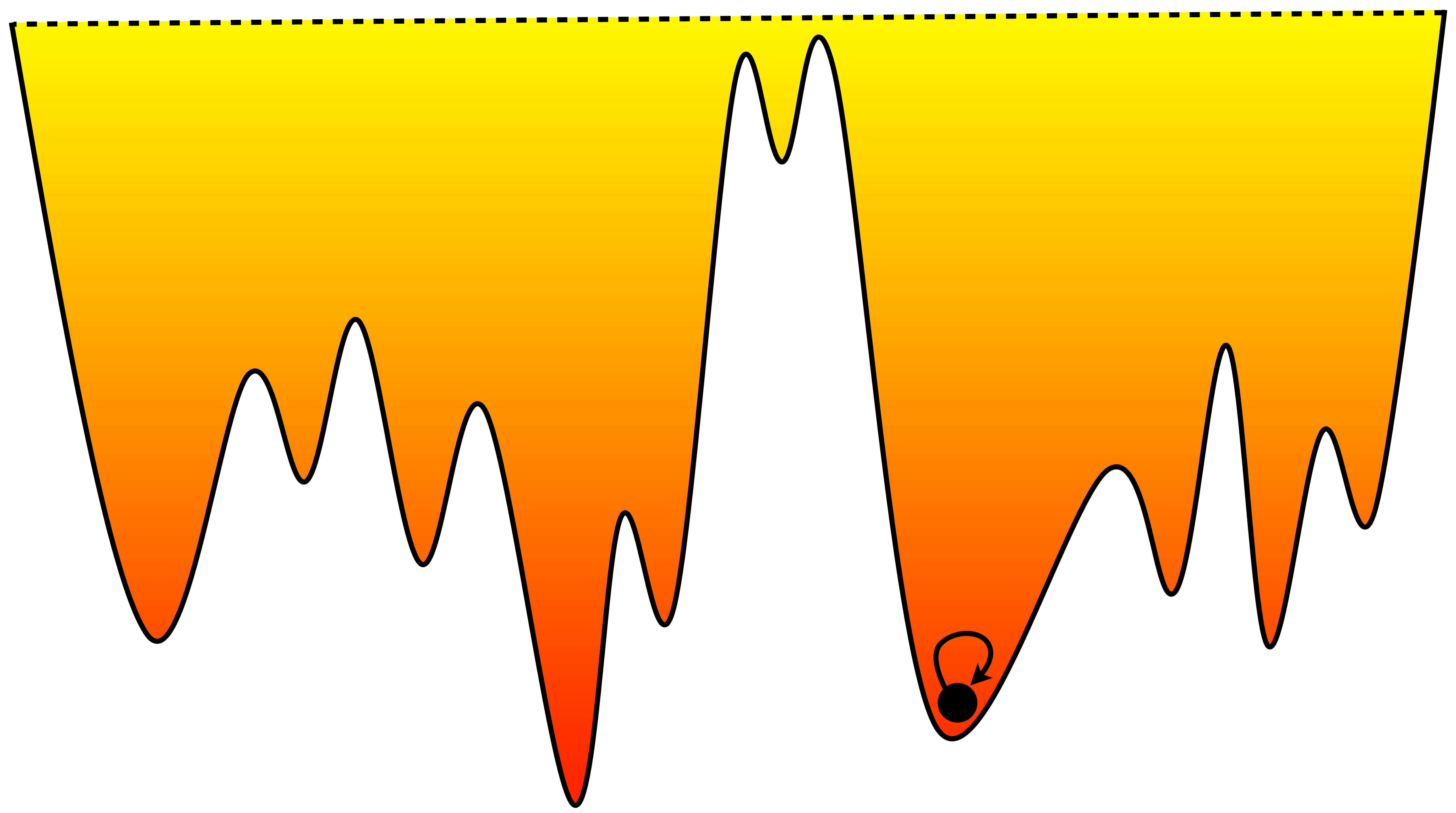

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