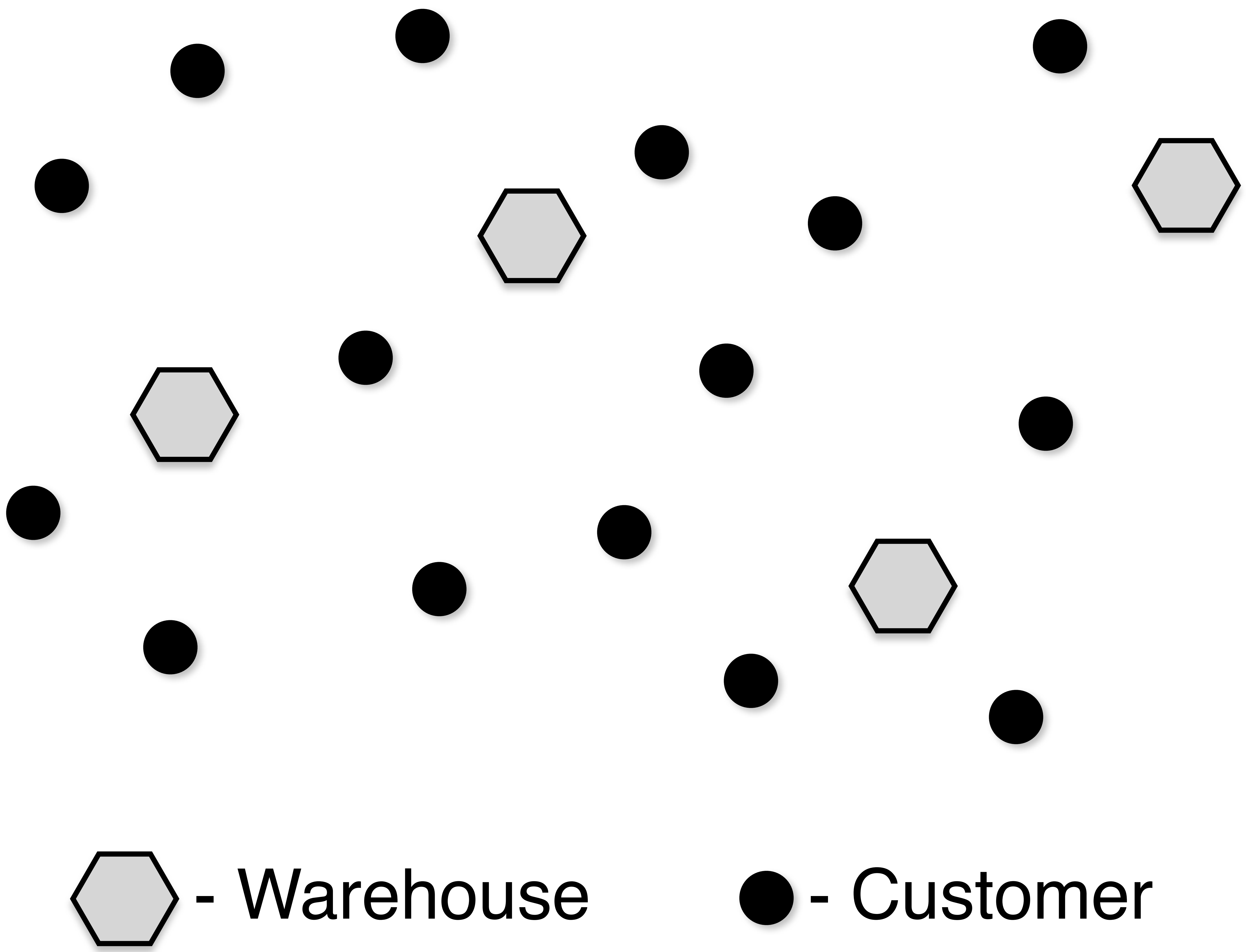
Discrete Optimization

Local Search: Part III

Goals of the Lecture

- Local search
  - optimization
  - warehouse location
  - traveling salesman problem

Warehouse Location



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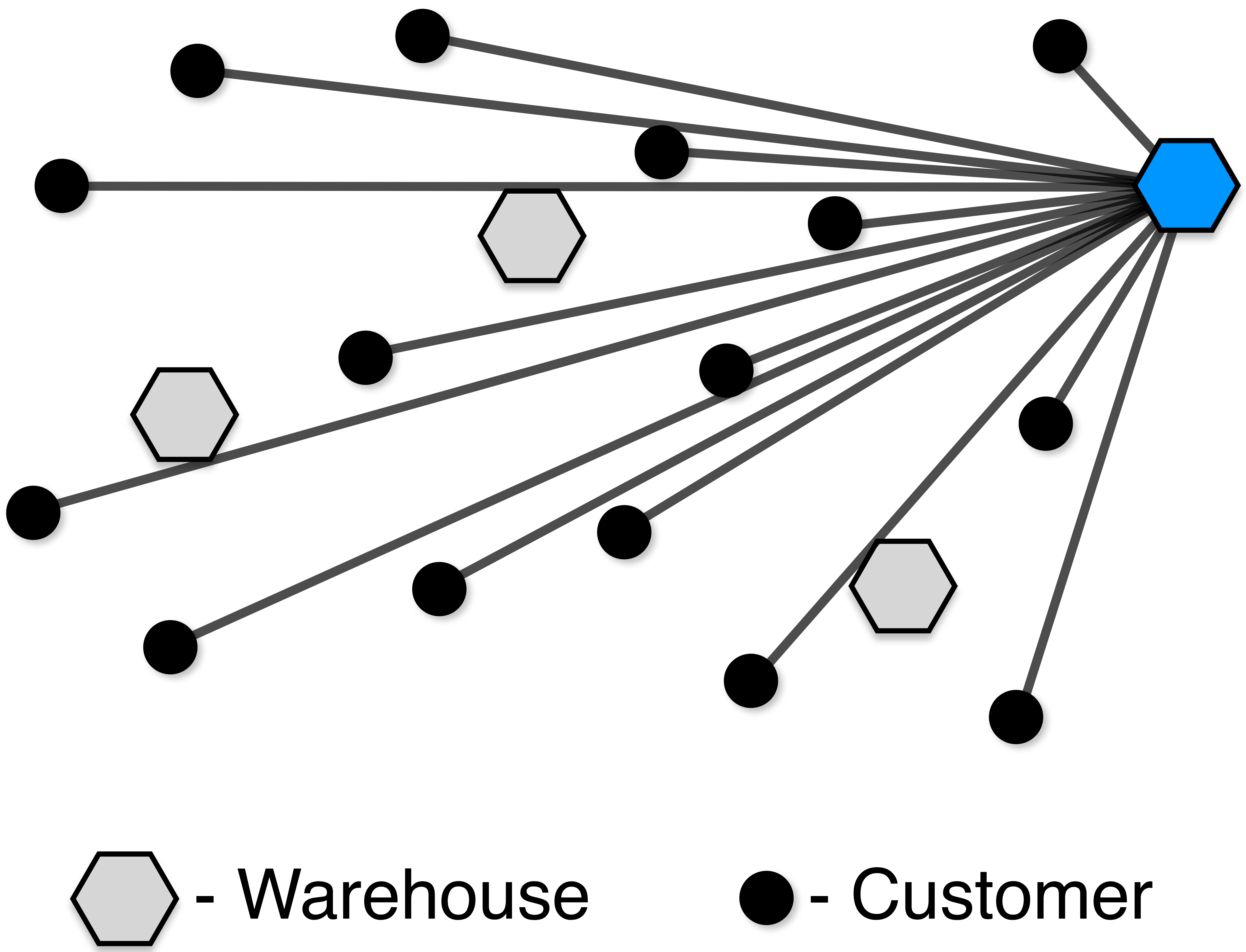

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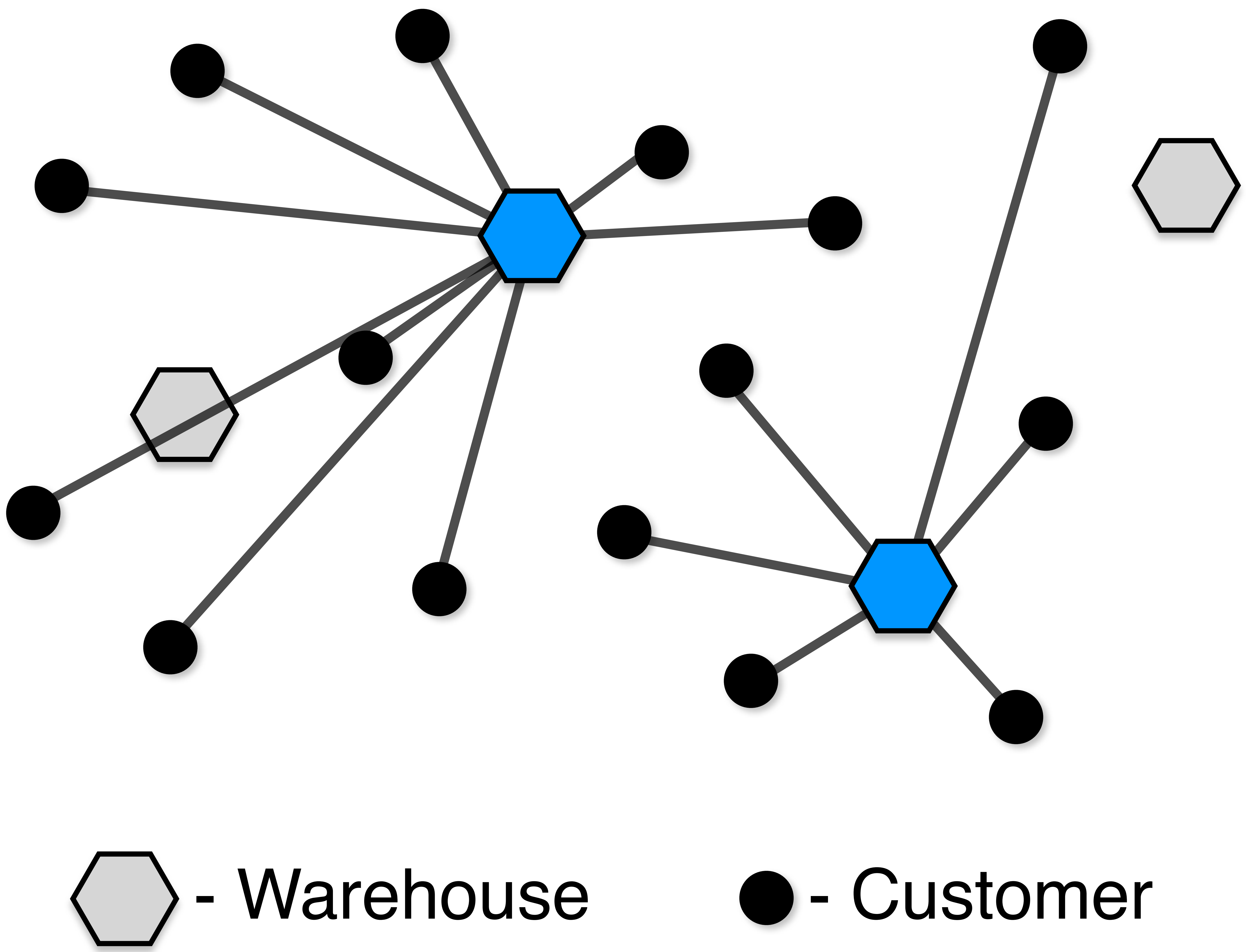

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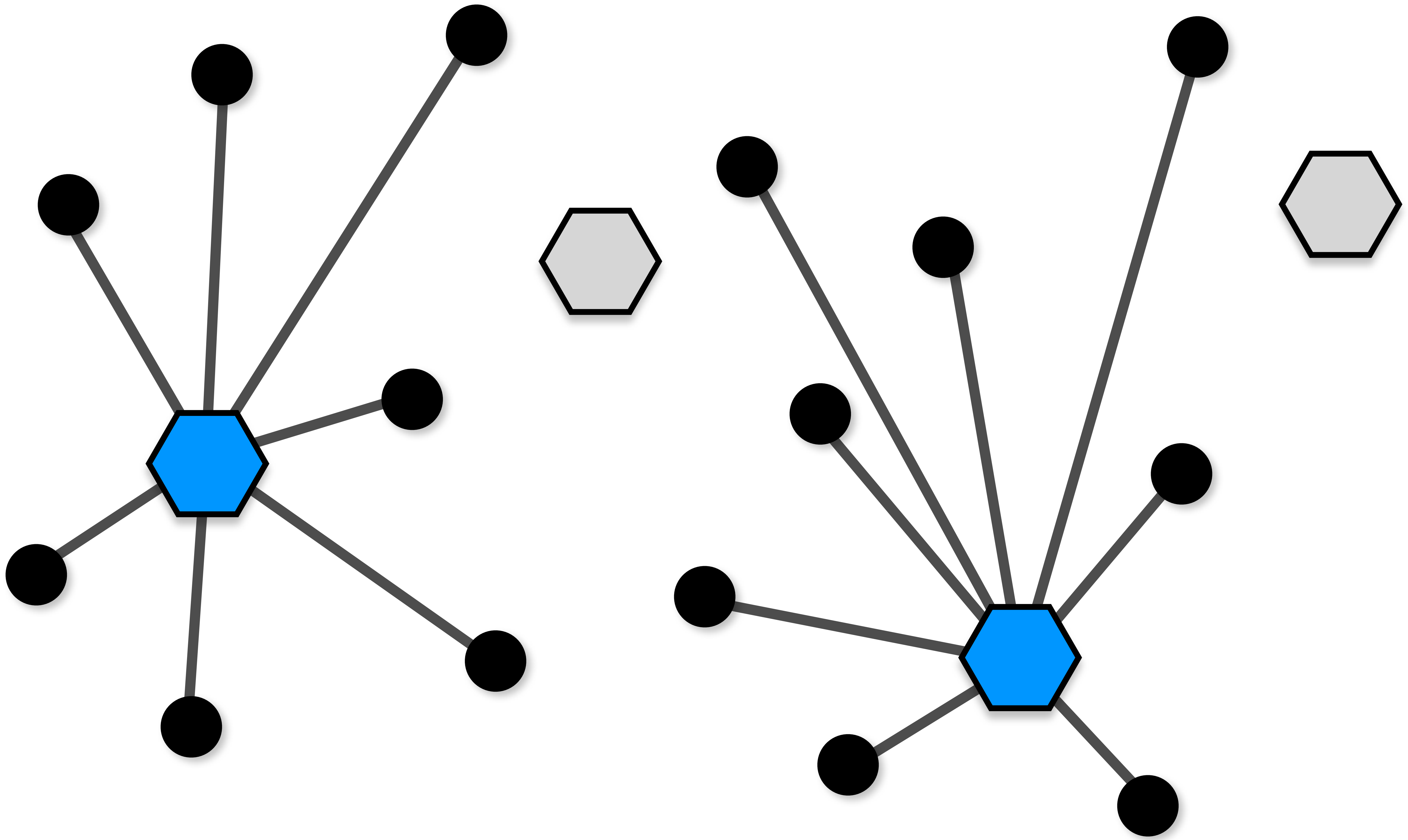

- Warehouse - Customer

Warehouse Location

- Given
  - a set of warehouses W, each warehouse with a fixed cost f_w
  - a set of customers C
  - a transportation cost $t_{w,c}$ from warehouse w to customer c
- Find which warehouses to open to minimize the fixed and transportation costs

Warehouse Location

- What are the decision variables?
  - o_w: whether warehouse w is open (0/1)
  - a[c]: the warehouse assigned to customer c

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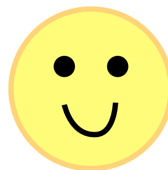
  - no constraints 

Warehouse Location

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  - no constraints
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$$\text{minimize} \sum_{w \in W} f_w o_w + \sum_{c \in C} t_{a[c],c}$$

Warehouse Location

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  - once the warehouse locations have been chosen, the problem is easy
  - it suffices to assign a customer to the open warehouse minimizing its transportation cost

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- What is the objective?

$$\text{minimize} \sum_{w \in W} f_w o_w + \sum_{c \in C} \min_{w \in W : o_w = 1} t_{w,c}$$

Warehouse Location

- Neighborhood
  - many possibilities

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  - open and close warehouses
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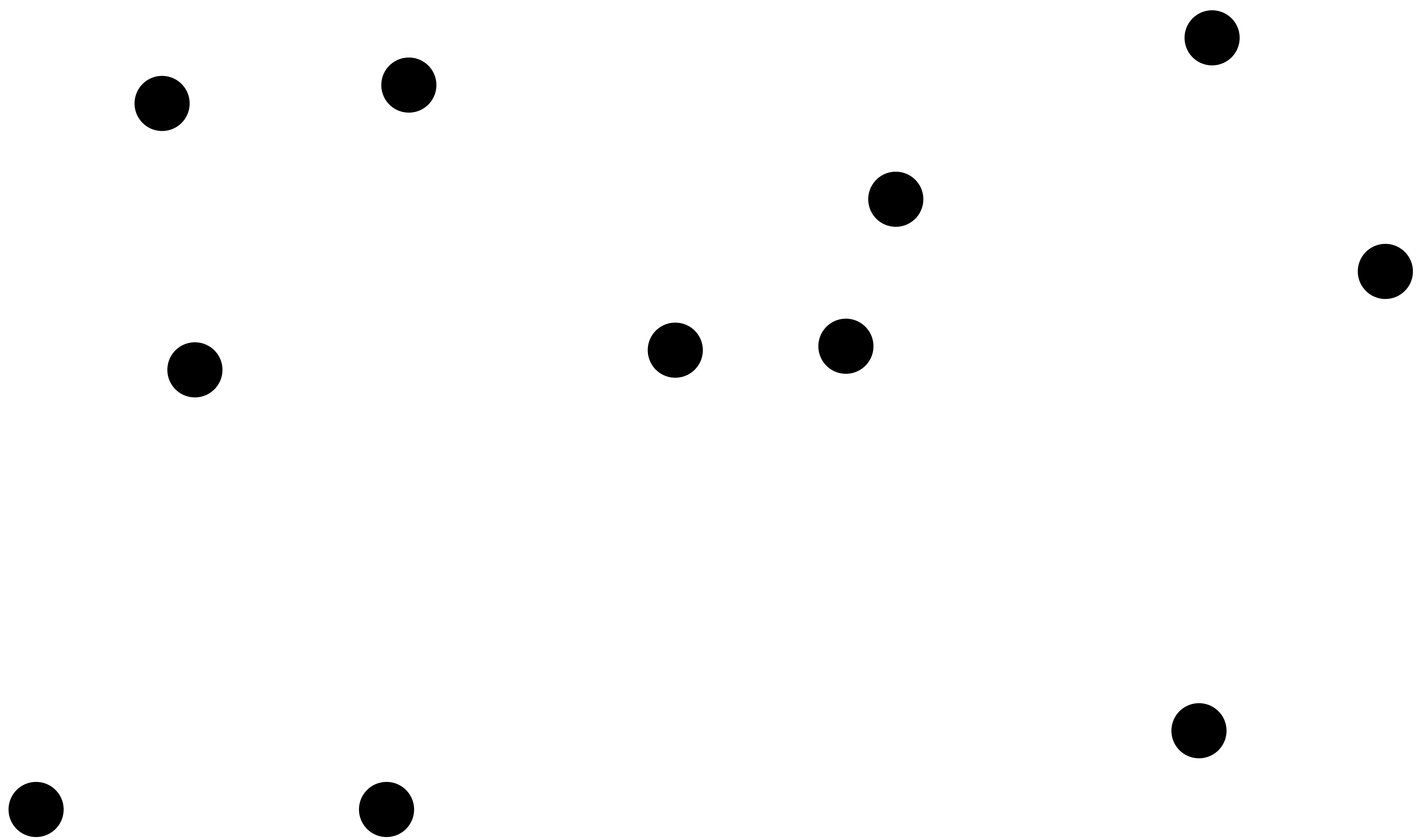
- Simplest neighborhood
  - open and close warehouses
  - that is, flip the value of o_w
- Union of neighborhoods
  - open and close a warehouse
  - swap two warehouses
    - close one and open the other

