Introductory Astronomy

Week 8: Cosmology

Clip 2: Homogeneous Isotropic Relativity

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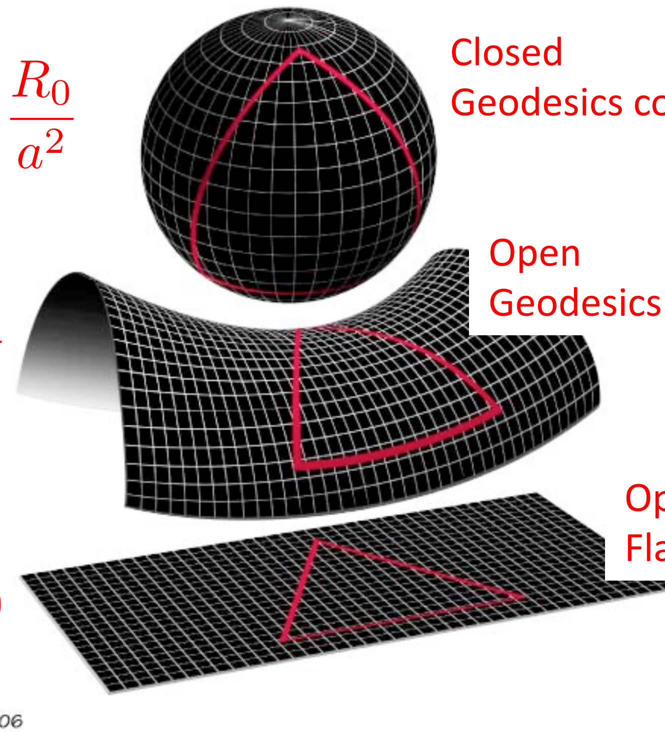
Isotropic Homogeneous Space

- In General Relativity isotropic and homogeneous means we can find coordinates in which curvature is constant
- This need not mean space is flat
- Three solutions



$R = \frac{R_0}{a^2}$ Closed
Geodesics converge

$R = -\frac{R_0}{a^2}$ Open
Geodesics diverge

$R = 0$ Open?
Flat

Robertson-Walker Geometry

- Space is homogeneous
- Scale factor $a(t)$ gives distance between comoving observers $D(t) = a(t)D_0$ where we choose $a(t_0) = 1$
- Light moves along geodesics of space at c as measured by any observer

- Cosmological redshift: light emitted at t_{em} at λ_{em} observed at t_{obs} at

$$\lambda_{obs} = \frac{a(t_{obs})}{a(t_{em})}\lambda_{em}$$

- Not Doppler. Both comoving
- Unlike gravitational: symmetric

Hubble and RW

- Approximate $a(t) \sim 1 + H_0(t - t_0)$ for $H_0|t - t_0| \ll 1$
- To this order $1 + z = \frac{\lambda_{obs}}{\lambda_{em}} = \frac{1}{a(t_{em})} \sim 1 + H_0(t_0 - t)$
- To this order $D \sim c(t_0 - t_{em})$
- So $z \sim (H_0/c)D$
- Hubble Constant: $a(t) \sim a(t^*) \left(1 + H(t^*)(t - t^*)\right)$
- Constant expansion means decreasing $H(t)$

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Distances (k=0)

- Coordinate distance D_0 "now"
- Angular size distance for small angle formula

$$\frac{\alpha}{206265"} = \frac{d(1+z)}{D_0} = \frac{d}{D_A}$$

$$D_A = \frac{D_0}{1+z}$$

- Luminosity distance

$$b = \frac{L}{4\pi D_0^2(1+z)^2} = \frac{L}{4\pi D_L^2}$$

$$D_L = D_0(1+z) = D_A(1+z)^2$$

- Redshifted blackbody spectrum is blackbody with $T_{obs} = \frac{T_{em}}{1+z}$

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More Robertson-Walker

- Like gravitational redshift means clocks in past observed to run slow
- Coordinate velocity of light $c/a(t)$
- Massive freely falling objects move along geodesics of space with peculiar velocity

$$v(t)a(t) = v_0 a(t_0)$$

Curvature of space, density of matter and evolution of $a(t)$ determined by Einstein's equation

All solutions have singularity at early or late times