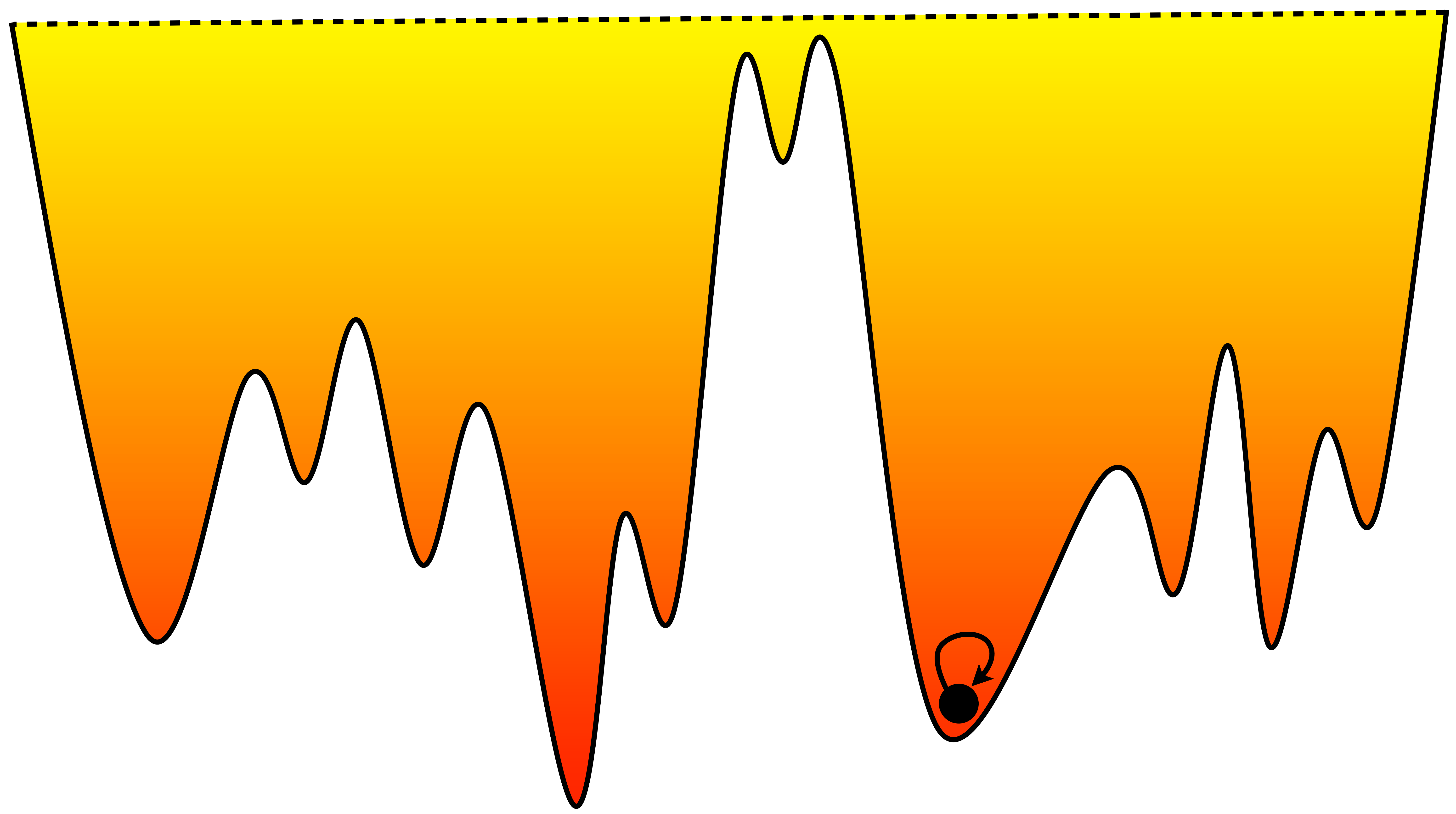
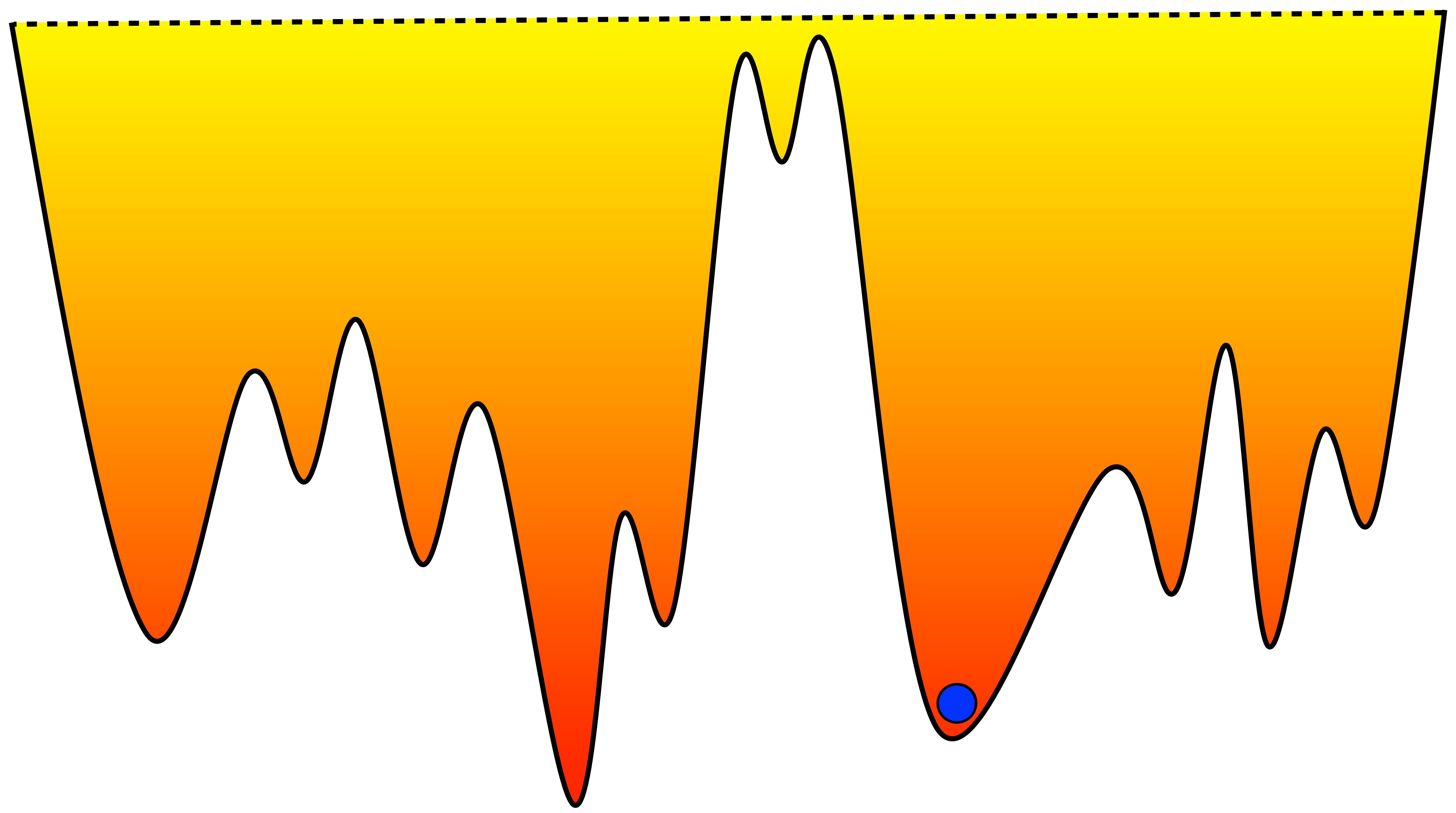