Traveling Salesman Problem



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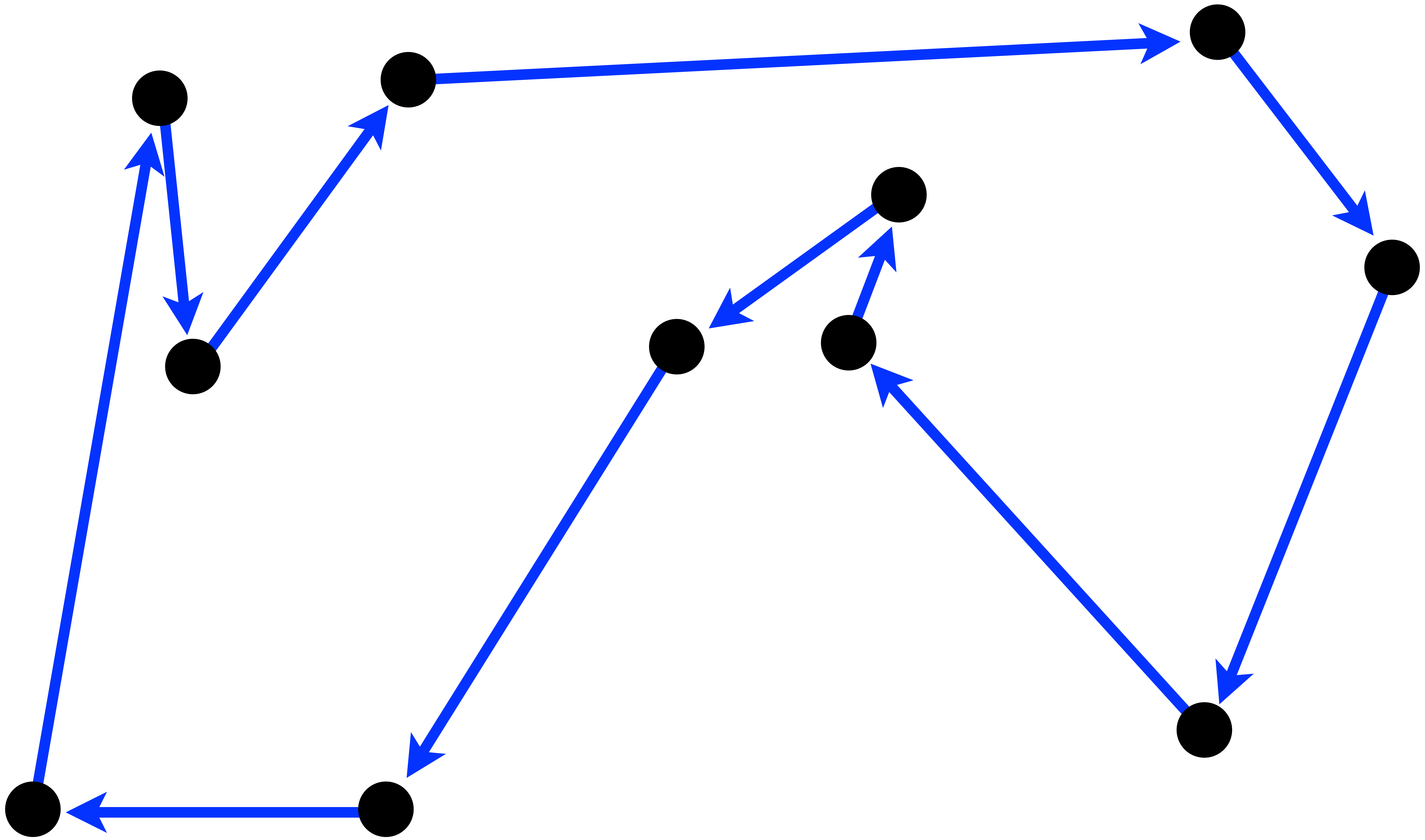

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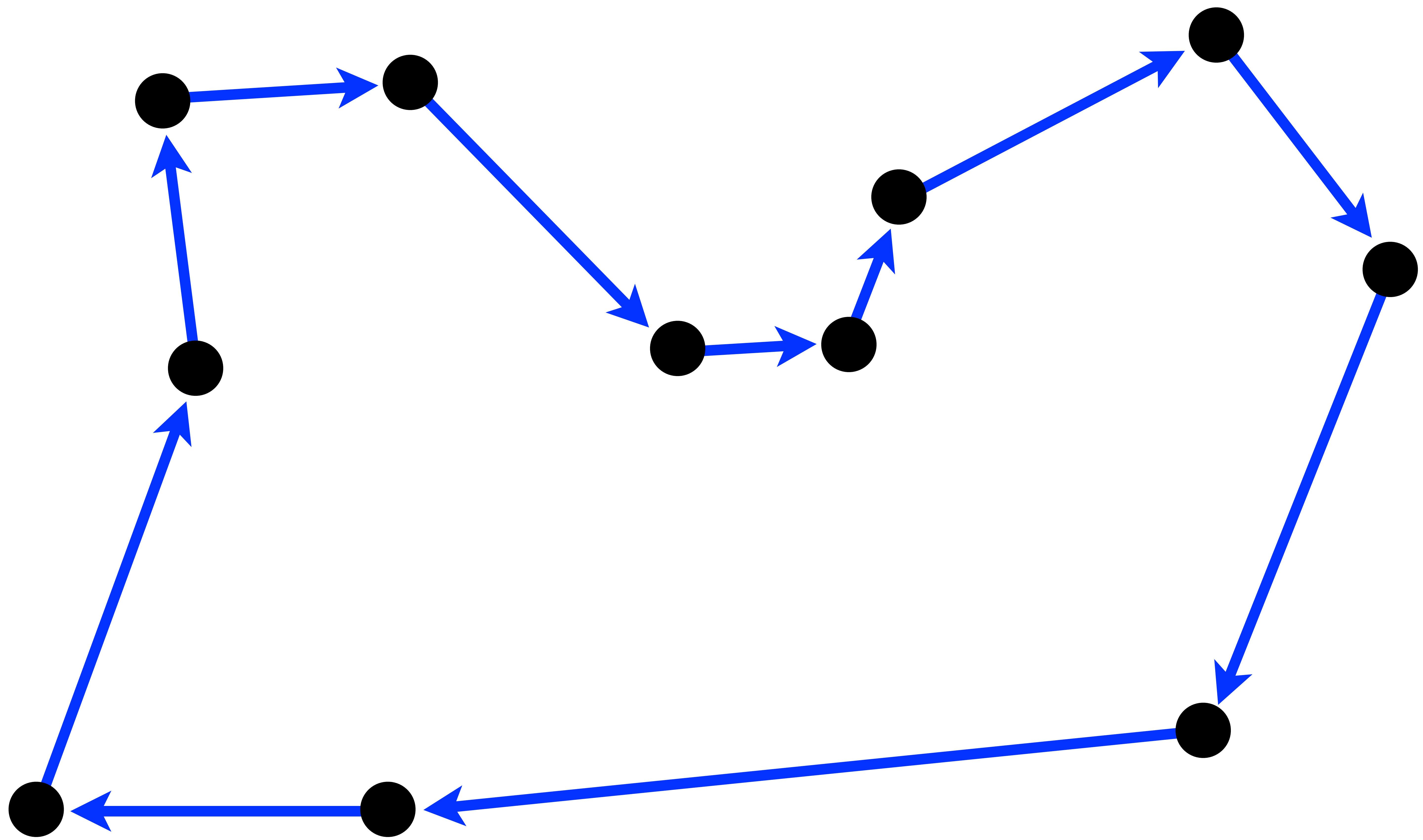

Traveling Salesman Problem

- Given
  - a set C of cities to visit
  - a symmetric distance matrix d between every two cities

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  - a set C of cities to visit
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- Find
  - a tour of minimal cost visiting each city exactly once
- The traveling salesman problem (TSP) is probably the most studied combinatorial problem

Traveling Salesman Problem

- Decision variables
  – like in the Euler tour
  – specify where to go next for every city

```
range Cities = 1..n;
int distance[Cities,Cities] = ...;
var{int} next[Cities] in Cities;
minimize
    sum(c in Cities) d[c,next[c]]
subject to
    circuit(next);
```

Local Search for the TSP

- 2-OPT neighborhood for the TSP
  - stay feasible, that is always maintain a tour
  - select two edges and replace them by two other edges

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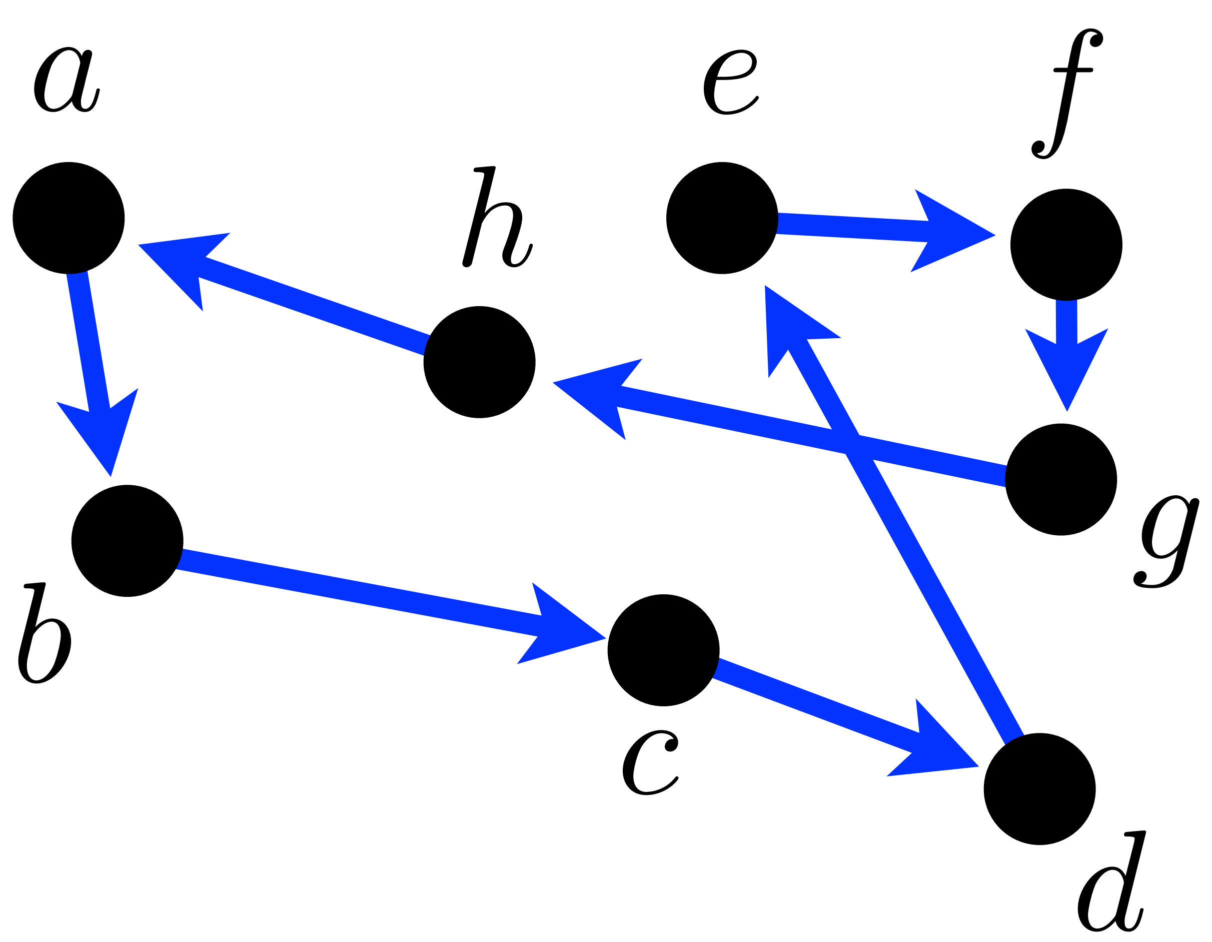

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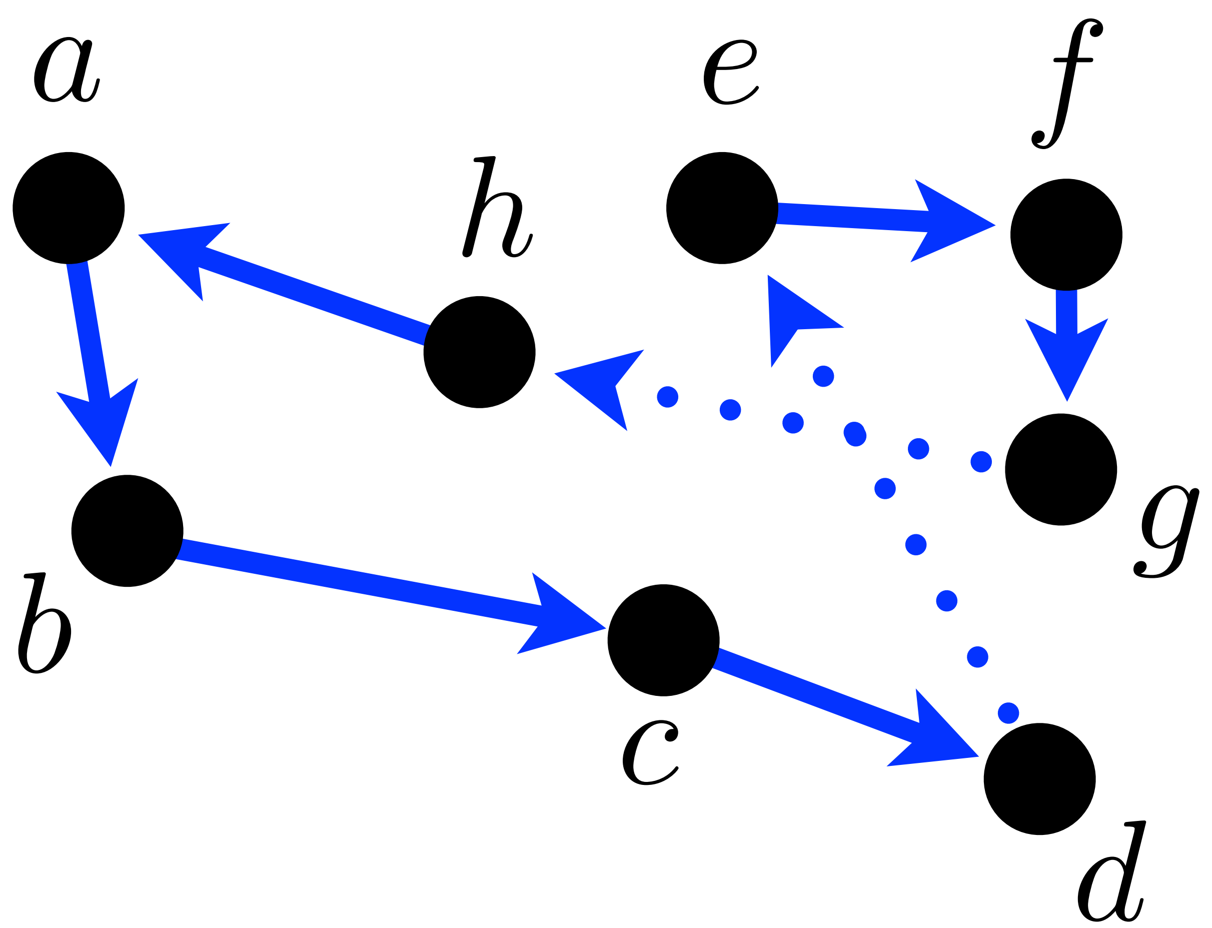

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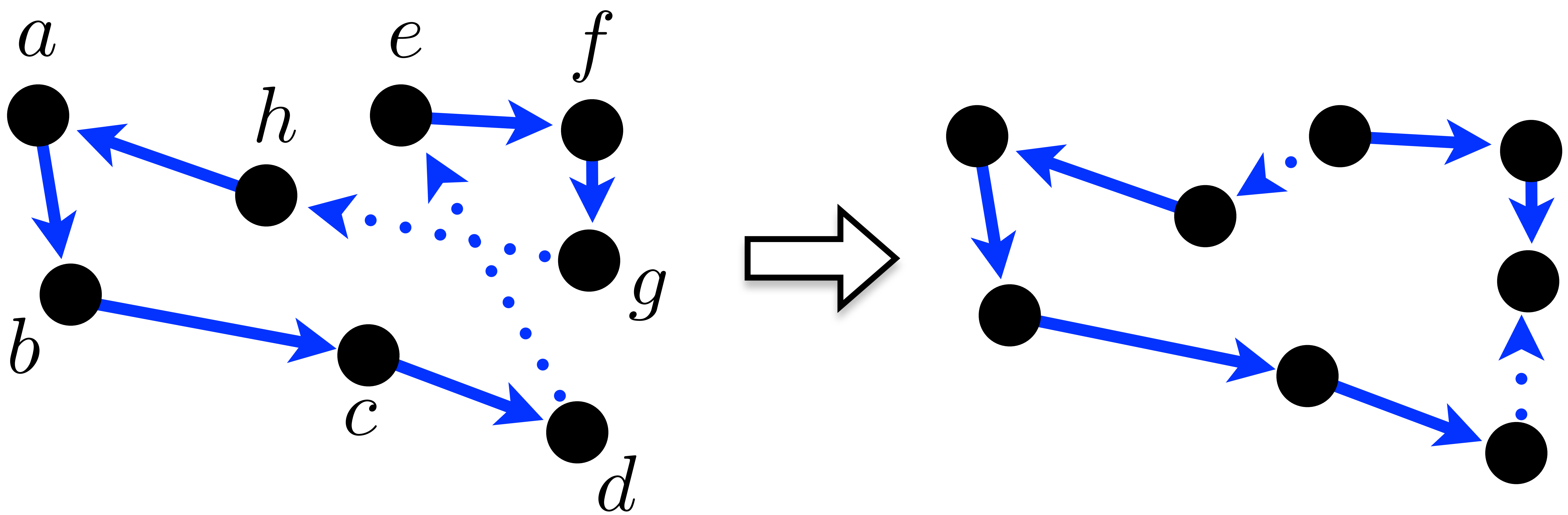

