Introductory Astronomy

Week 8: Cosmology

Clip 3: Friedmann Equations

Isotropic Homogeneous Matter

- Matter can be described by a constant energy density $\rho(t)$ and a constant pressure $P(t)$
- Pressure and density related by equation of state
- Extreme limits:
  - Dust is slow massive particles interacting only via gravitation: $P = 0$. Isotropic in comoving frame $\vec{v} = 0$
  - Radiation refers to gas of massless particles $P = \rho c^2/3$. Isotropic in frame where spectrum isotropic

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Friedmann Equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- RHS $R = \dfrac{kR_0}{a(t)^2}$
- LHS $\rho = \rho(t)\ \ P = P(\rho)$

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho(t) - \frac{kR_0 c^2}{a(t)^2}$$

$$-H^2 q = \left(\frac{\ddot{a}}{a}\right) = \dot{H} + H^2 = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right)$$

$$a(t) \sim a(t^*)\left[1 + H\,(t - t^*) - H^2 q/2\,(t - t^*)^2\right]$$

- Curvature of space determined by density and velocity
- Deceleration due to gravitating energy
- Energy Conservation:

$$\rho_D(t) = \rho_{D0}\, a(t)^{-3}$$
$$\rho_R(t) = \rho_{R0}\, a(t)^{-4}$$

Einstein's Blunder

- Can modify equation adding cosmological constant

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\frac{\Lambda}{c^2} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- Alternatively think of this as changing $T_{\mu\nu}$

$$\rho_\Lambda = \frac{c^2}{8\pi G}\Lambda \qquad P_\Lambda = -\rho_\Lambda c^2$$

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kR_0c^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$-qH^2 = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{\Lambda c^2}{3}$$

- Static Einstein universe

$$P = 0 \qquad k = 1$$

$$R_0 = \Lambda = \frac{4\pi G}{c^2}\rho$$

- Hubble: Λ unnecessary!