Tabu node ● : node I have already visited

Tabu Search



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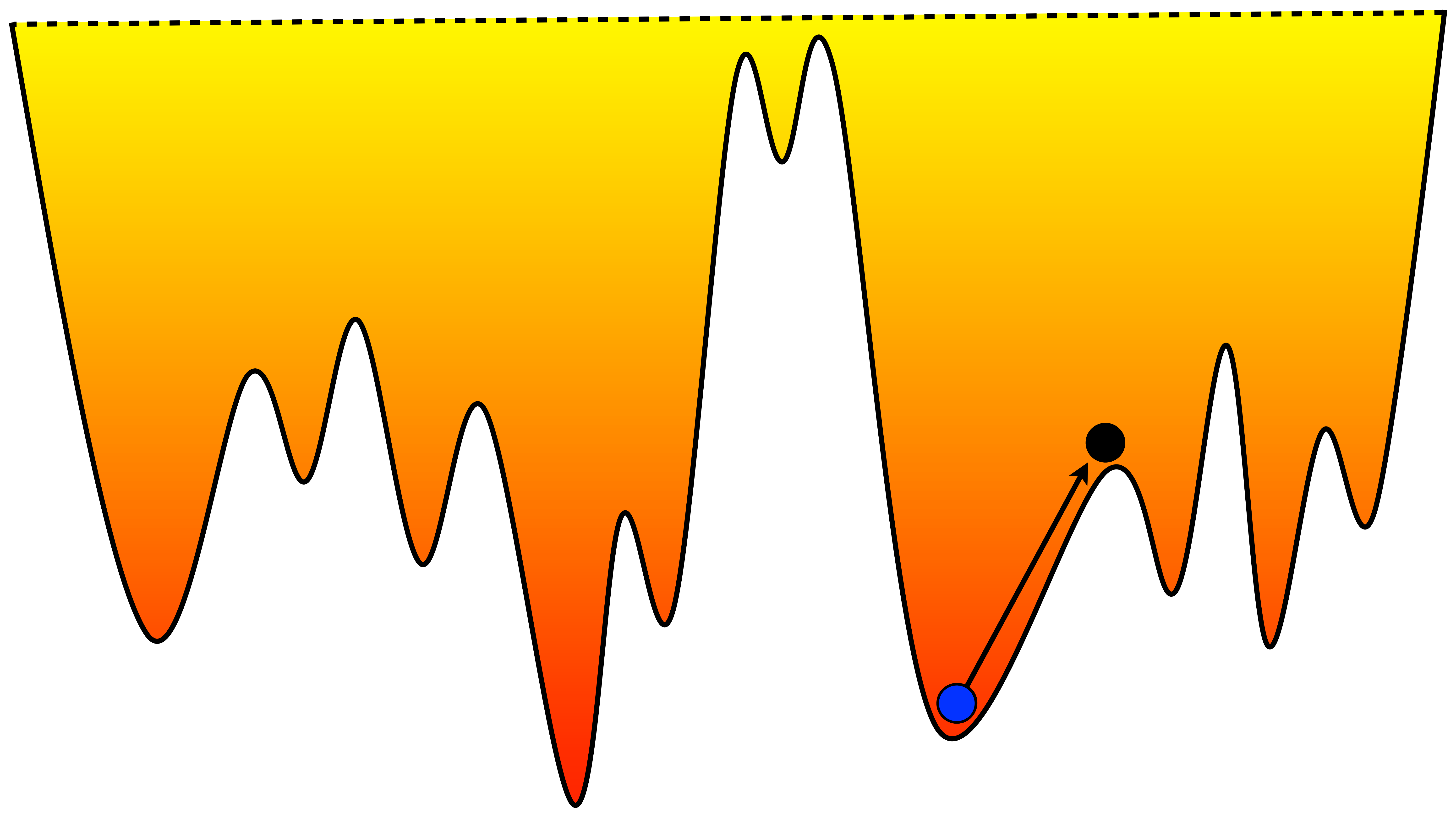

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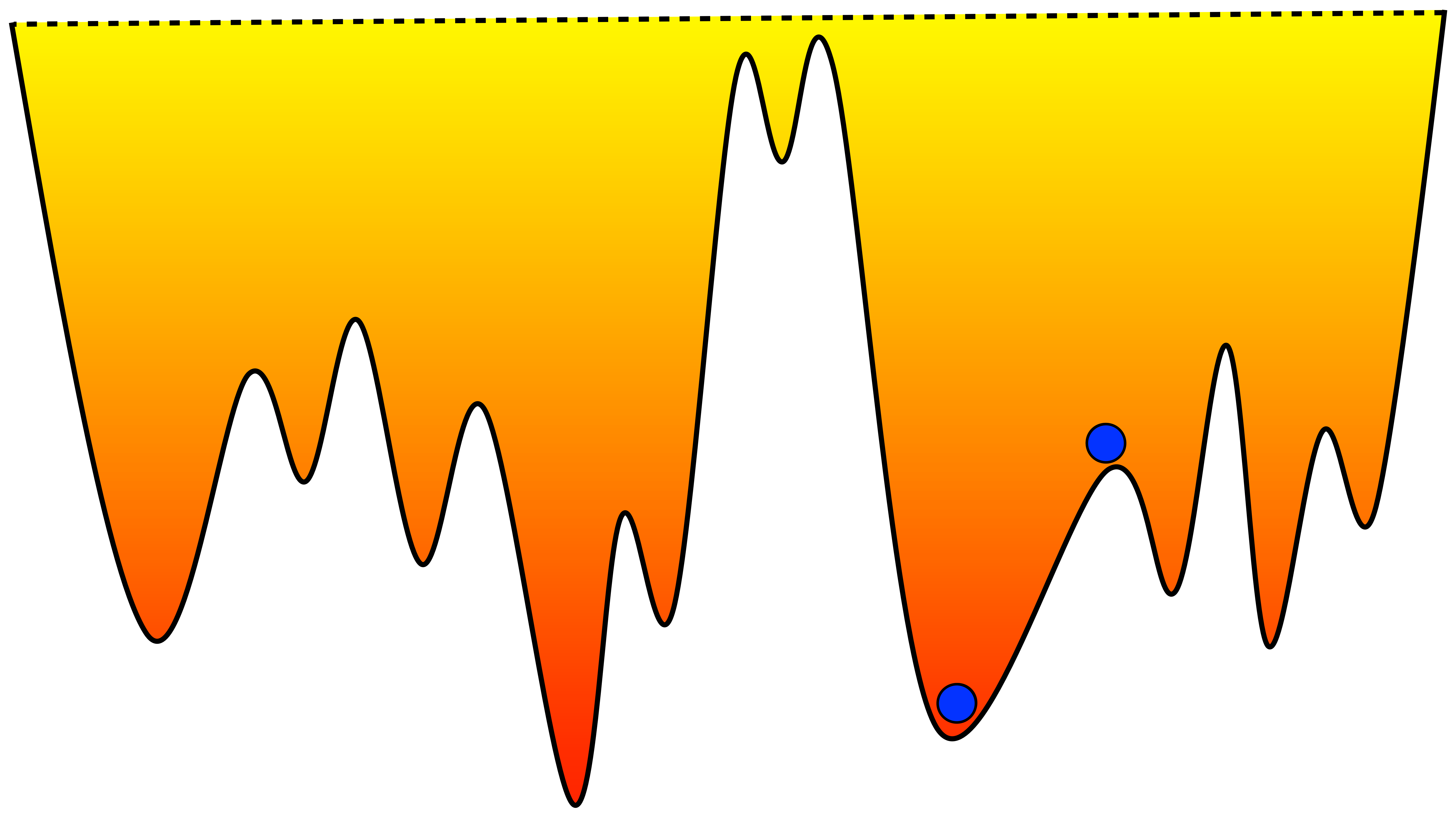