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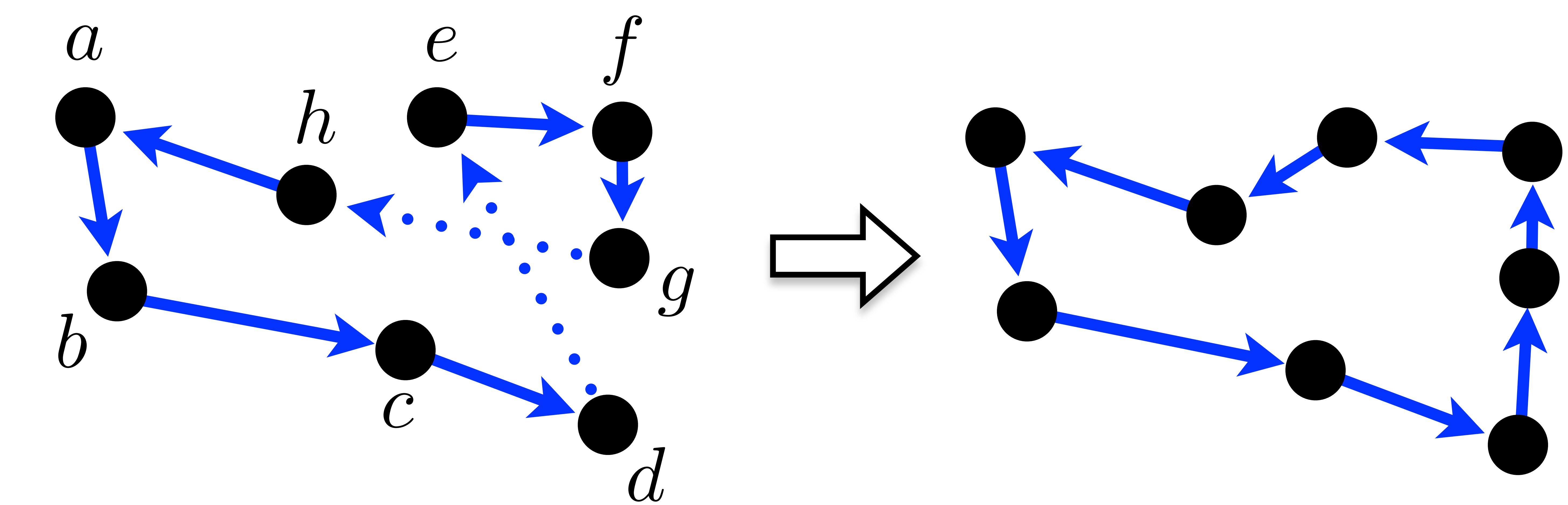

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  - the neighborhood is the set of all tours that can be reached by swapping two edges
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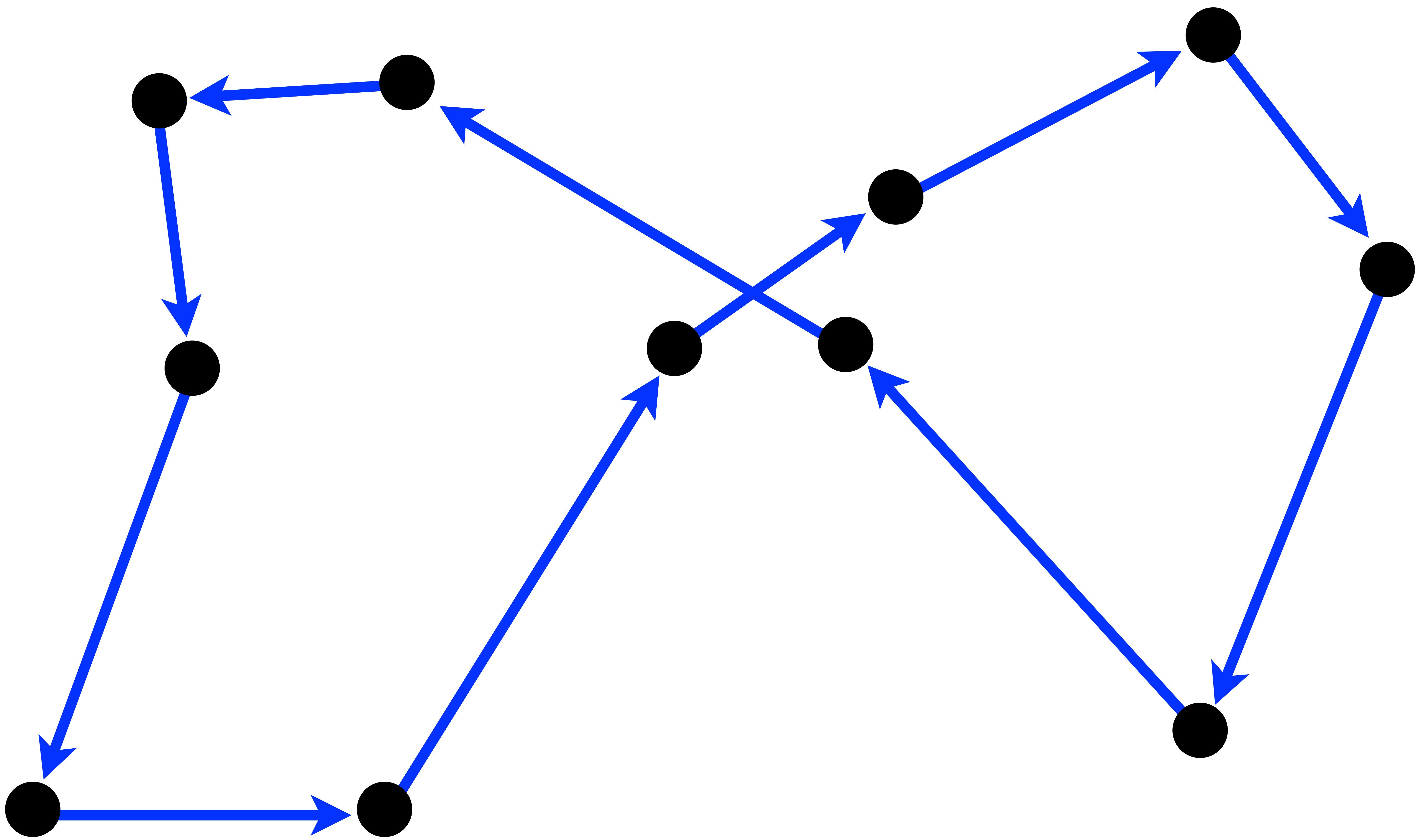

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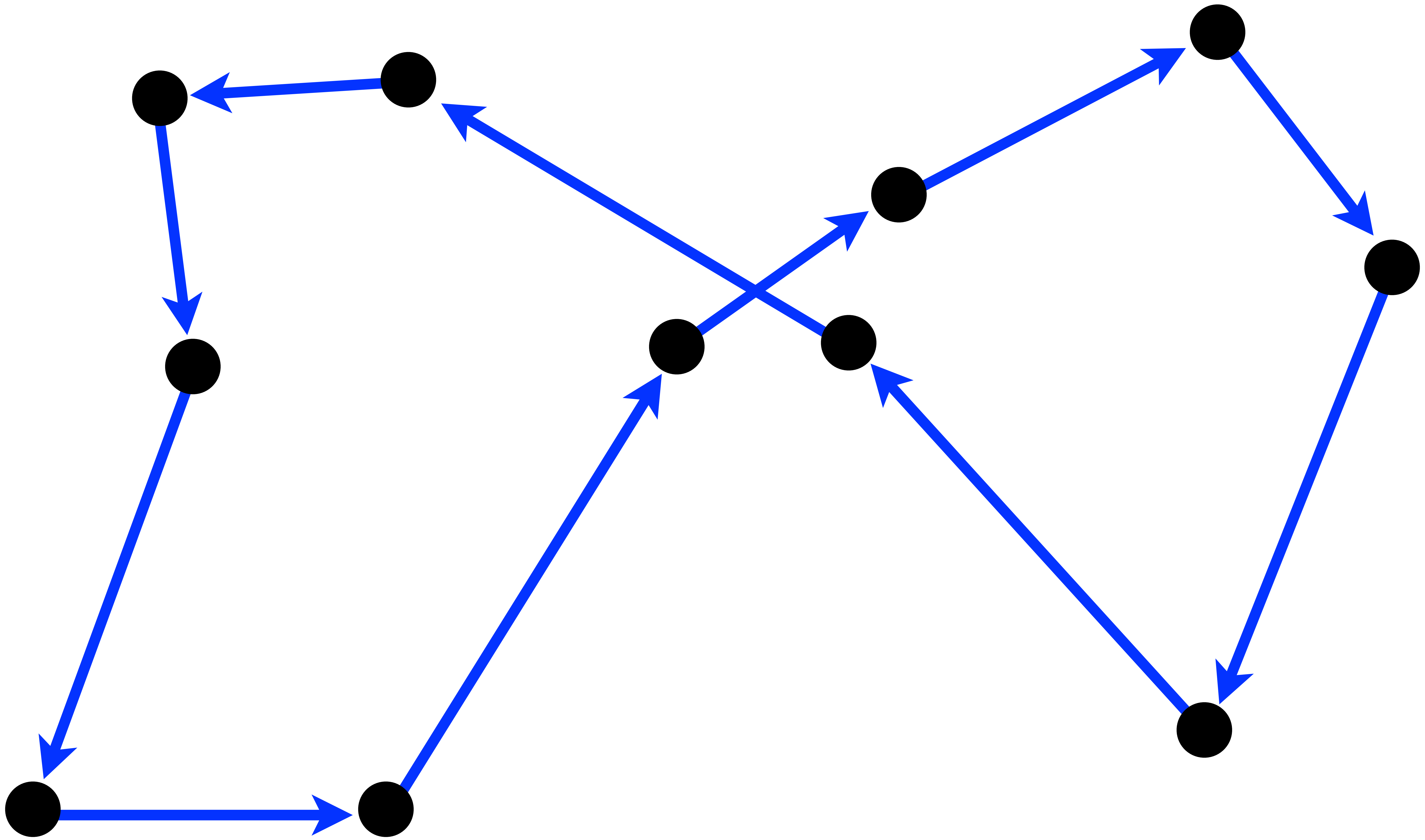

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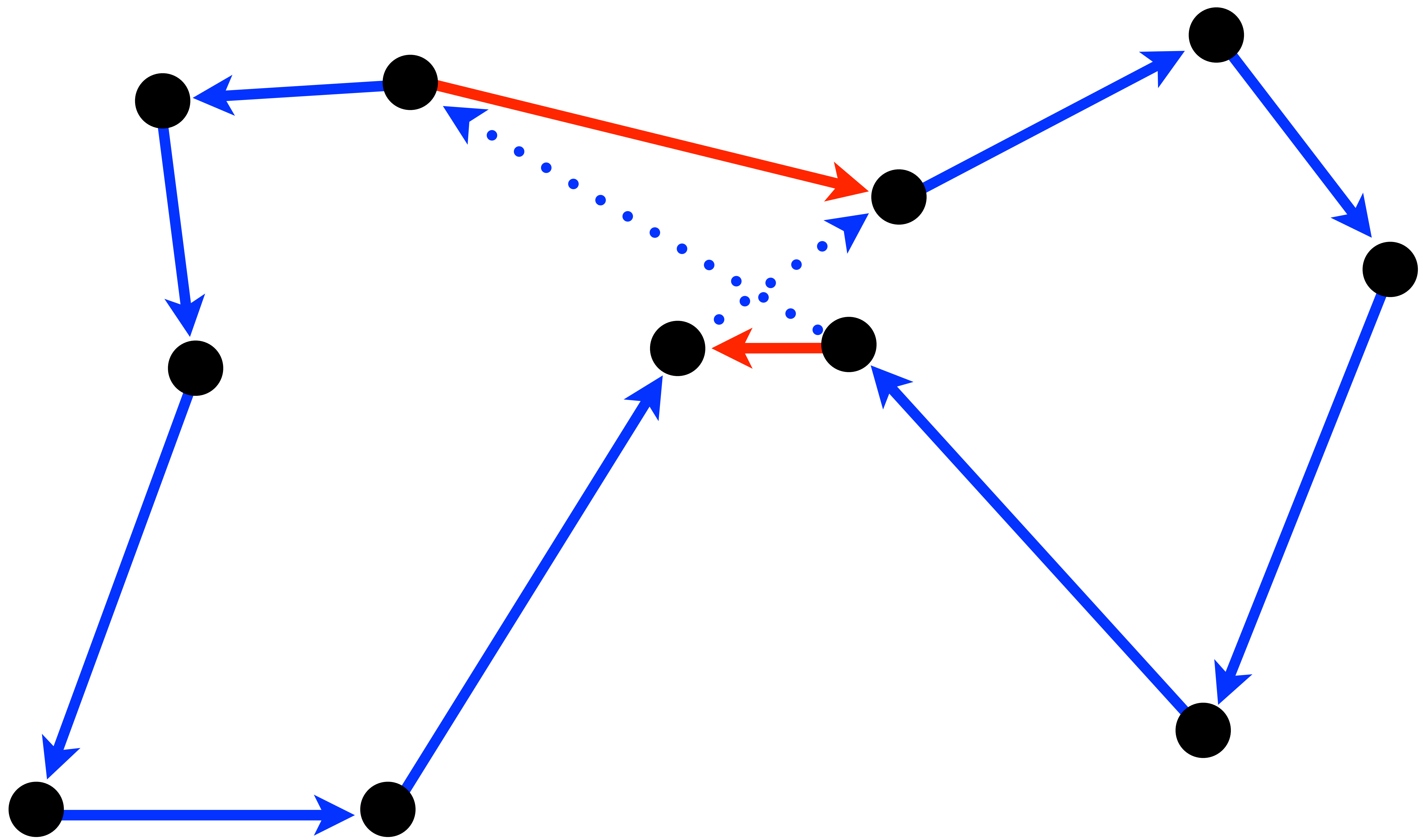

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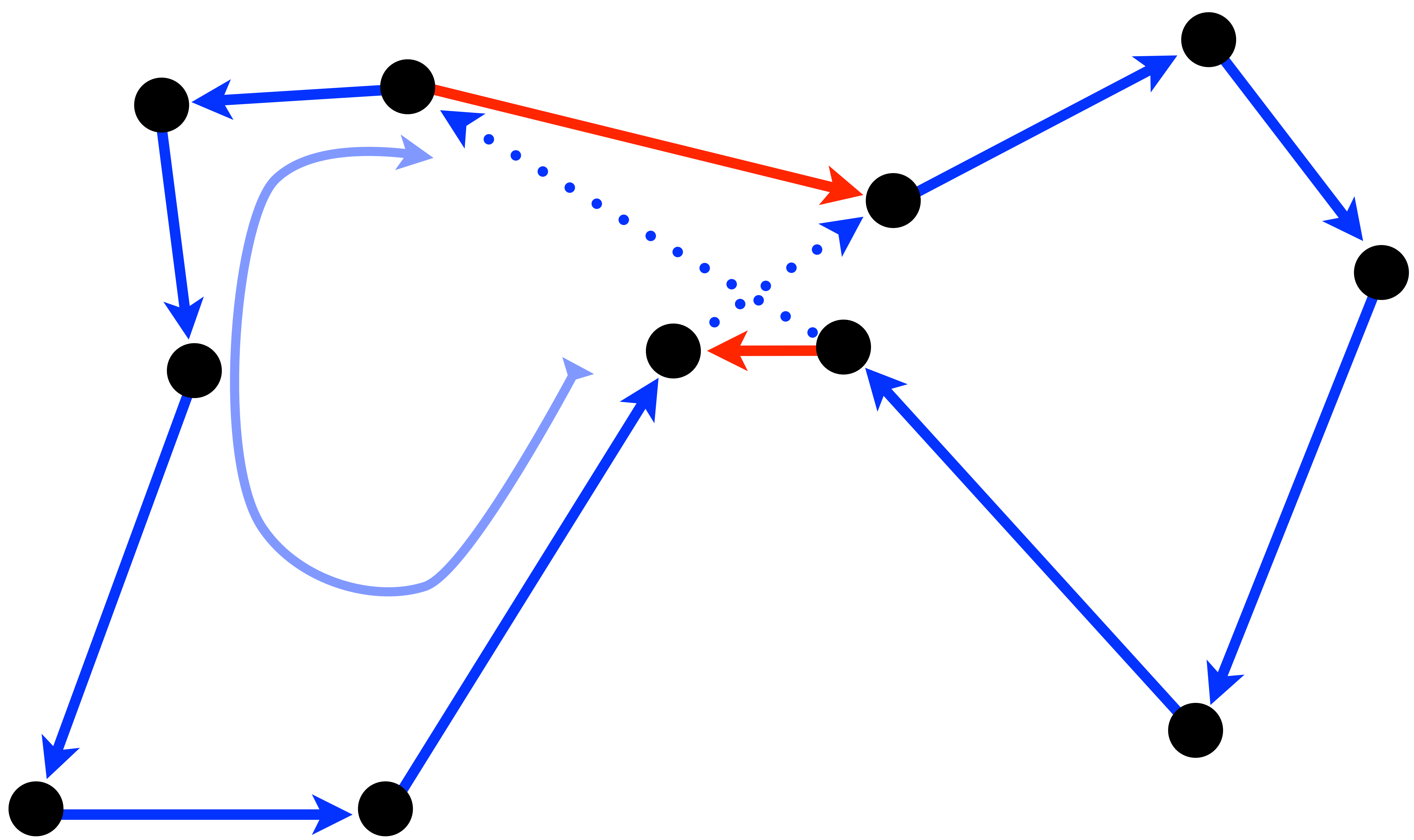

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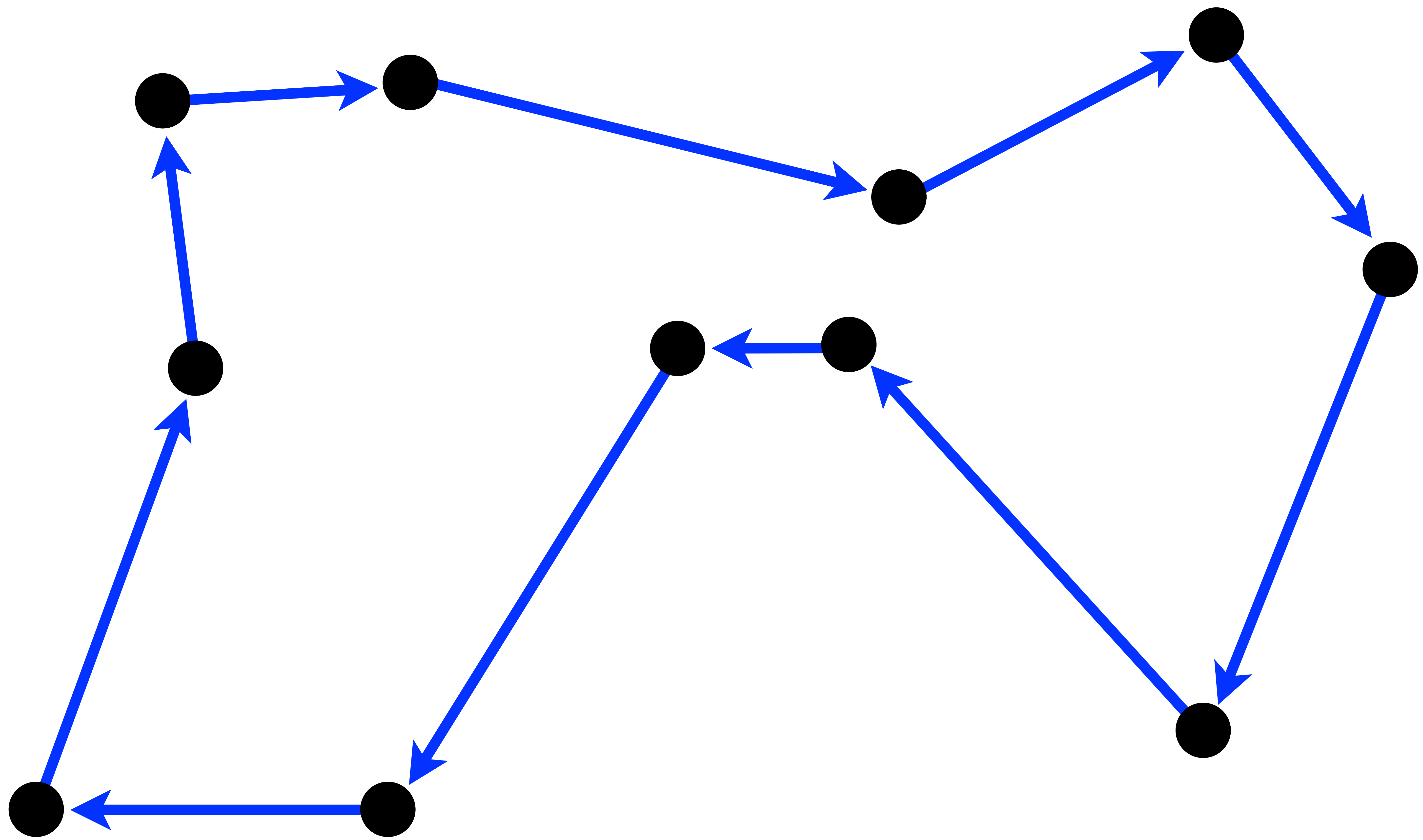