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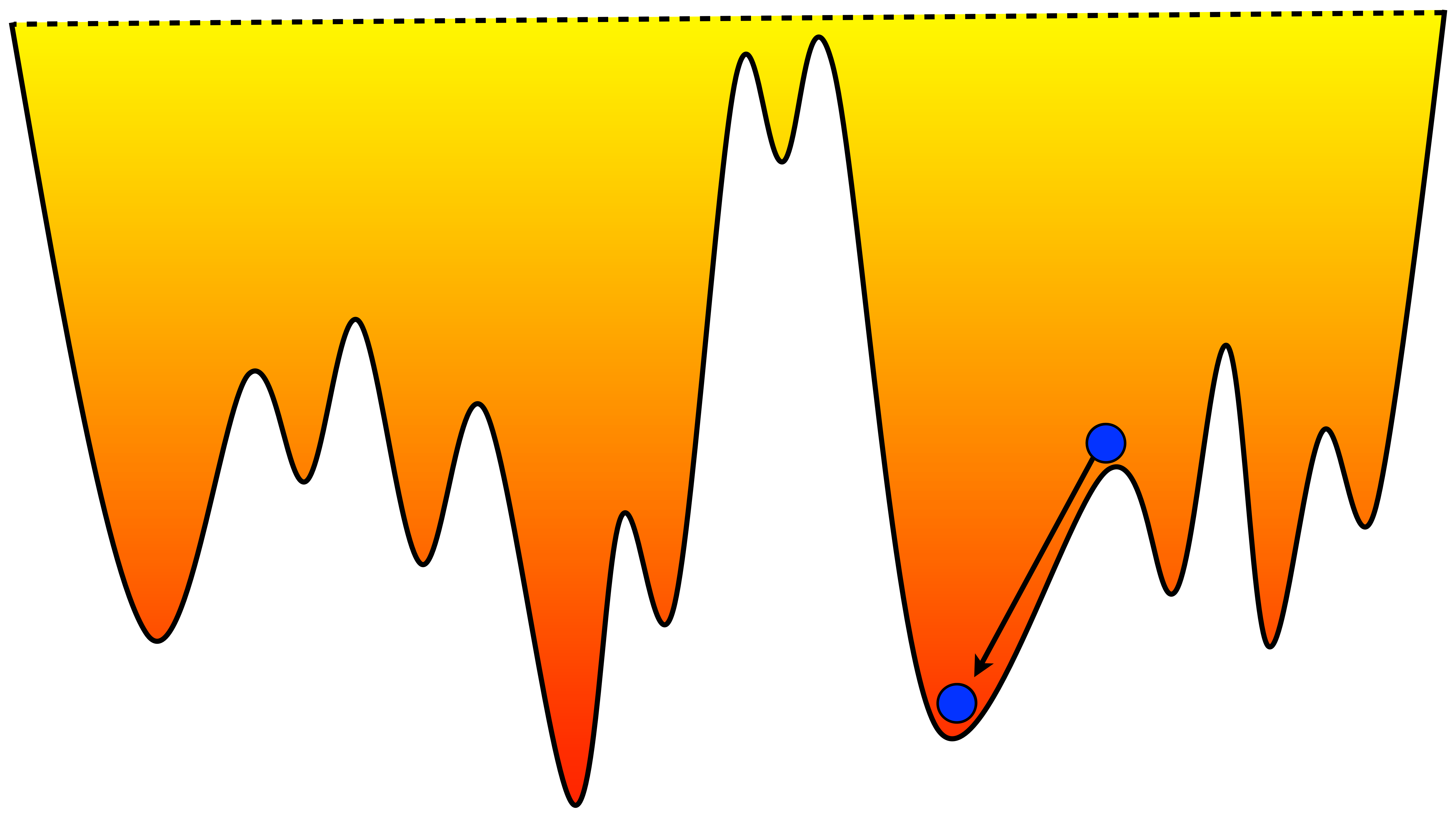

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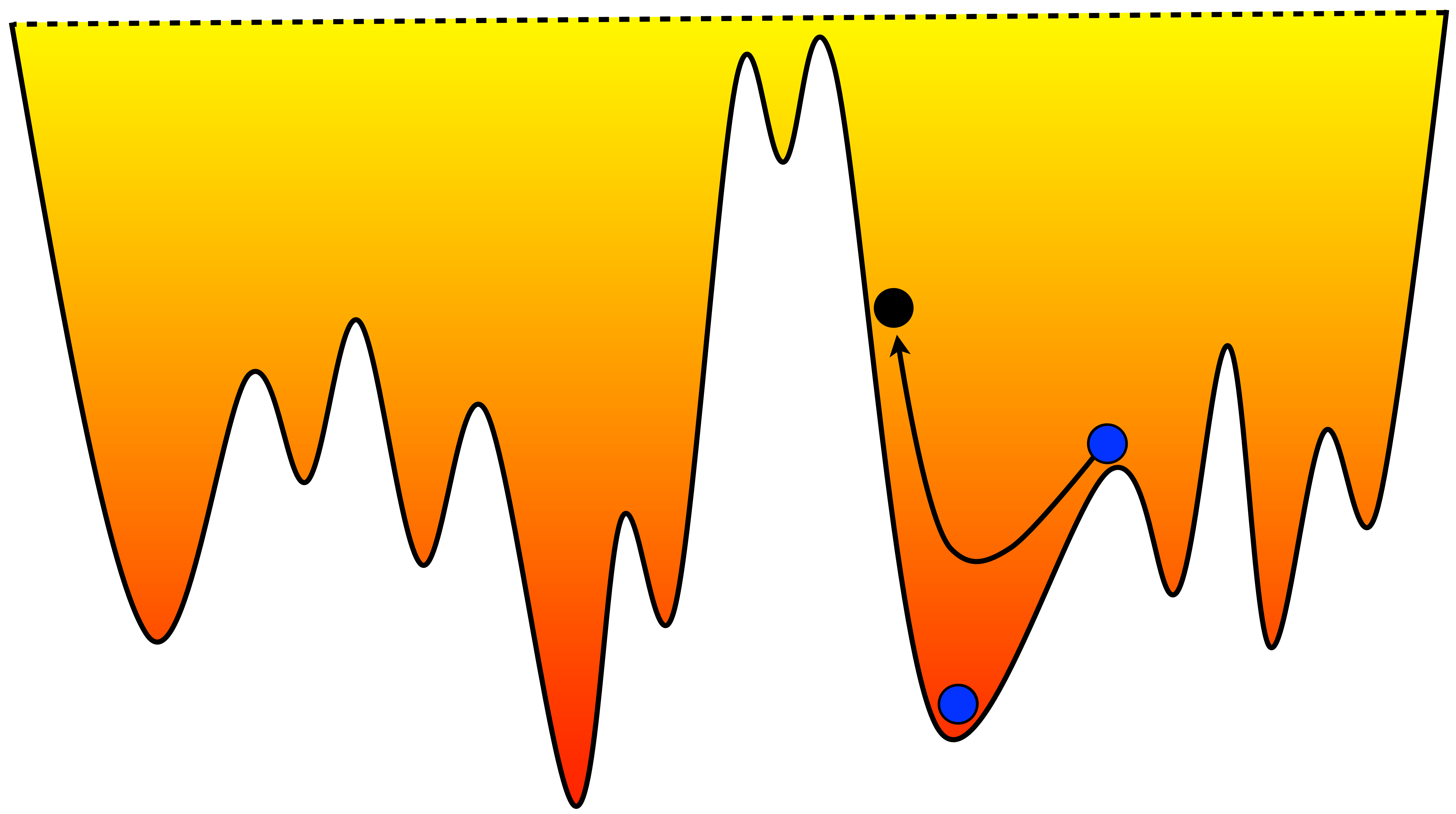