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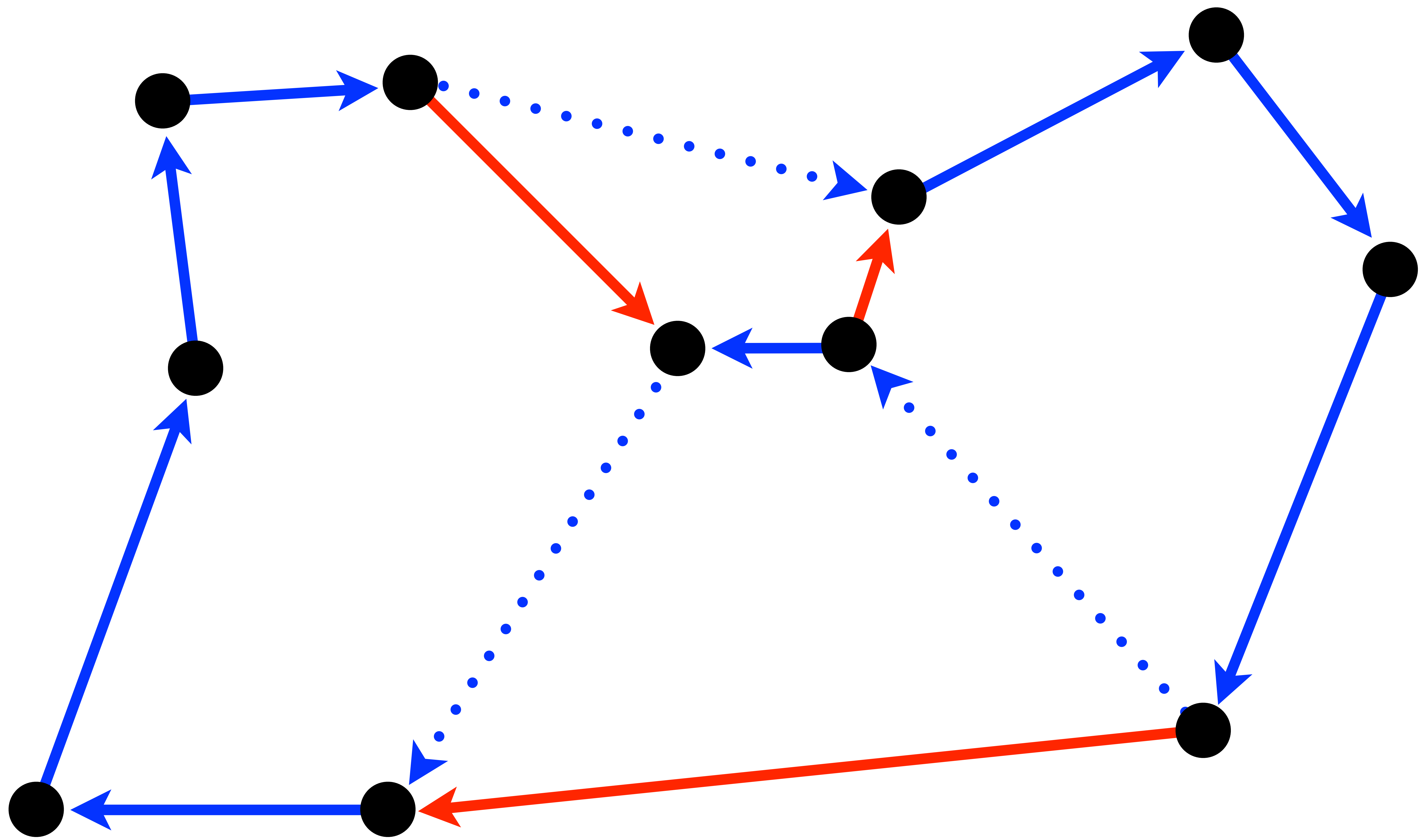

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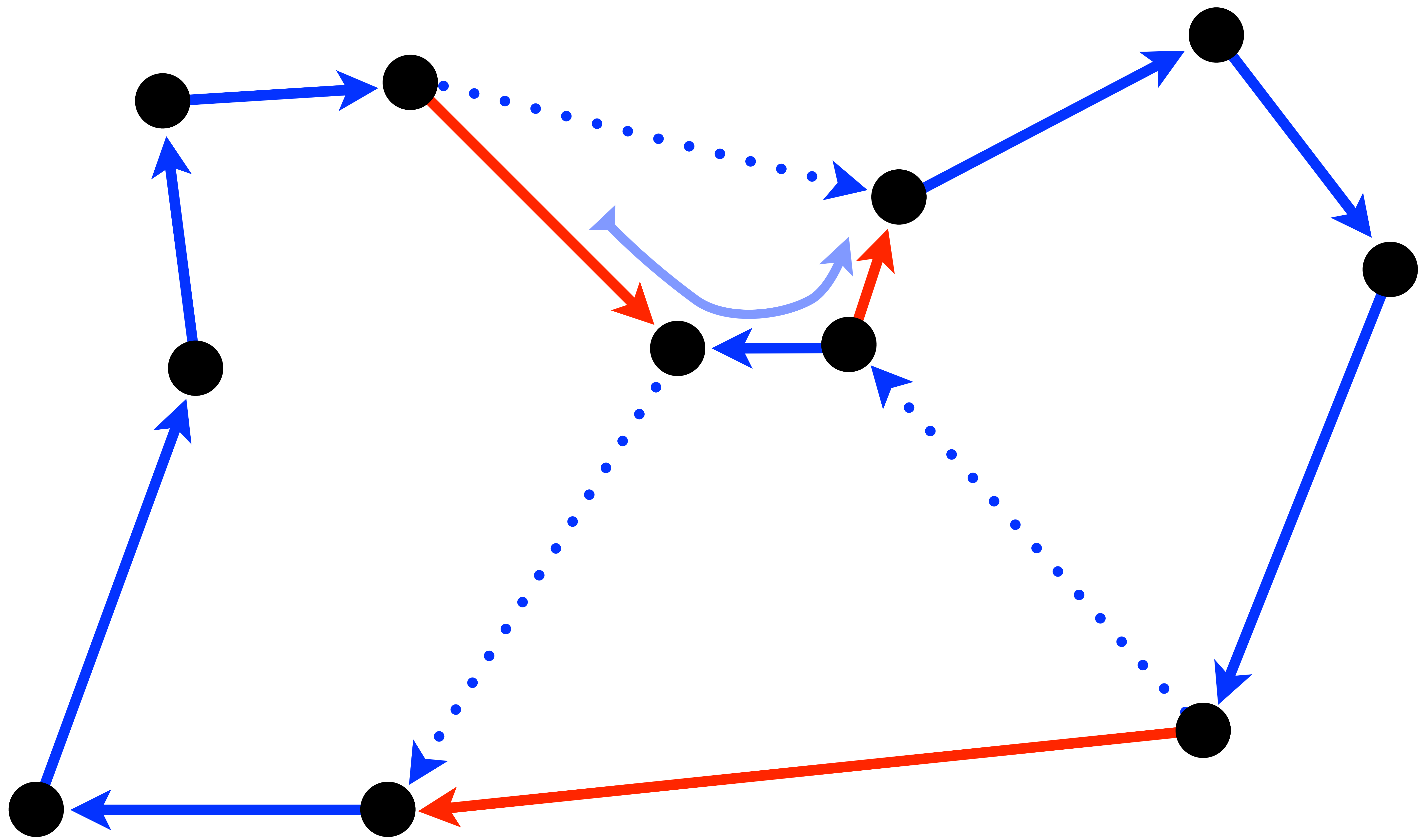

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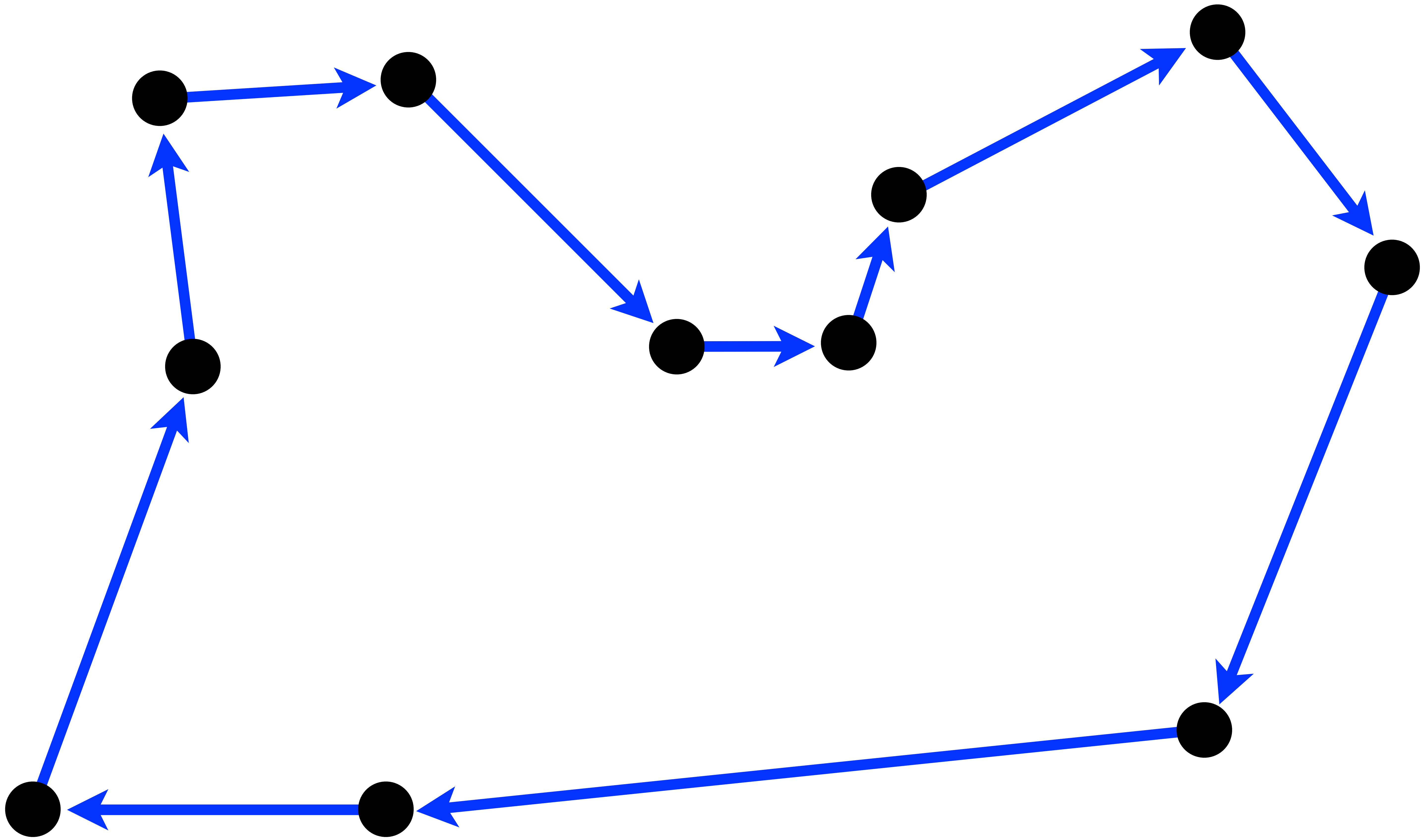

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  - much better than 2-OPT in quality but more expensive
- 4-OPT
  - often marginally better but much more expensive

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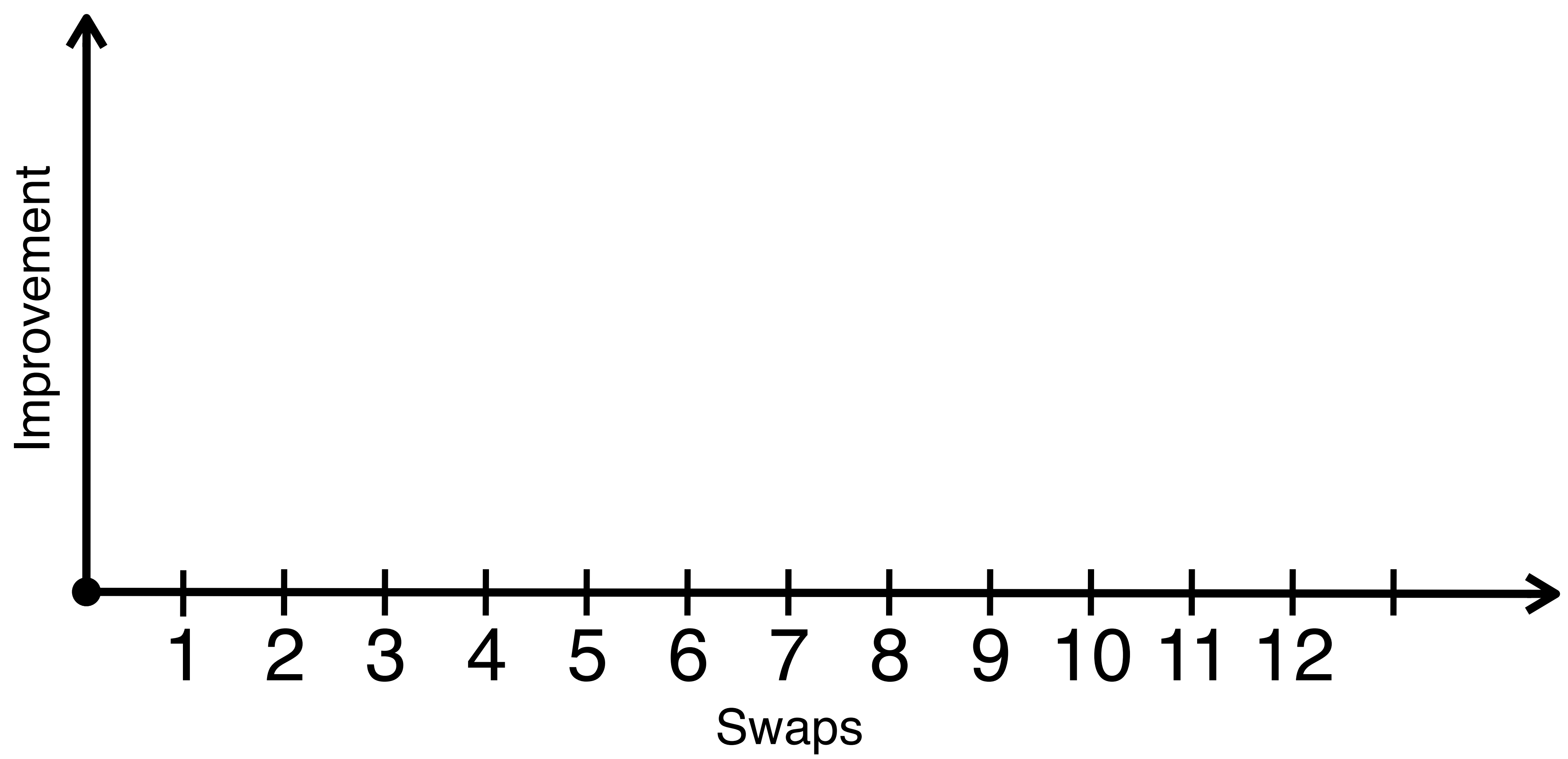

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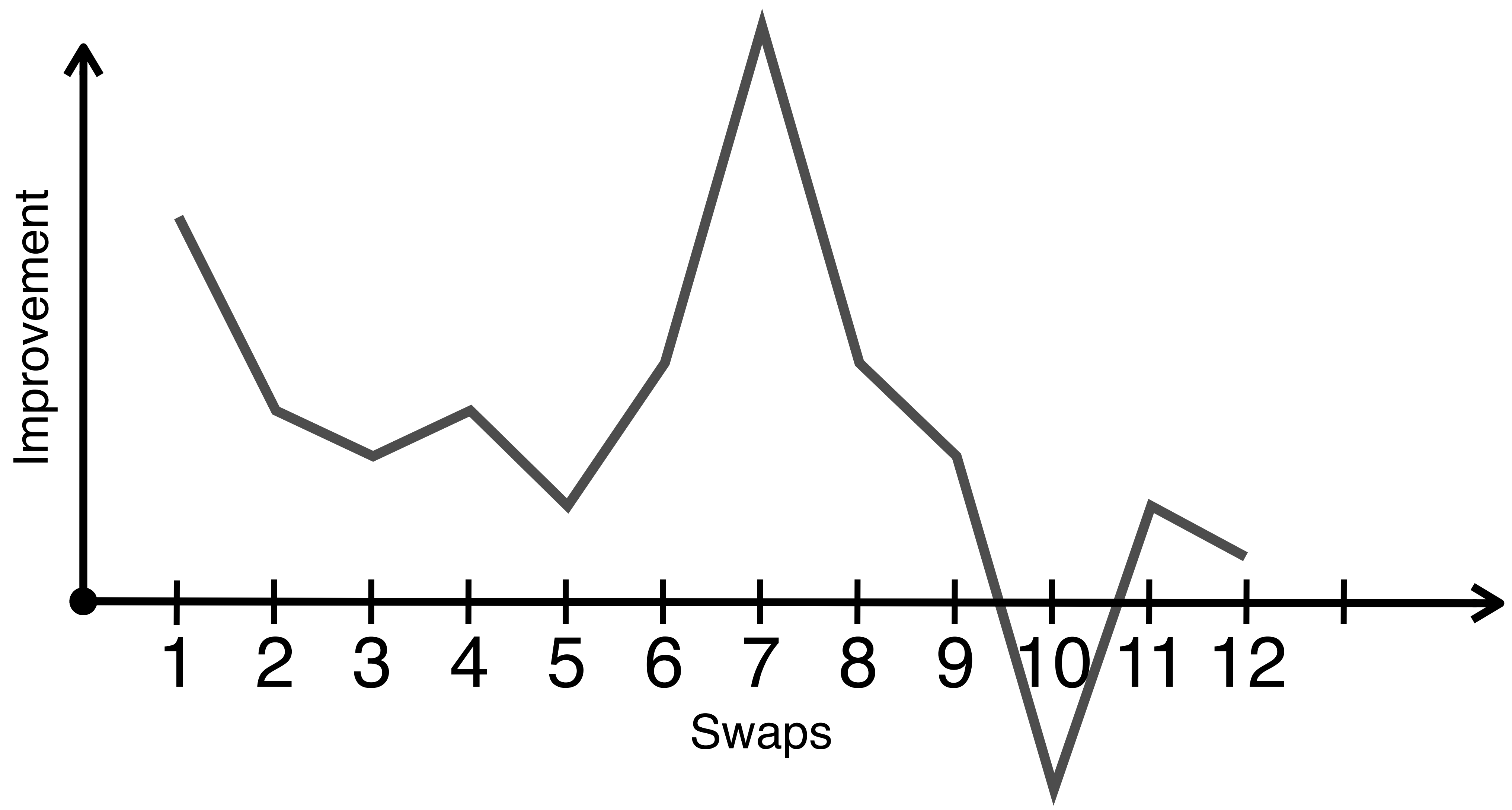