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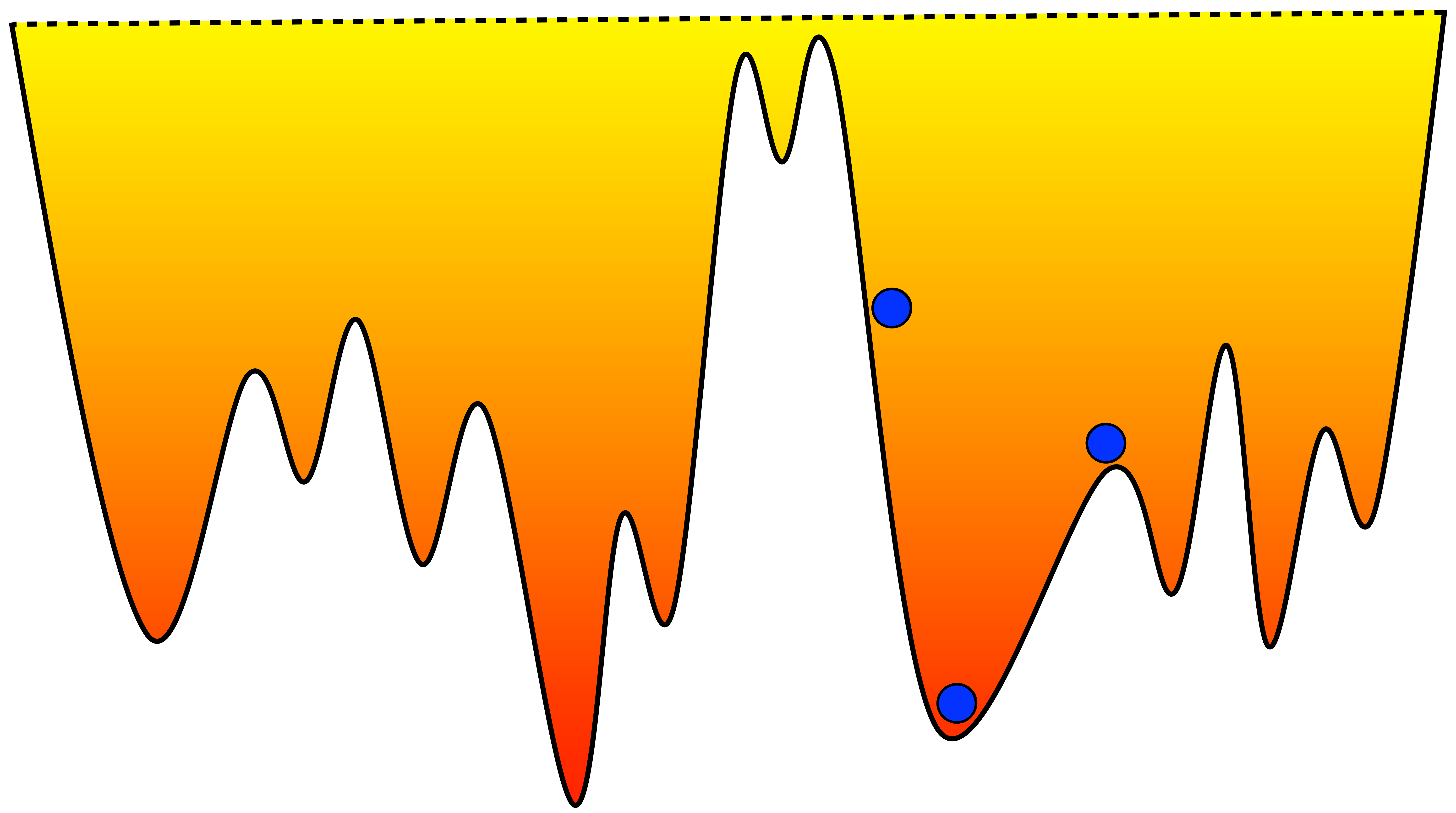