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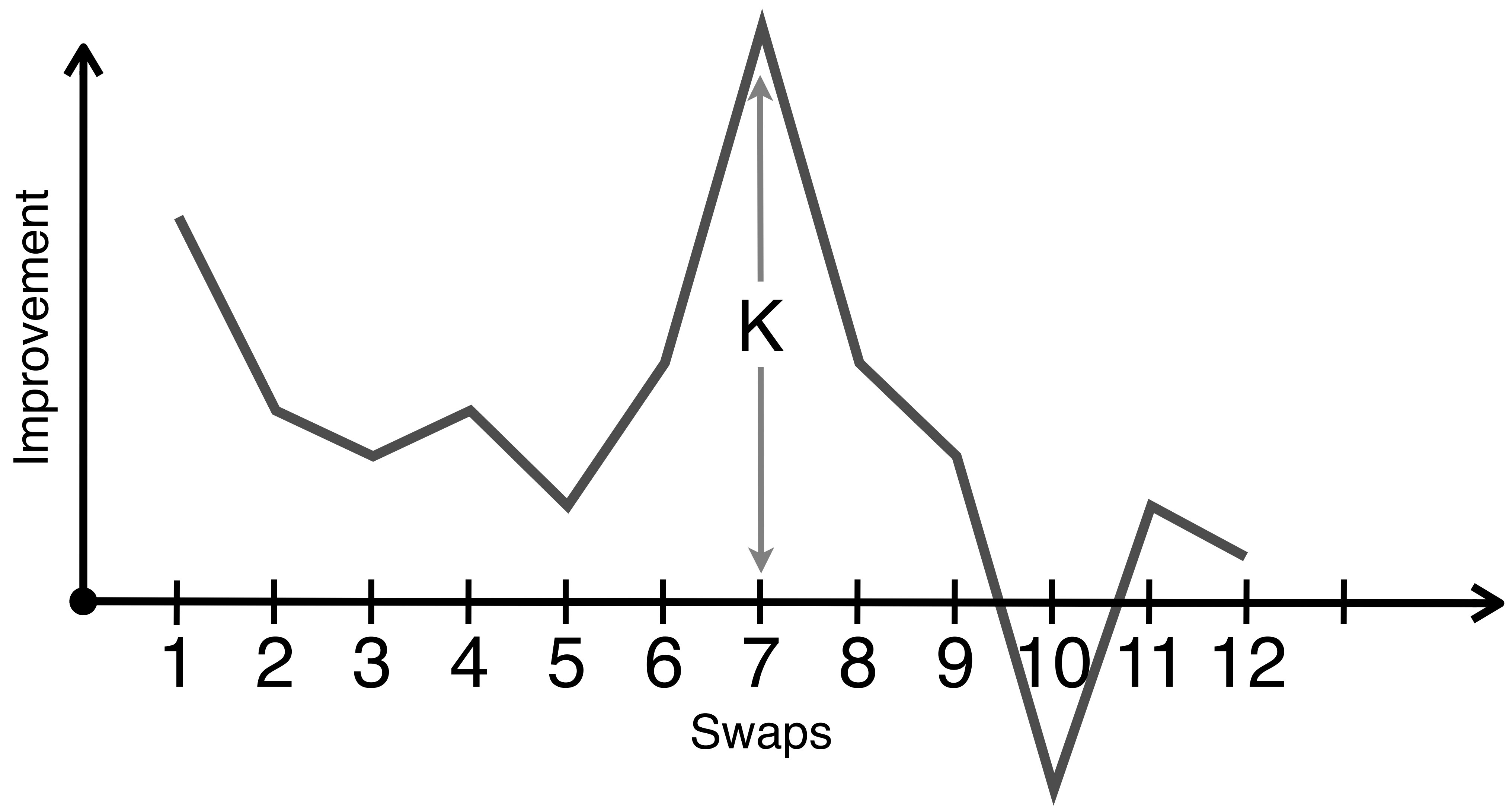
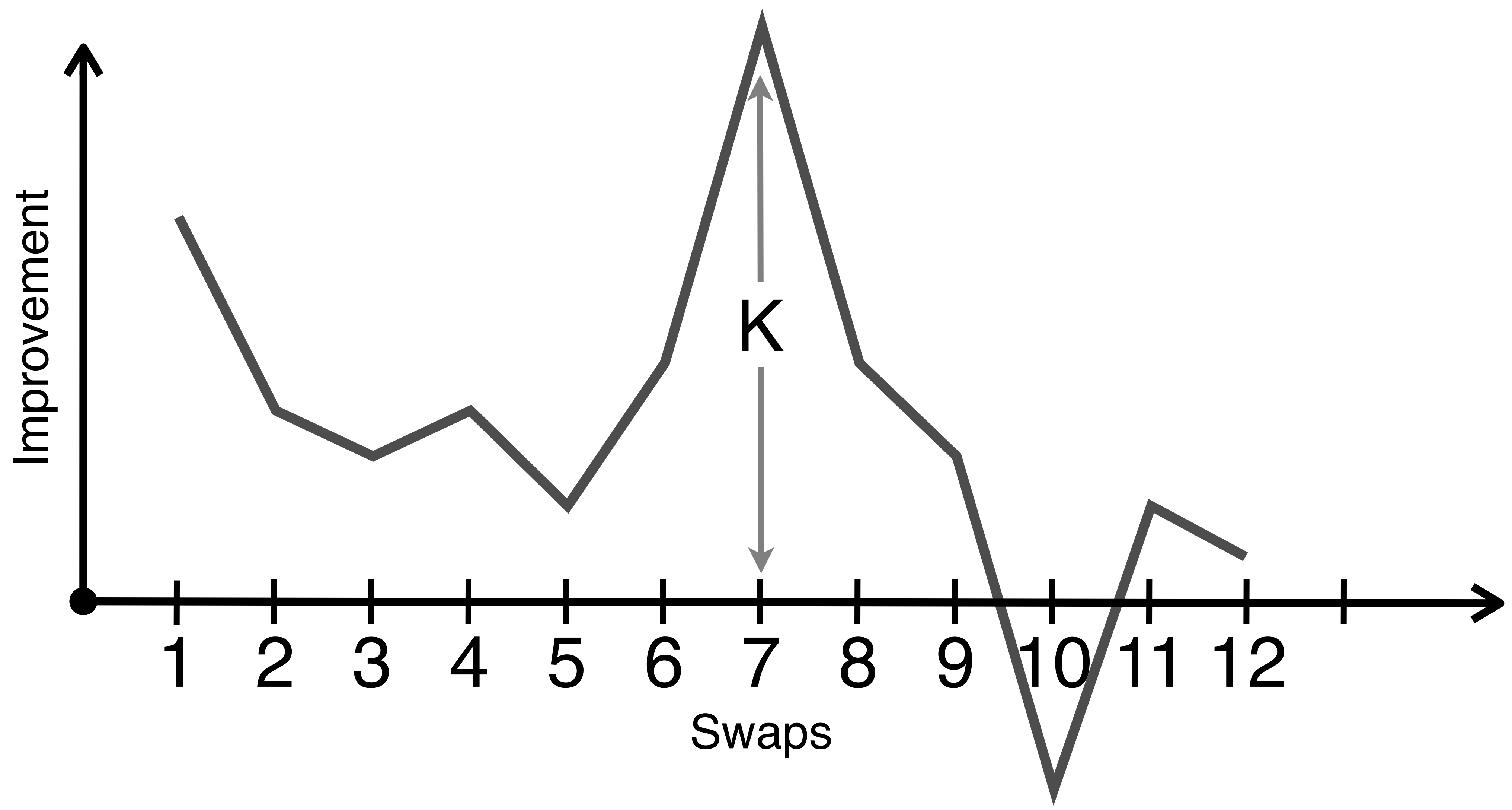


K-OPT

- K-OPT
  - find a good k dynamically at a fraction of the cost
  - explore a sequence of swaps of increasing sizes



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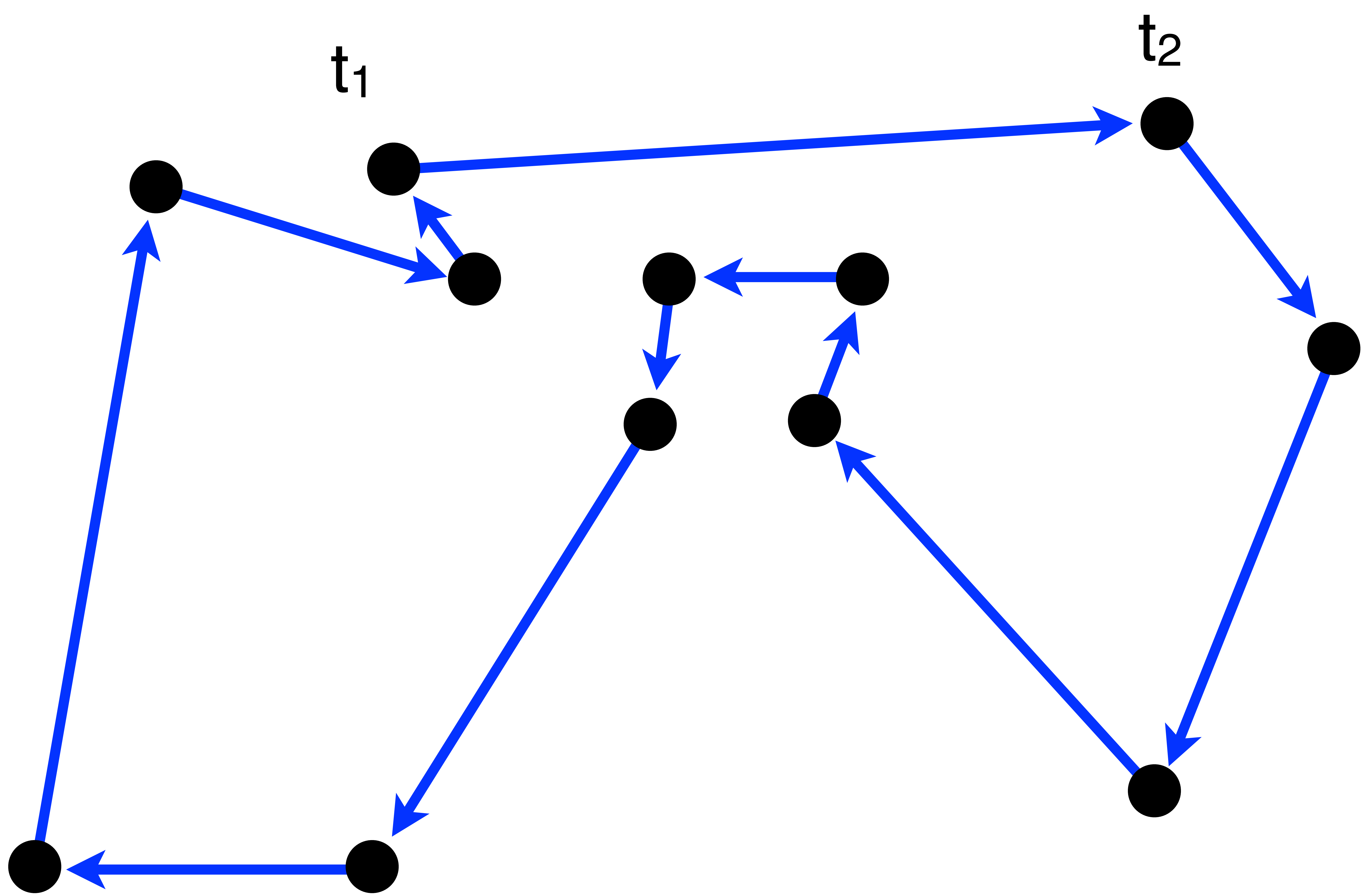

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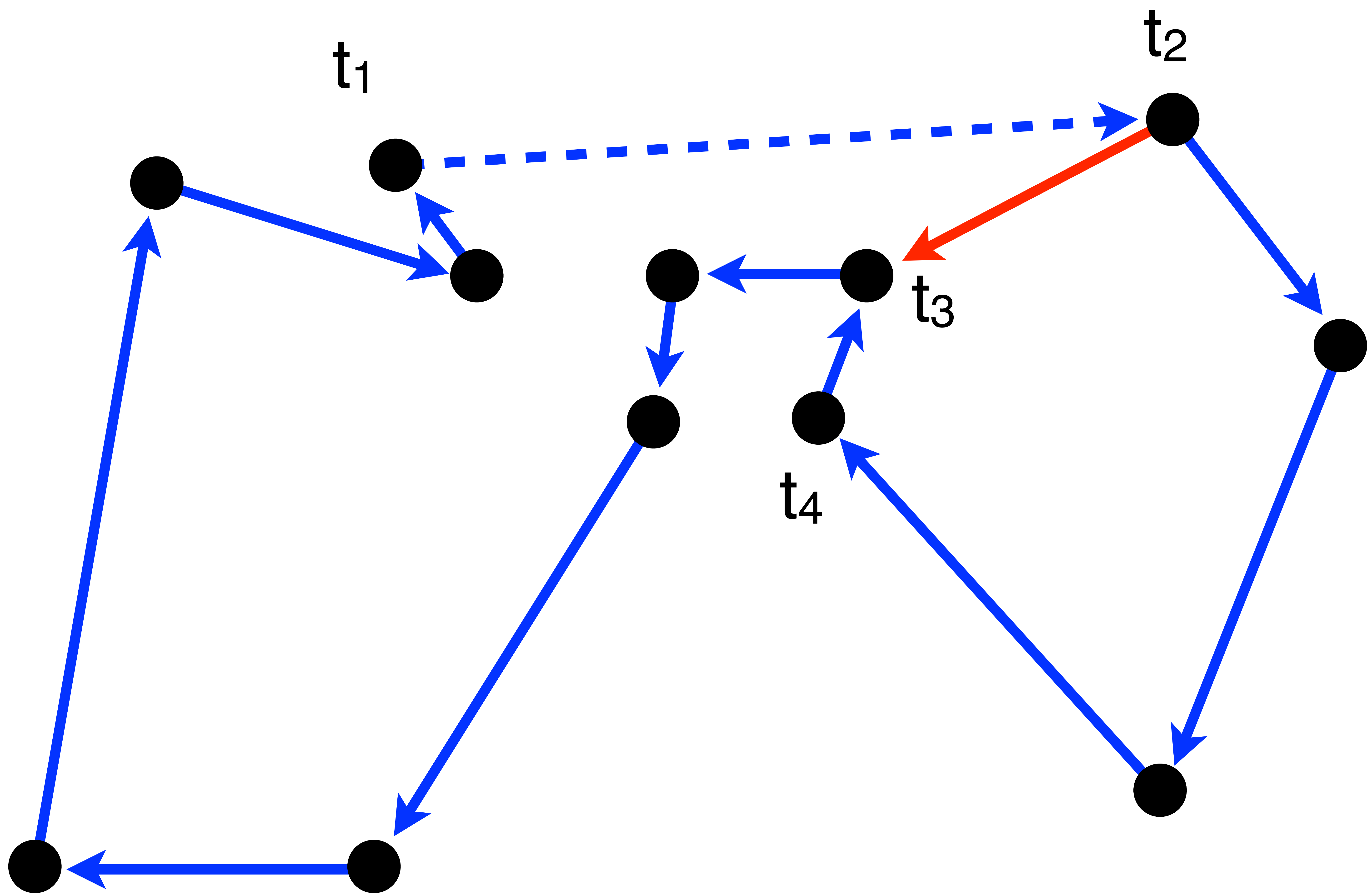

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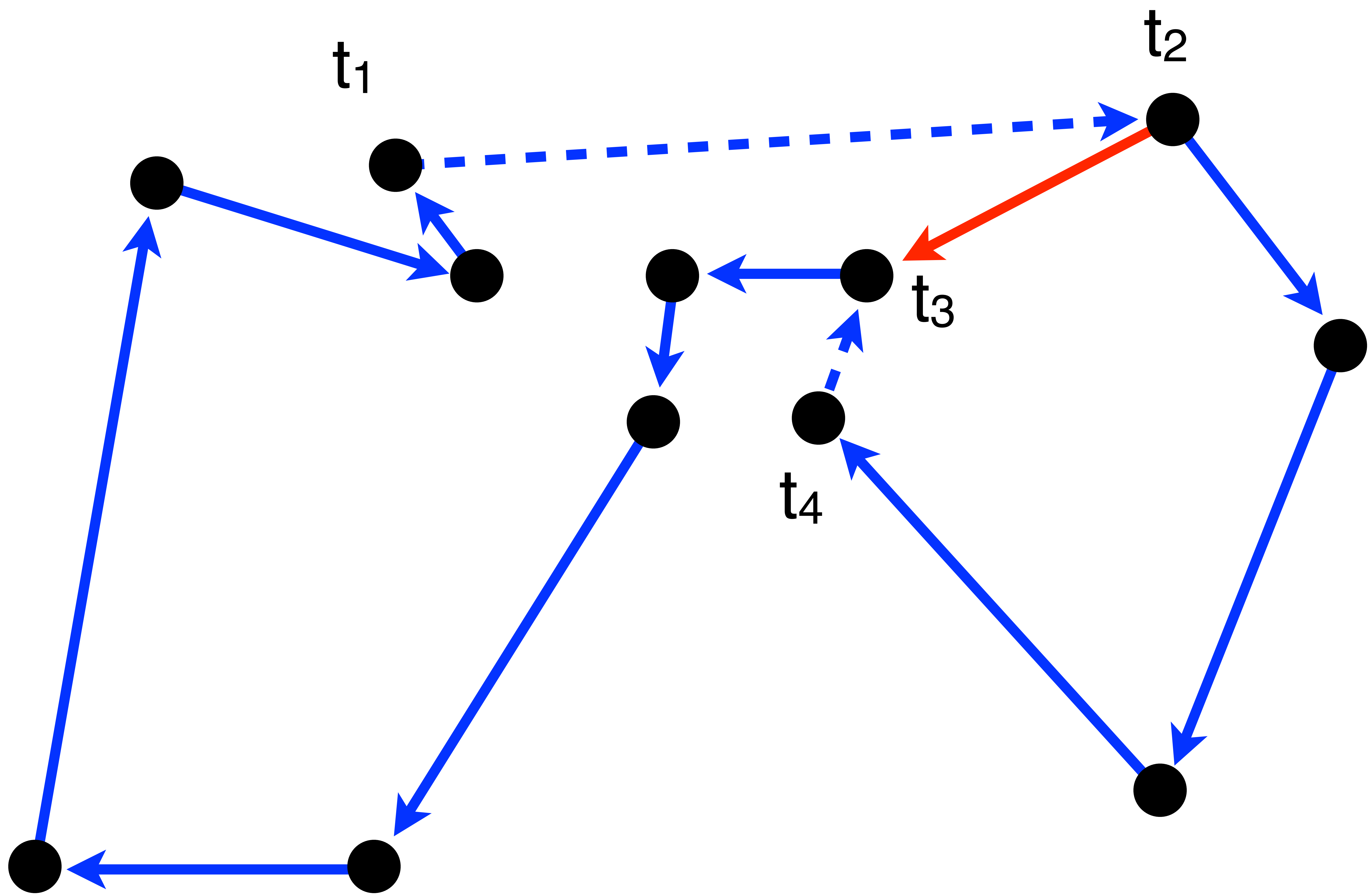