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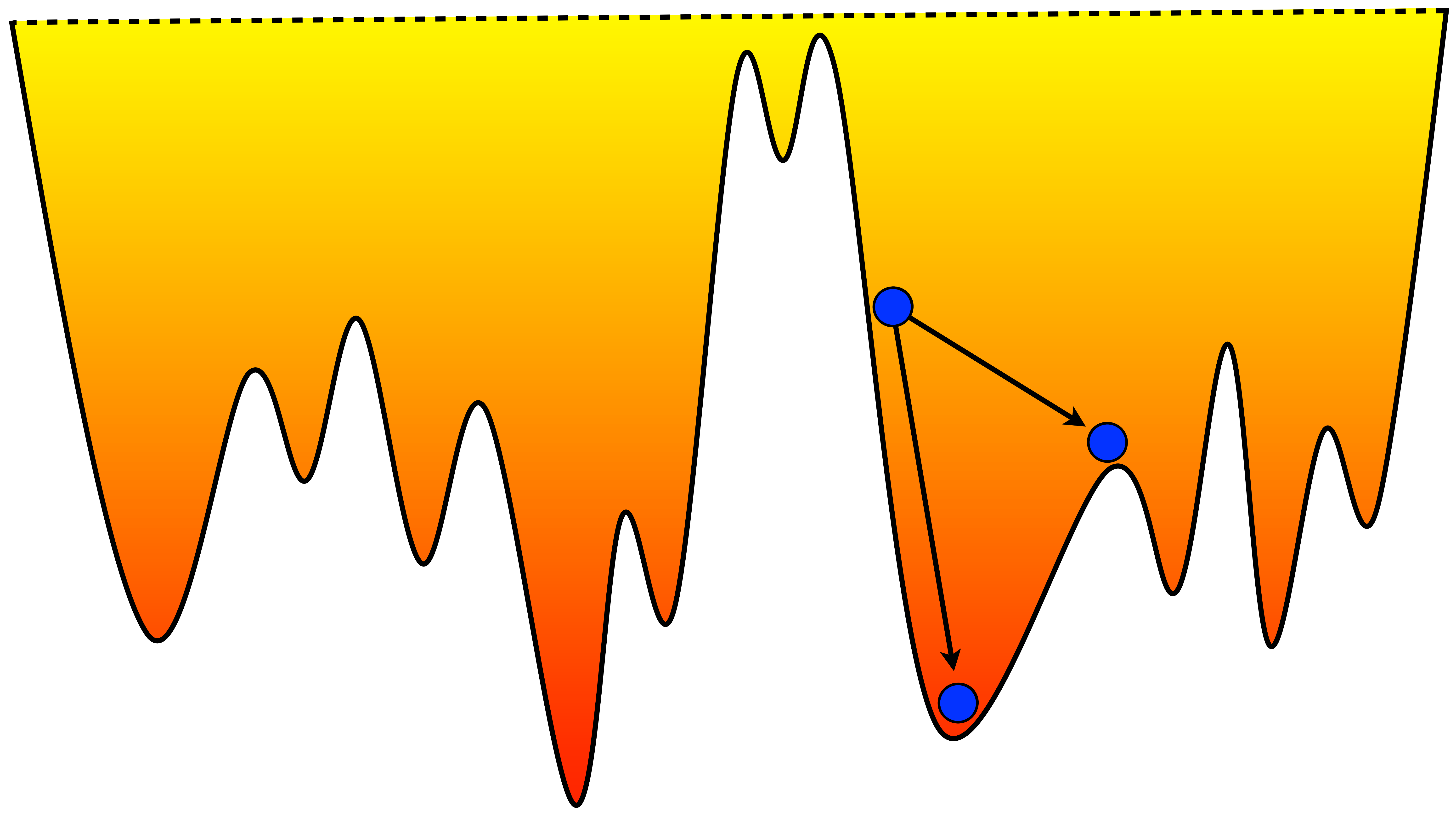

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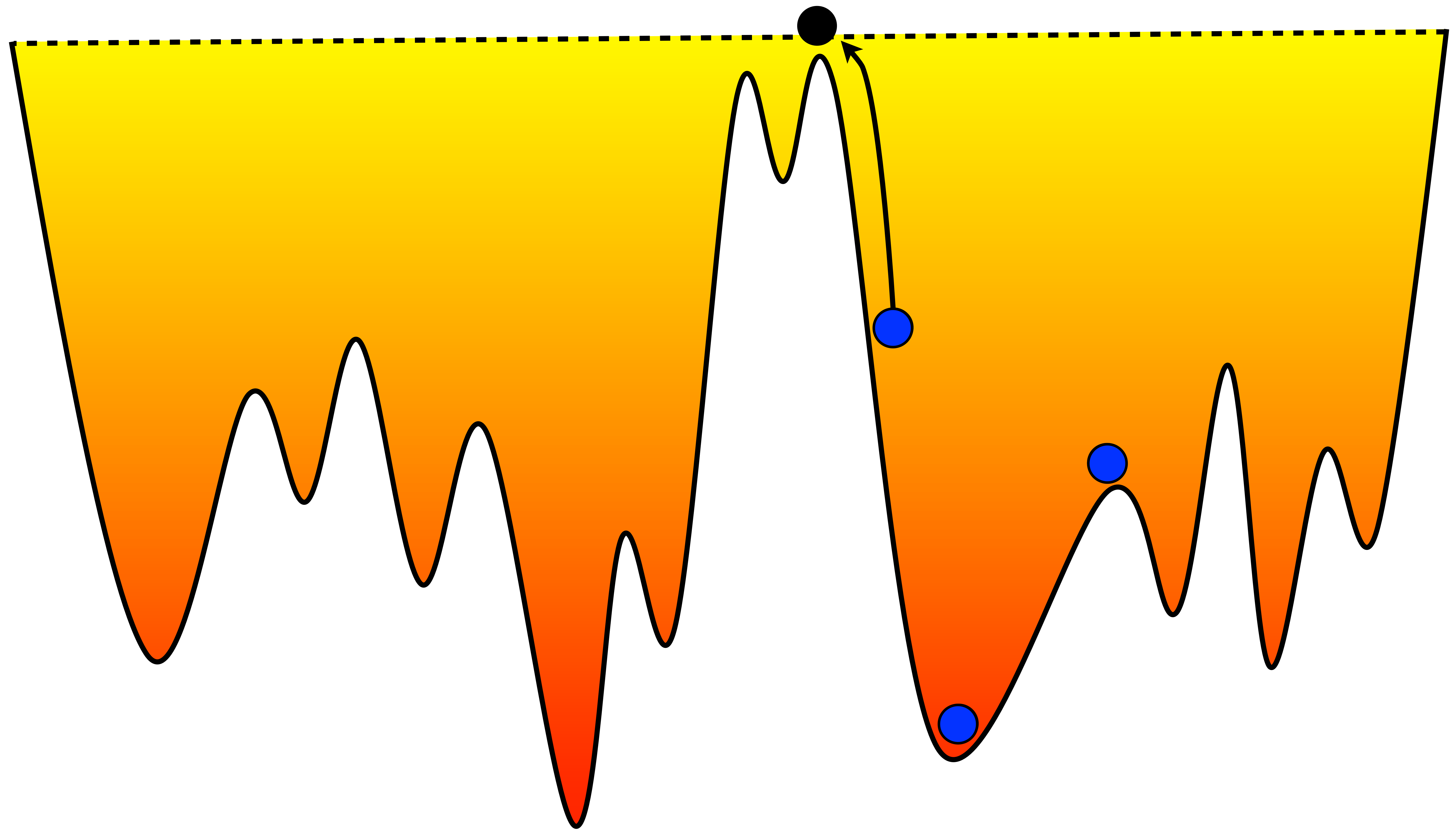

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Tabu Search

- Key abstract idea
  - maintain the sequence of nodes already visited
    - tabu list and tabu nodes

```
1.  function LocalSearch($f, N, L, S, s_1$) {
2.      $s^* := s_1$;
3.      $\tau := \langle s_1 \rangle$;
4.      for $k := 1$ to $MaxTrials$ do
5.          if $satisfiable(s) \wedge f(s_k) < f(s^*)$ then
6.              $s^* := s_k$;
7.          $s_{k+1} := S(L(N(s_k), \tau), \tau)$;
8.          $\tau := \tau :: s_{k+1}$;
9.      return $s^*$;
10. }
```

Tabu Search

- Basic abstract tabu-search
  - select the best configurations that is not tabu, i.e., has not been visited before

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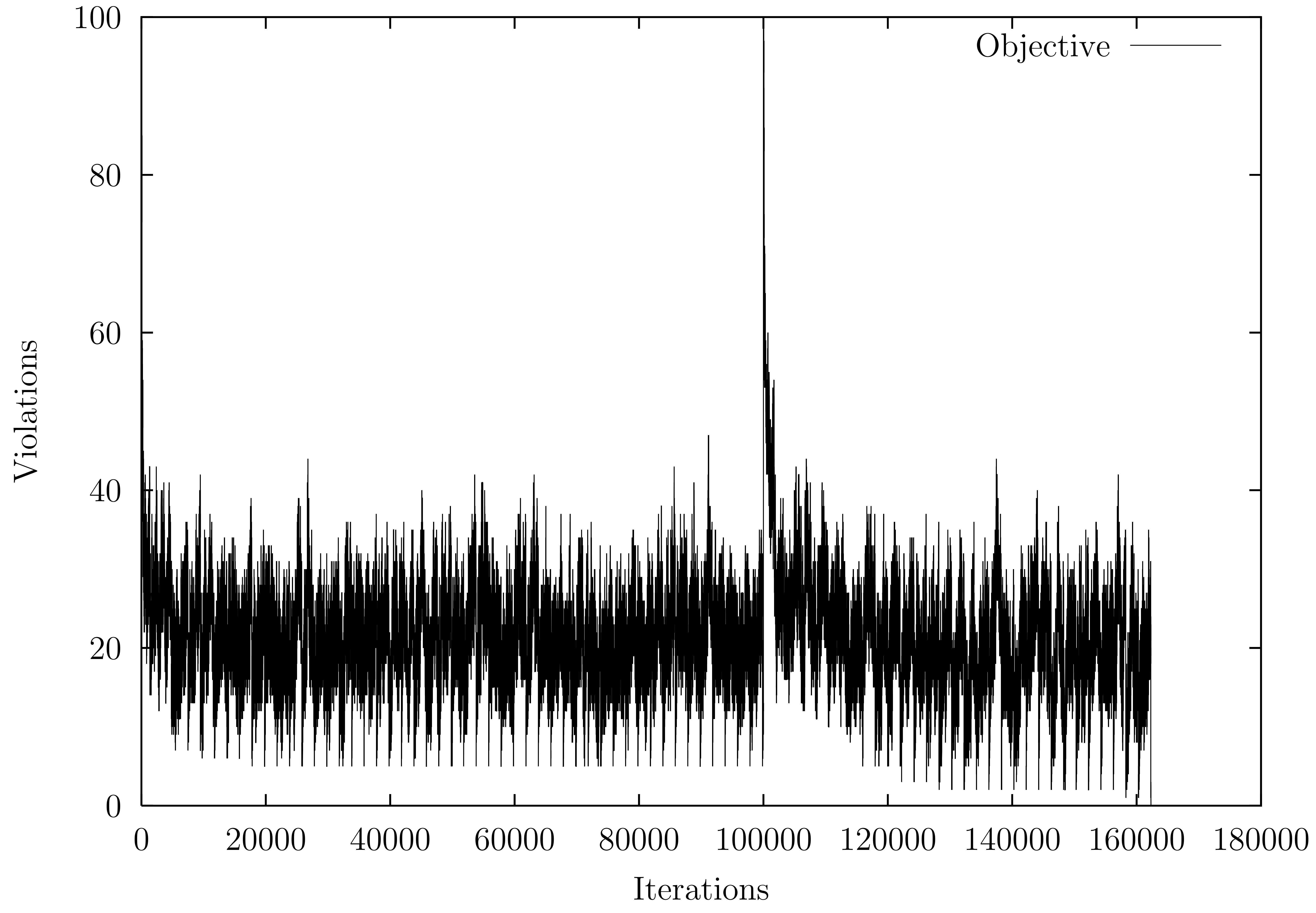
```
1.   **function** TabuSearch($f$,$N$,$s$)
2.       **return** LocalSearch($f$,$N$,L-NotTabu,S-Best);
```