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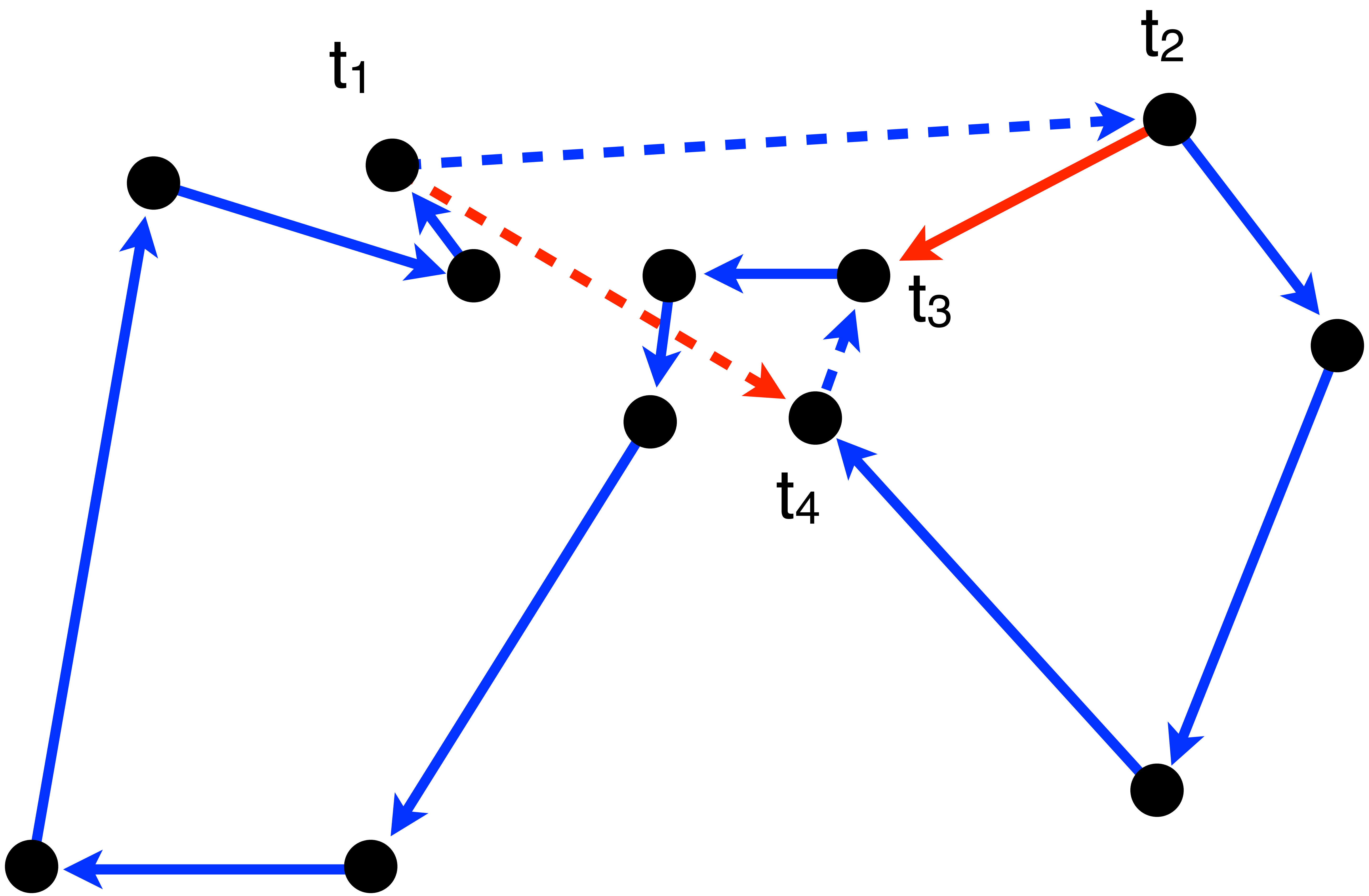

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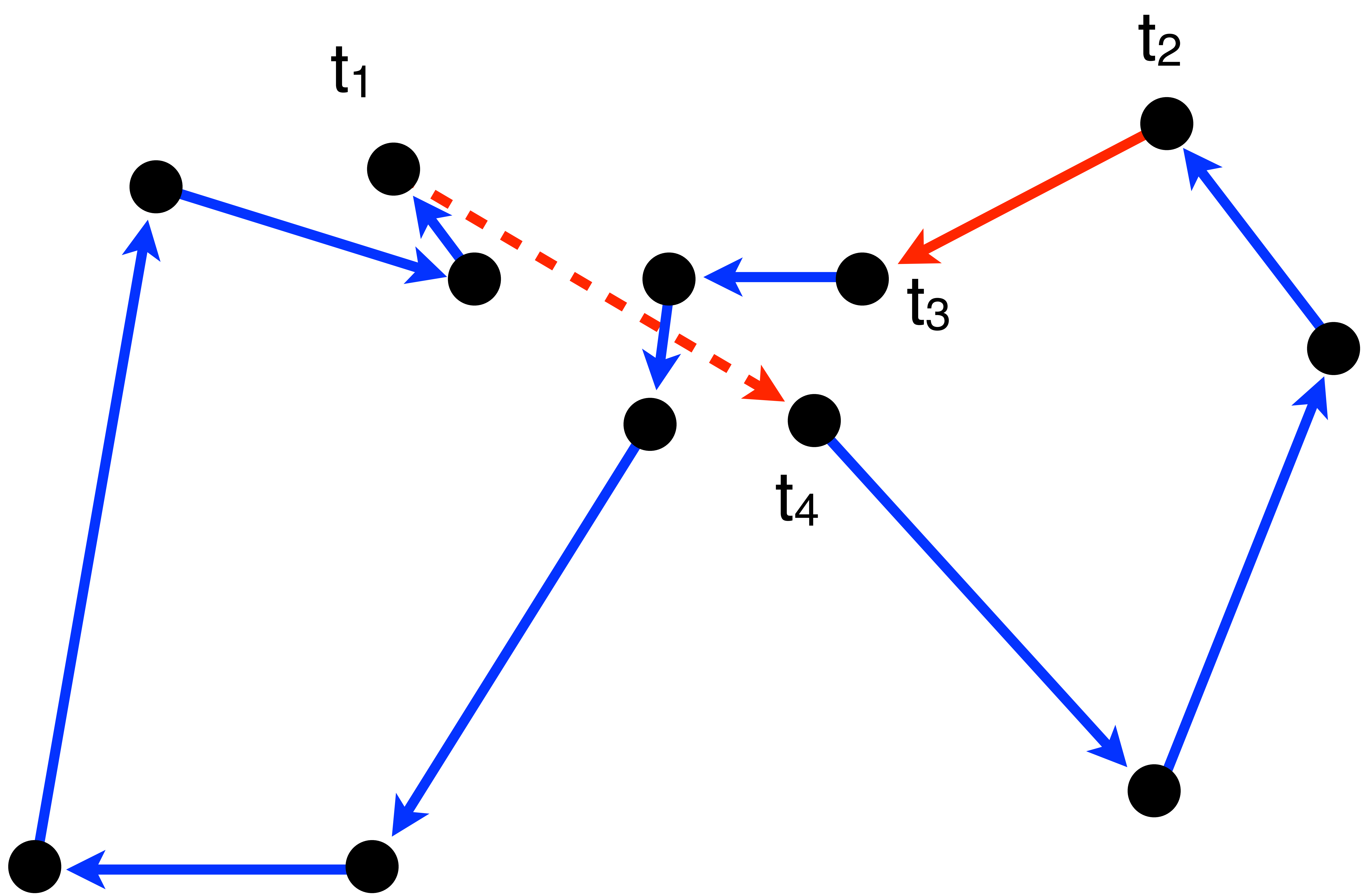

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  - else we have a solution by removing the edge (t_4,t_3) and connecting (t_1,t_4)
  - compute the cost but do not connect
  - instead restart with t_1 and its (pretended) edge (t_1,t_4)

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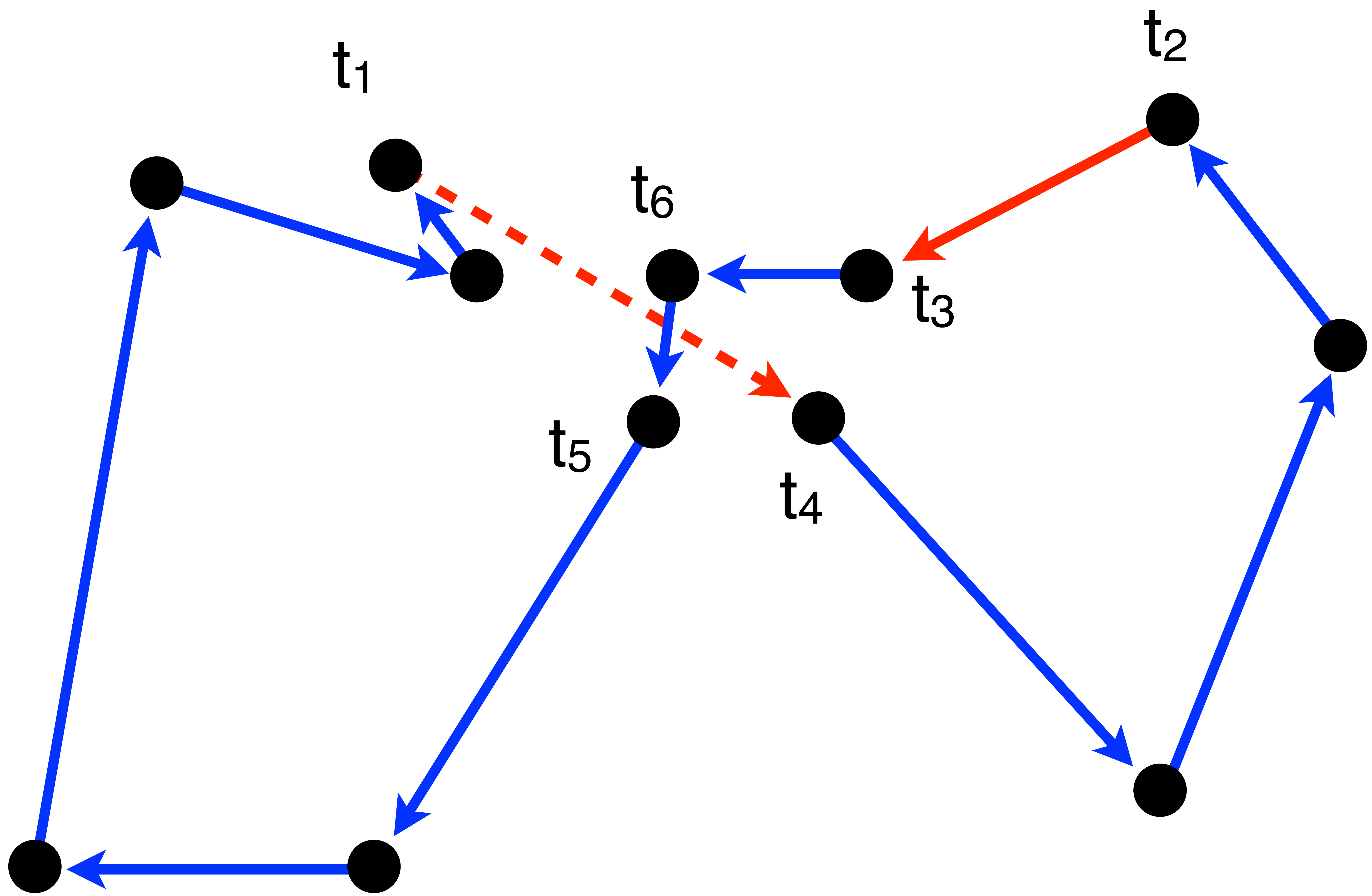

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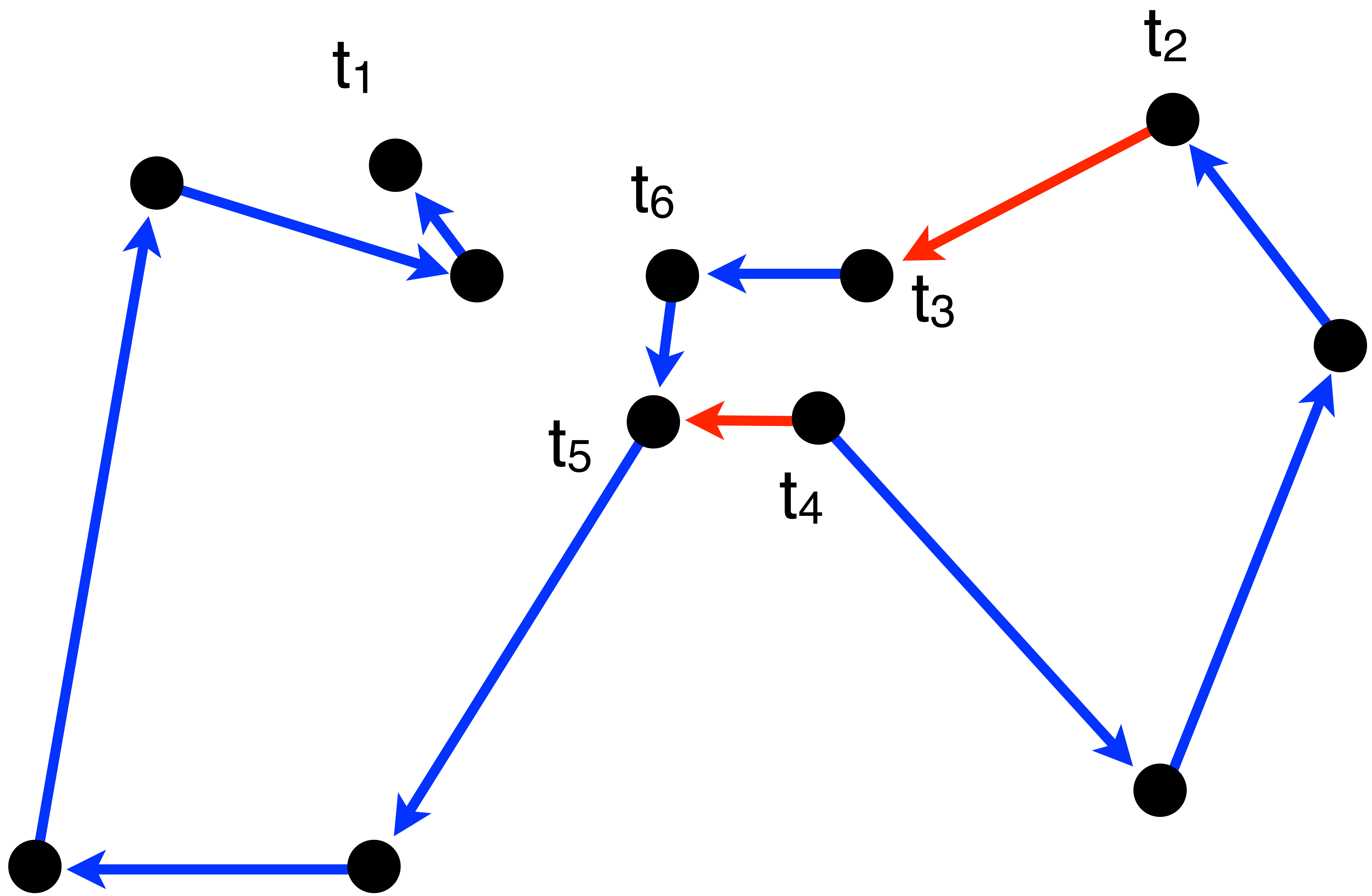

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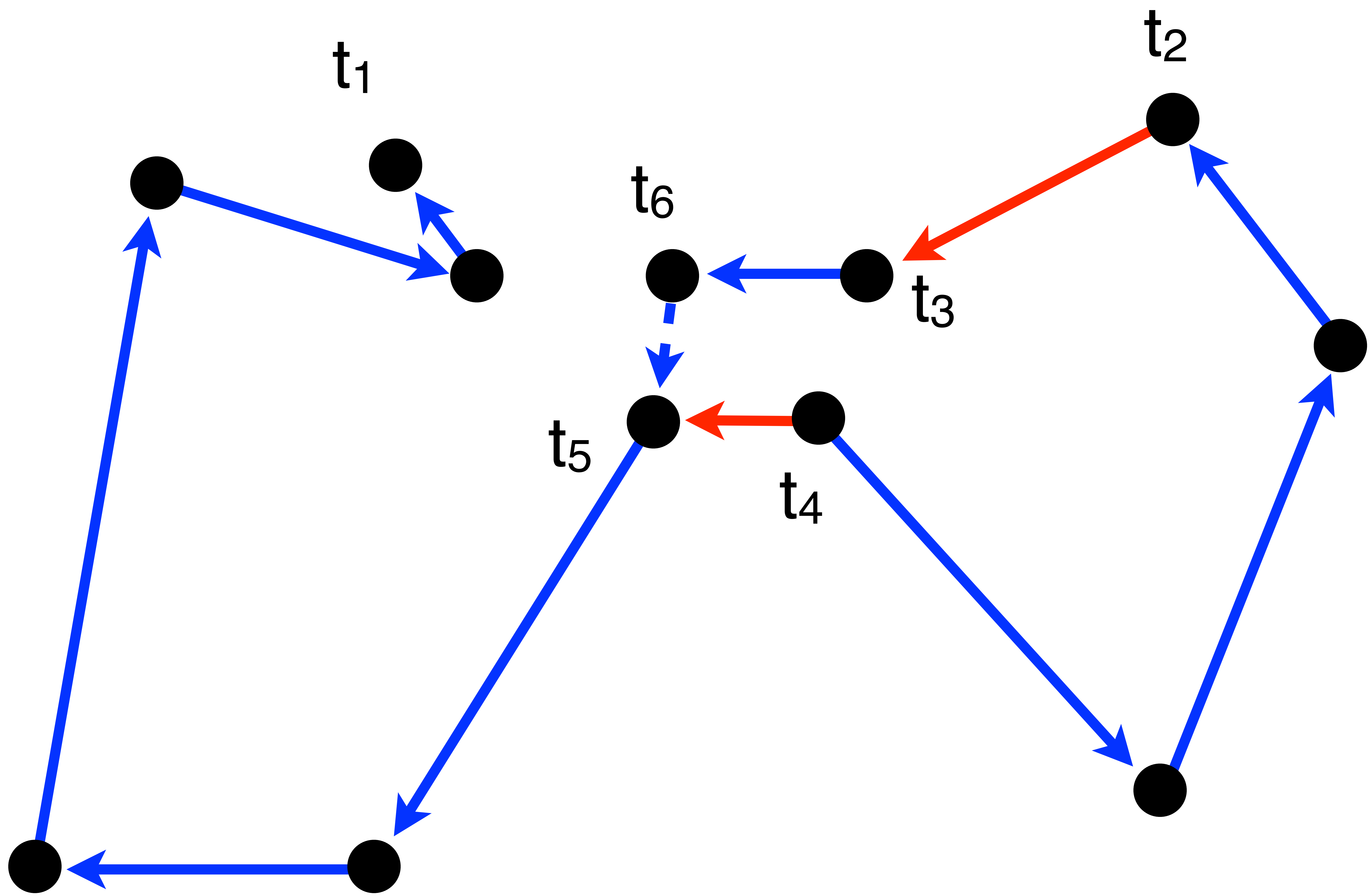