where

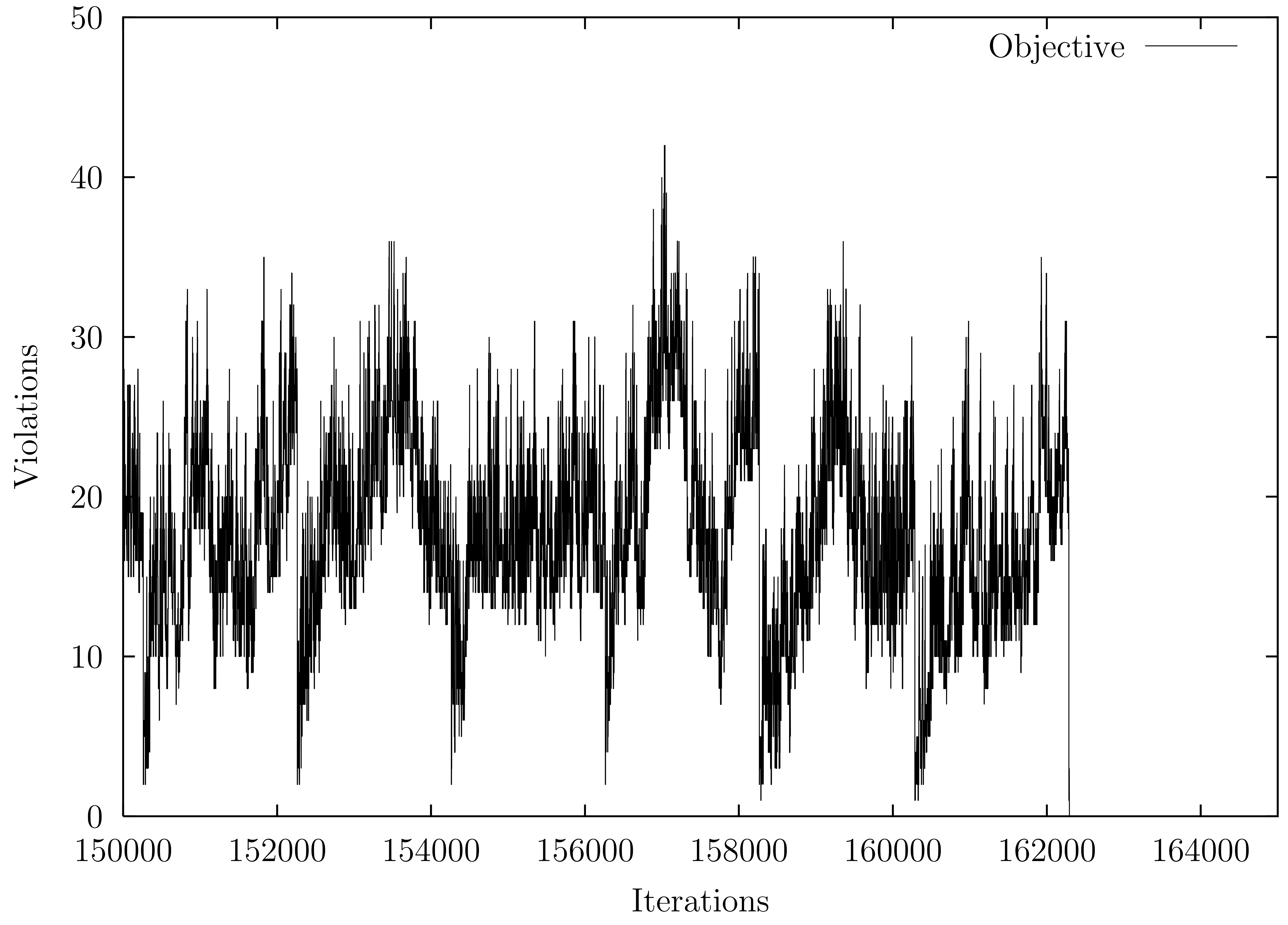
```
1.   **function** L-NotTabu($N$,$\tau$)
2.       **return** $\{ n \in N \mid n \notin \tau \}$;
```

Tabu Search



Objective
Violations
Iterations
0
20
40
60
80
100
0
20000
40000
60000
80000
100000
120000
140000
160000
180000

Tabu Search



Objective
Violations
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0
10
20
30
40
50
150000
152000
154000
156000
158000
160000
162000
164000

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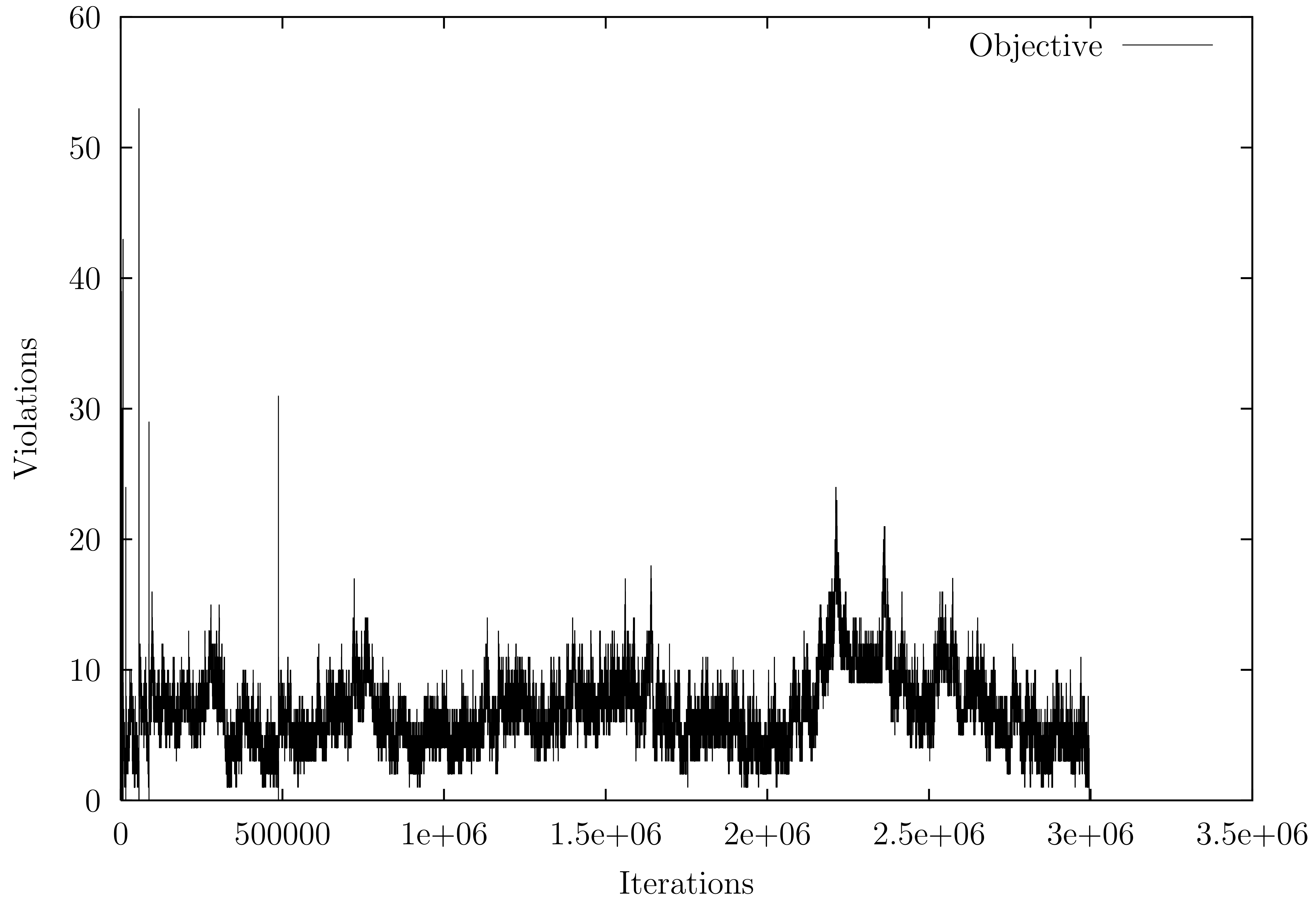

Objective
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0
10
20
30
40
50
60
0
500000
1e+06
1.5e+06
2e+06
2.5e+06
3e+06
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Metaheuristics

- Many others
  - variable neighborhood search
  - guided local search
  - ant-colony optimization
  - hybrid evolutionary algorithms
  - scatter search
  - ...

Until Next Time