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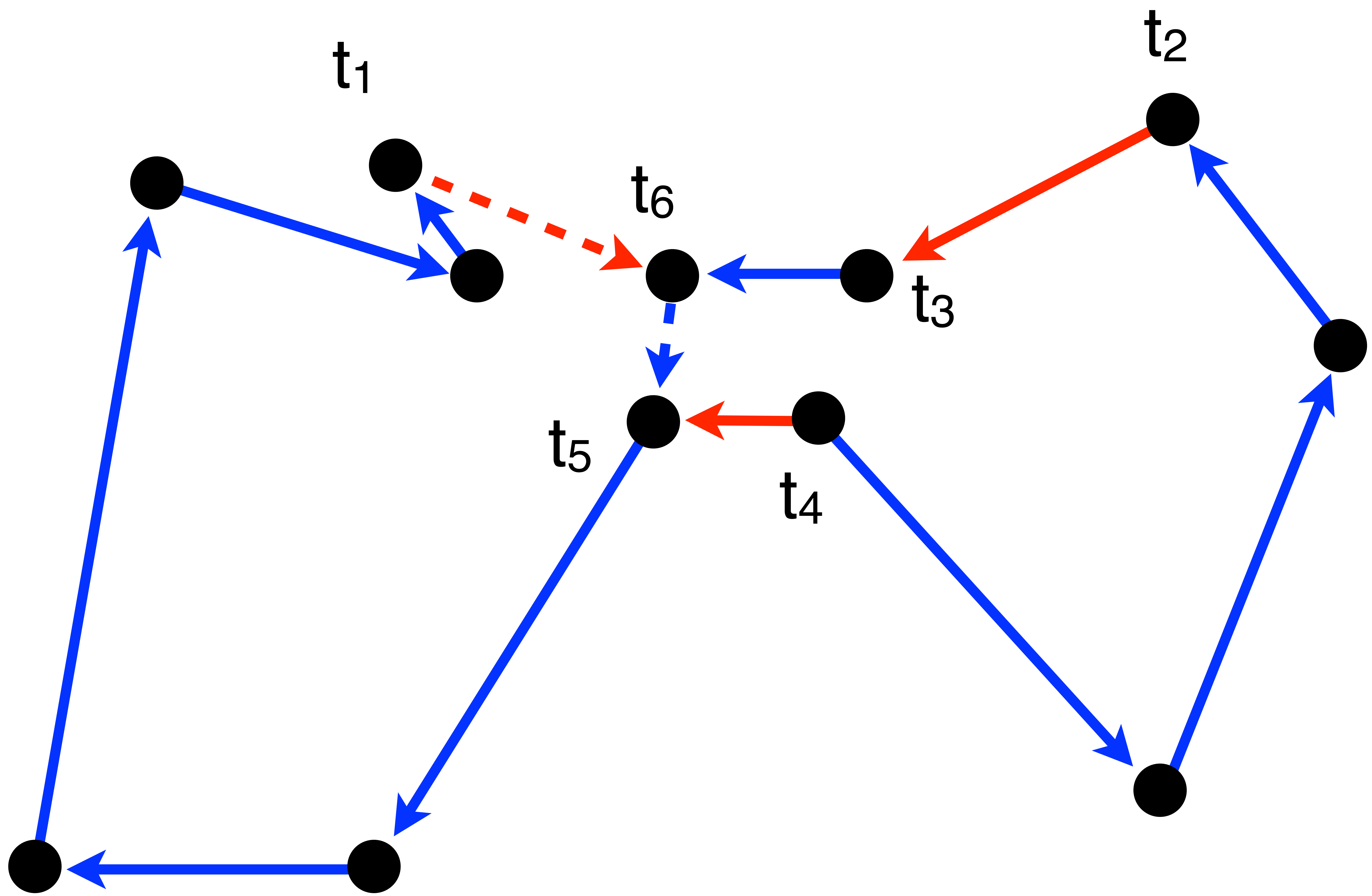

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  - choose a vertex t_1 and its edge $y_1 = (t_1, t_4)$
  - choose an edge $x_2 = (t_4, t_5)$ with $d(y_2) < d(y_1)$
  - if none exist, restart with another vertex
  - else we have a solution by removing the edge (t_6, t_5) and connecting (t_1, t_6)
  - compute the cost but do not connect
  - instead restart with t_1 and its (pretended) edge (t_1, t_6)

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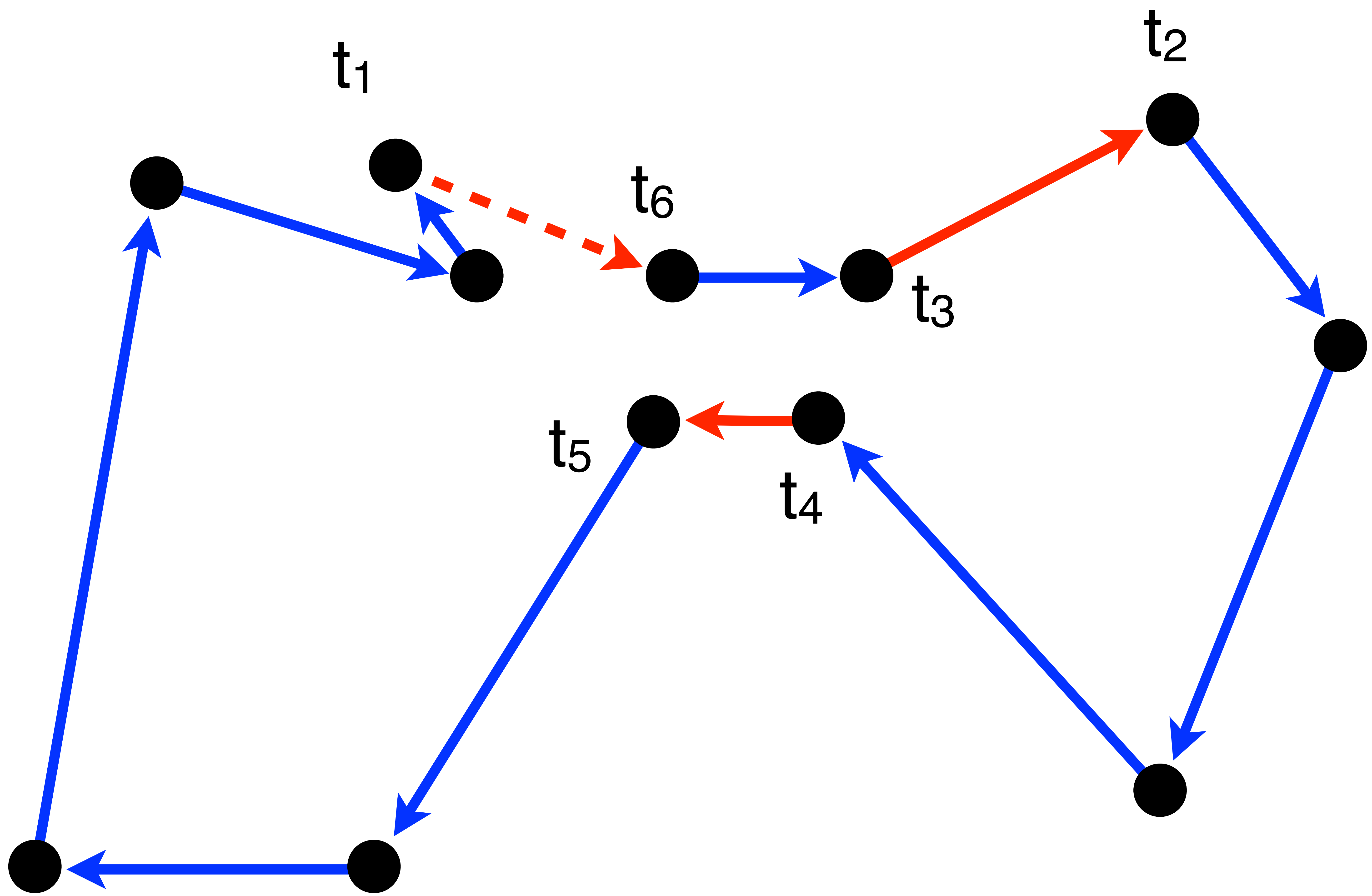

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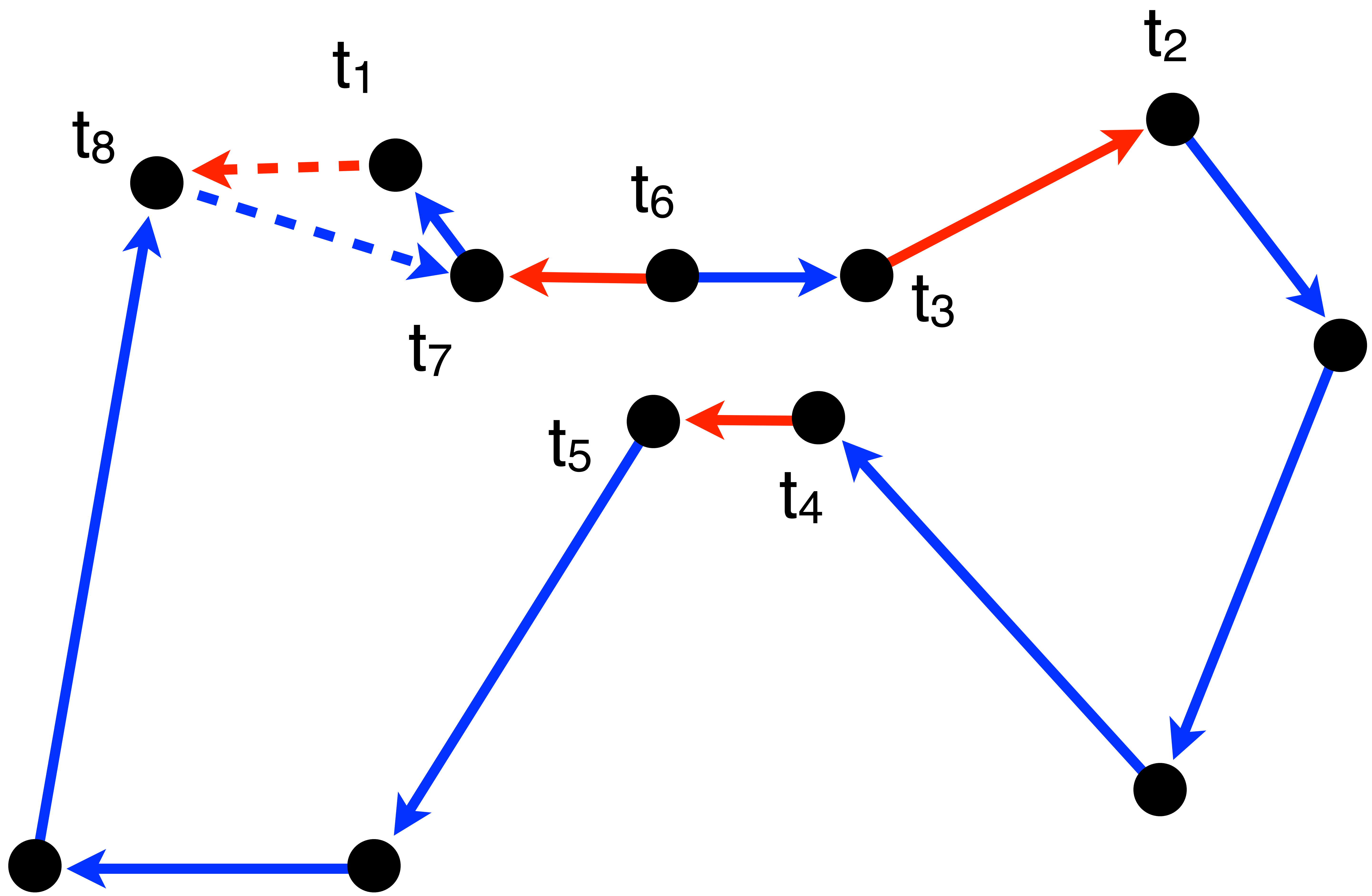

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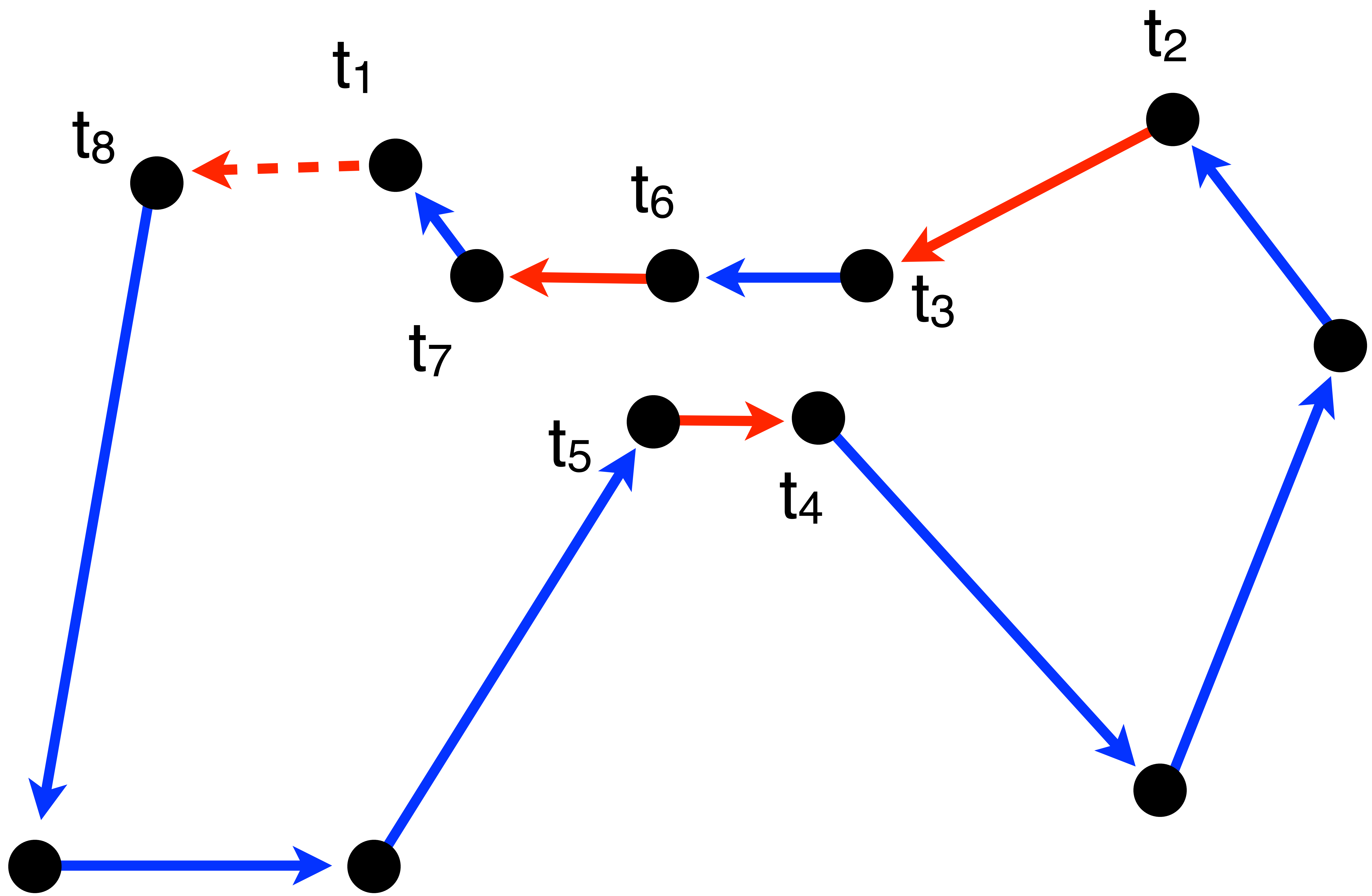

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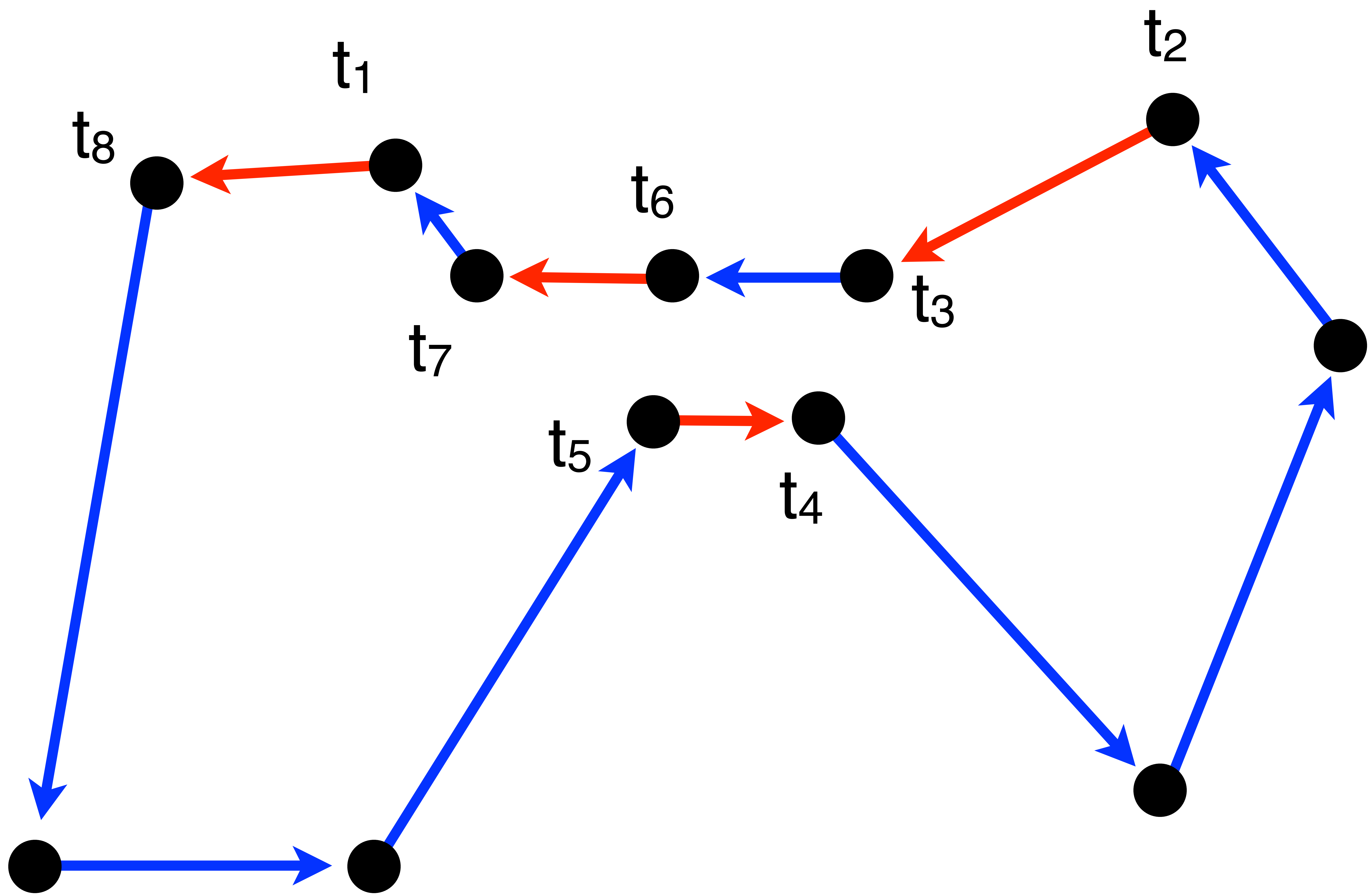

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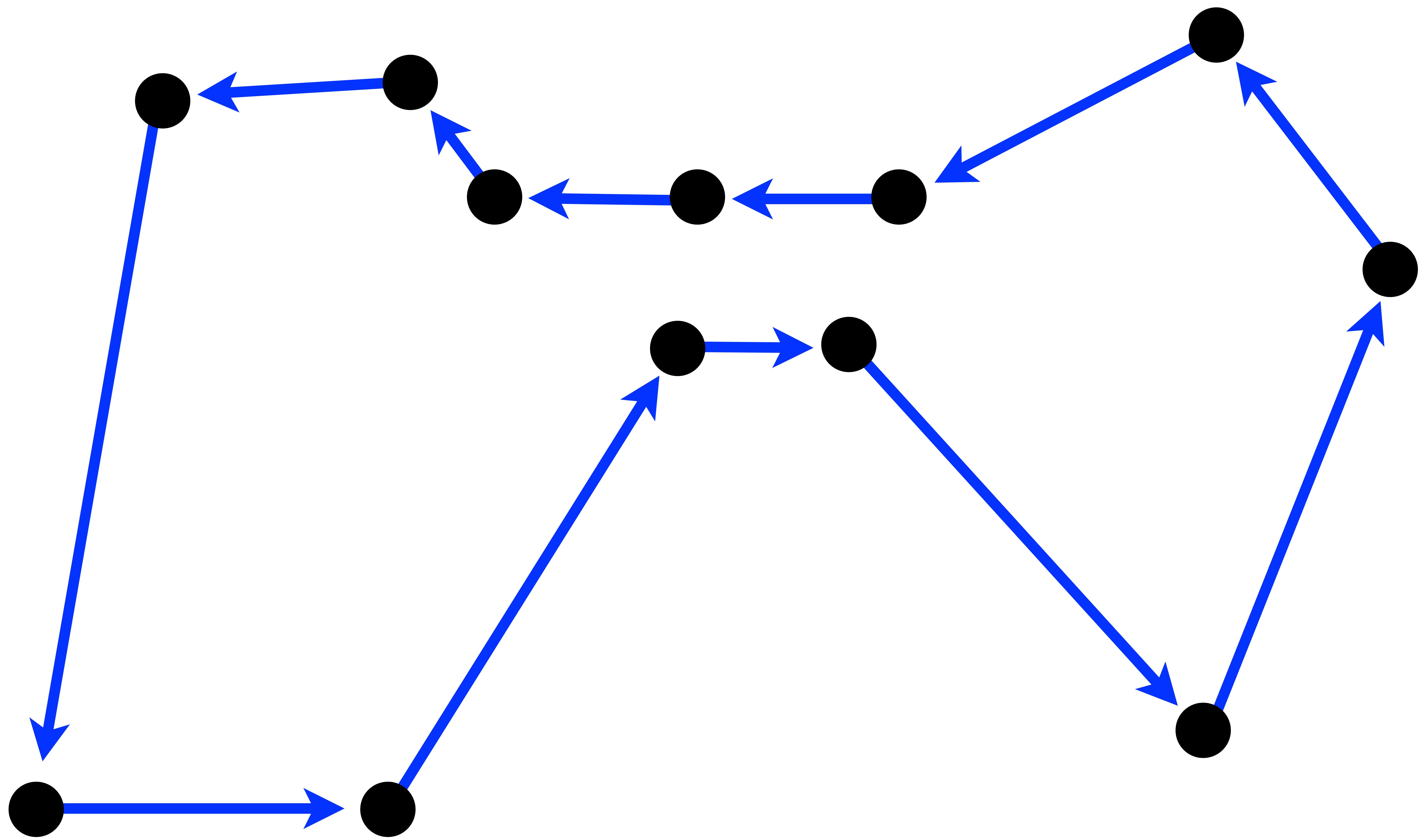

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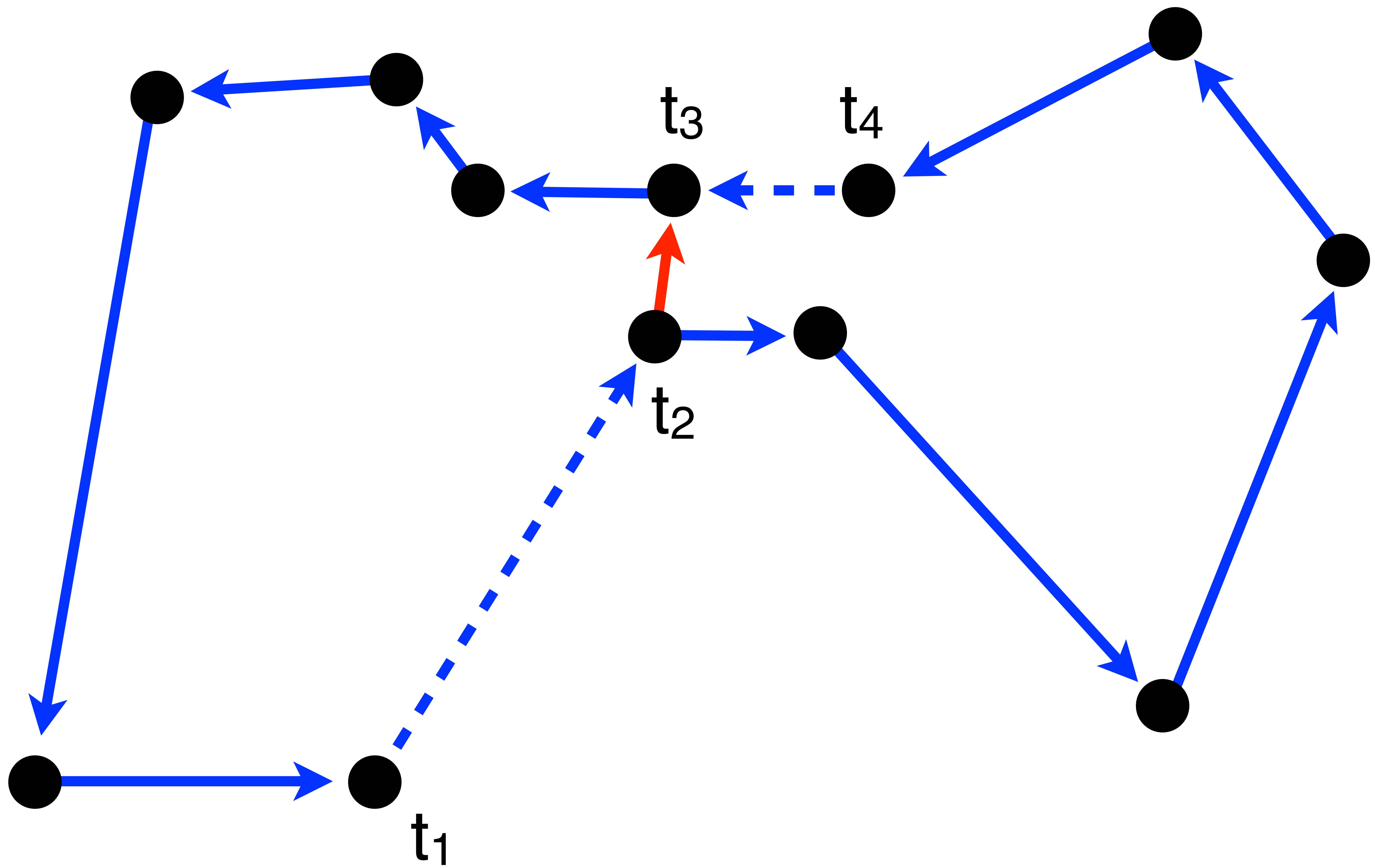

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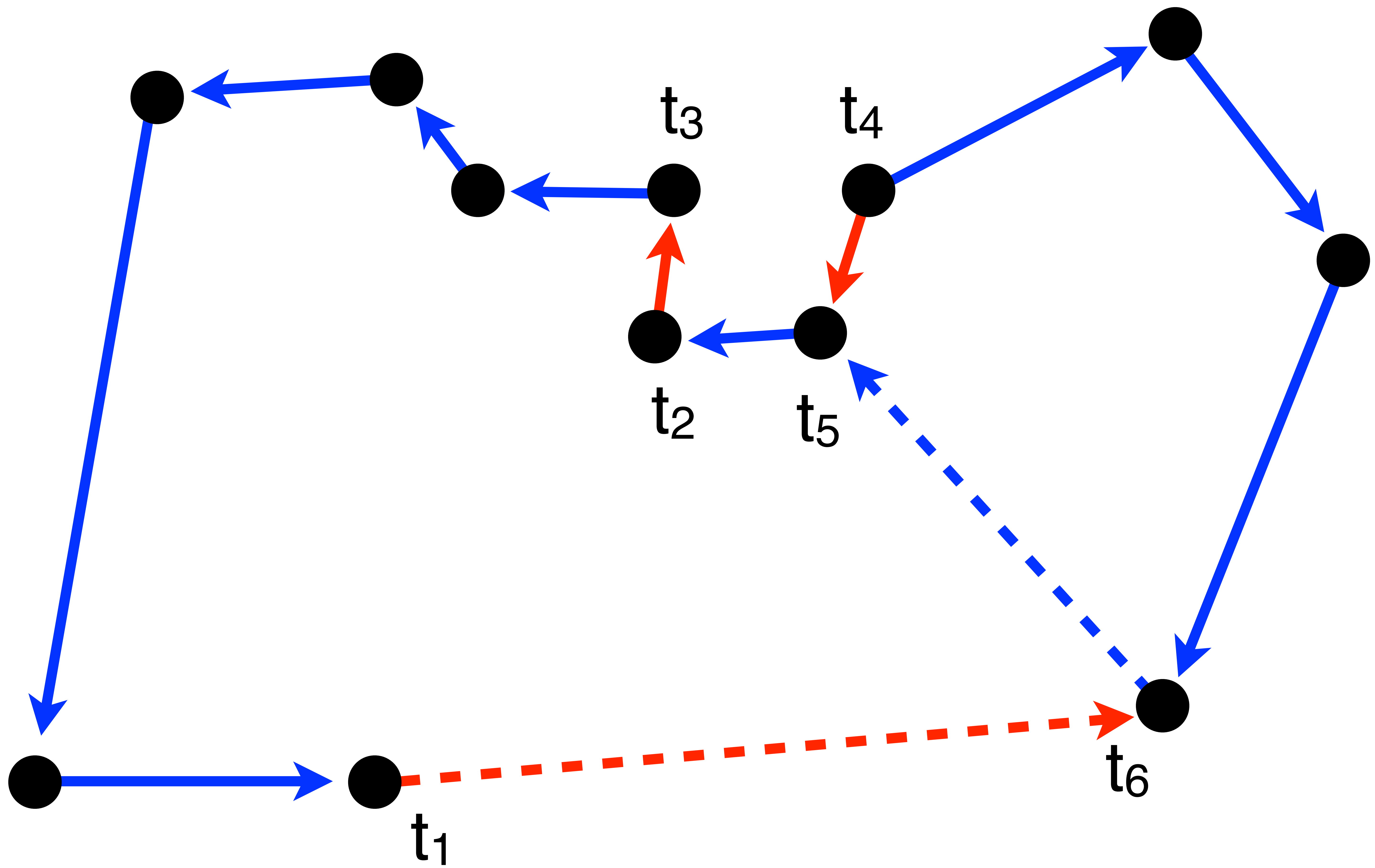

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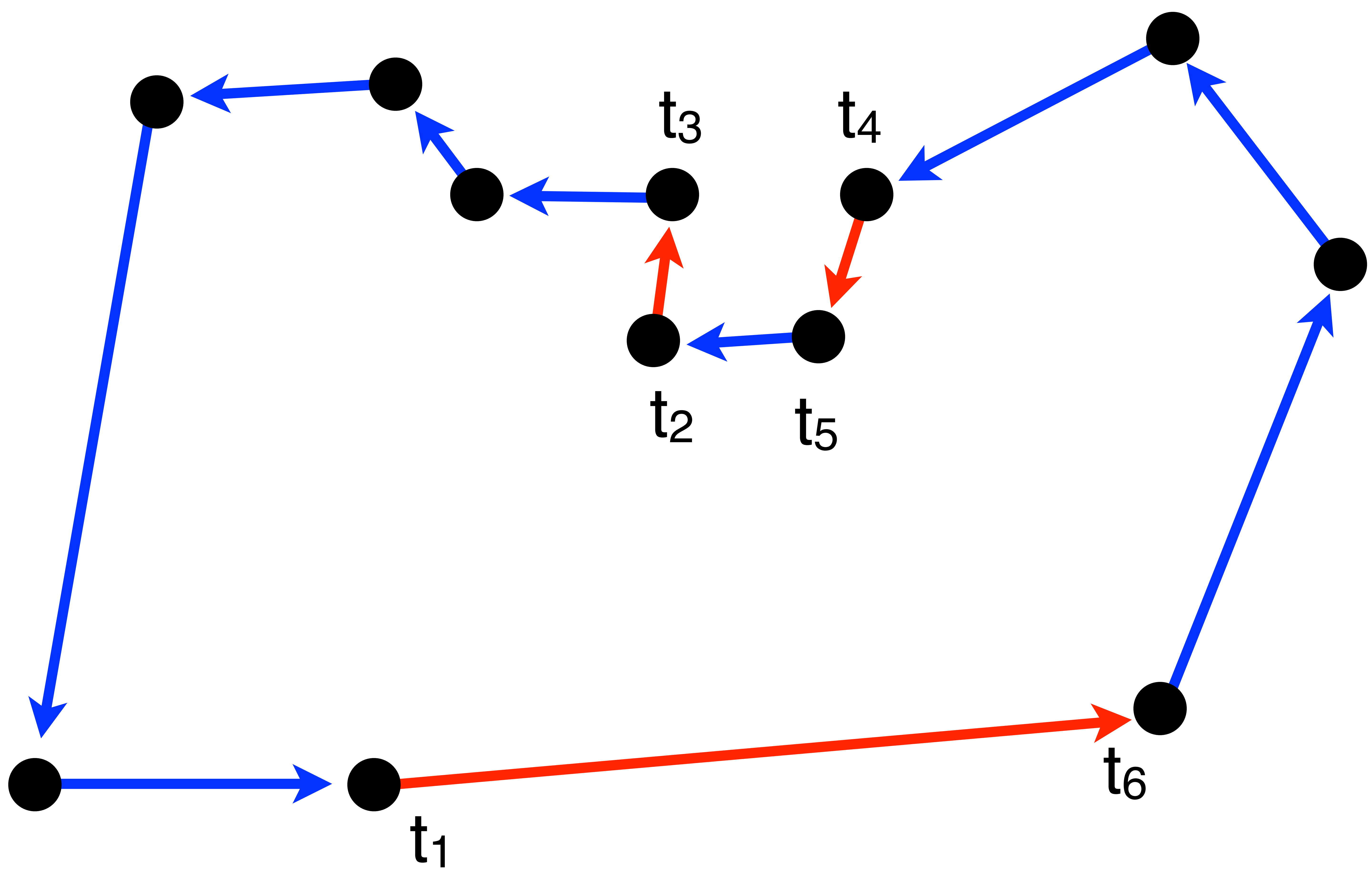

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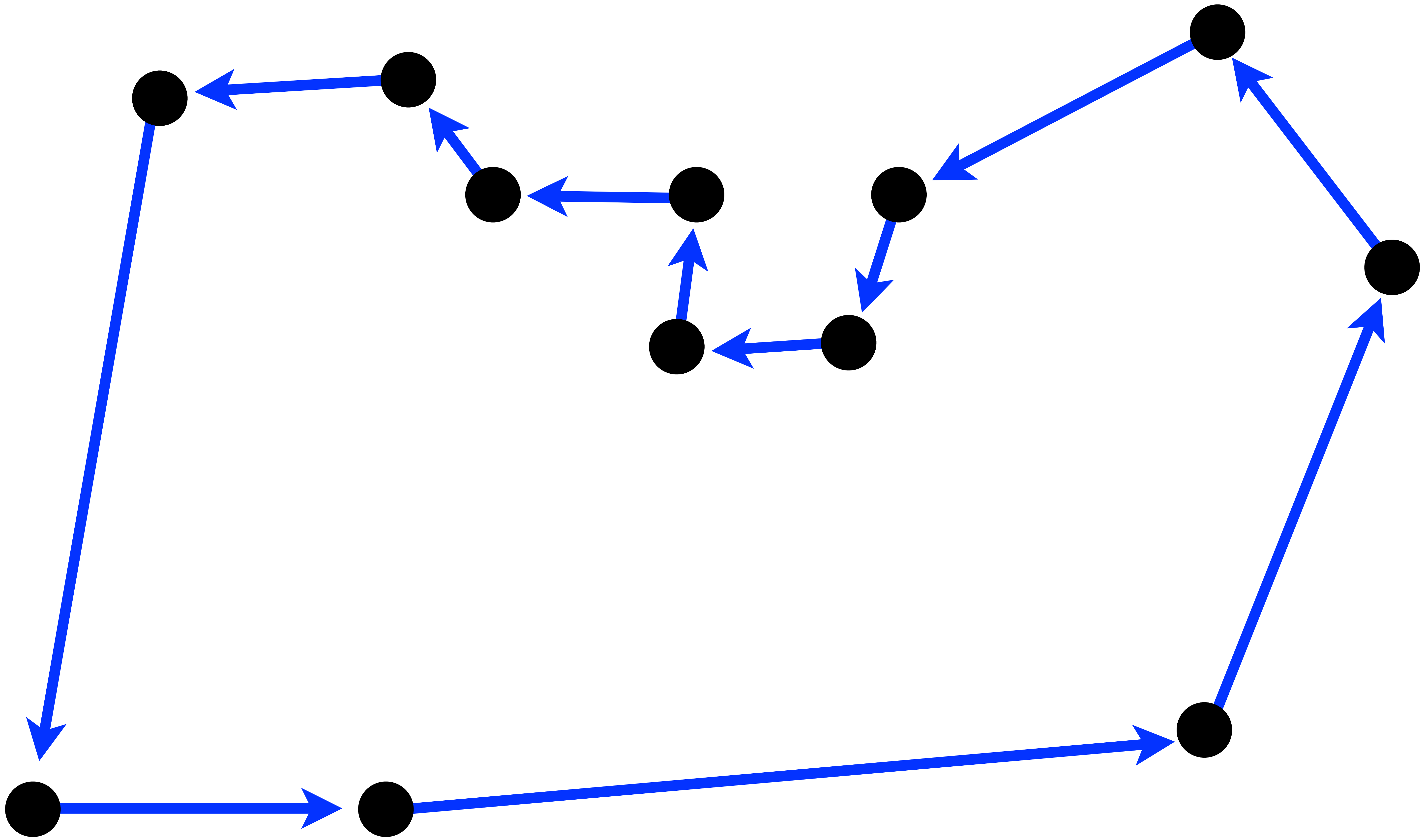

Until Next Time