Introductory Astronomy

Week 6: Relativity and Black Holes

Clip 10: Gravity is Geometry

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Gravity

- Small regions of spacetime, if observed by inertial (freely falling) observers, appear free of gravity
- Inertial determines acceleration. At any event are inertial observers with all velocities and their experiences are related by Lorentz transformations
- Tidal effects mean that inertial here and inertial there do not share same acceleration
- Given a velocity there is a unique inertial worldline starting with that velocity

Geometry

- Small regions of a curved space appear like flat space and can be described with usual coordinates
- Through any point we can draw straight lines going off in all directions and they are related by rotations
- Curvature means straight lines here and there are not related by rotation
- Given a point and an initial direction, there is a unique geodesic (straight line) starting at that point in that direction
- Shape of a space is encoded in distances between points as coordinate-invariant information - curvature

A Simple and Familiar Example

- Earth's surface is a two-dimensional curved space
- To a good approximation, it is spherical. Every point is like any other
- Start at pole (might as well) in any direction and head due South along meridian
- Initially paths behave like straight lines on plane
- Farther out notice they are too near each other – Earth is curved – and eventually all meet at opposite pole
- Great circles are geodesics of a sphere
- Positive curvature means geodesics diverge less than in flat space

The Analogy

- Inertial worldlines
- Lorentz transformations
- Interval

$$s^2 = c^2t^2 - x^2$$

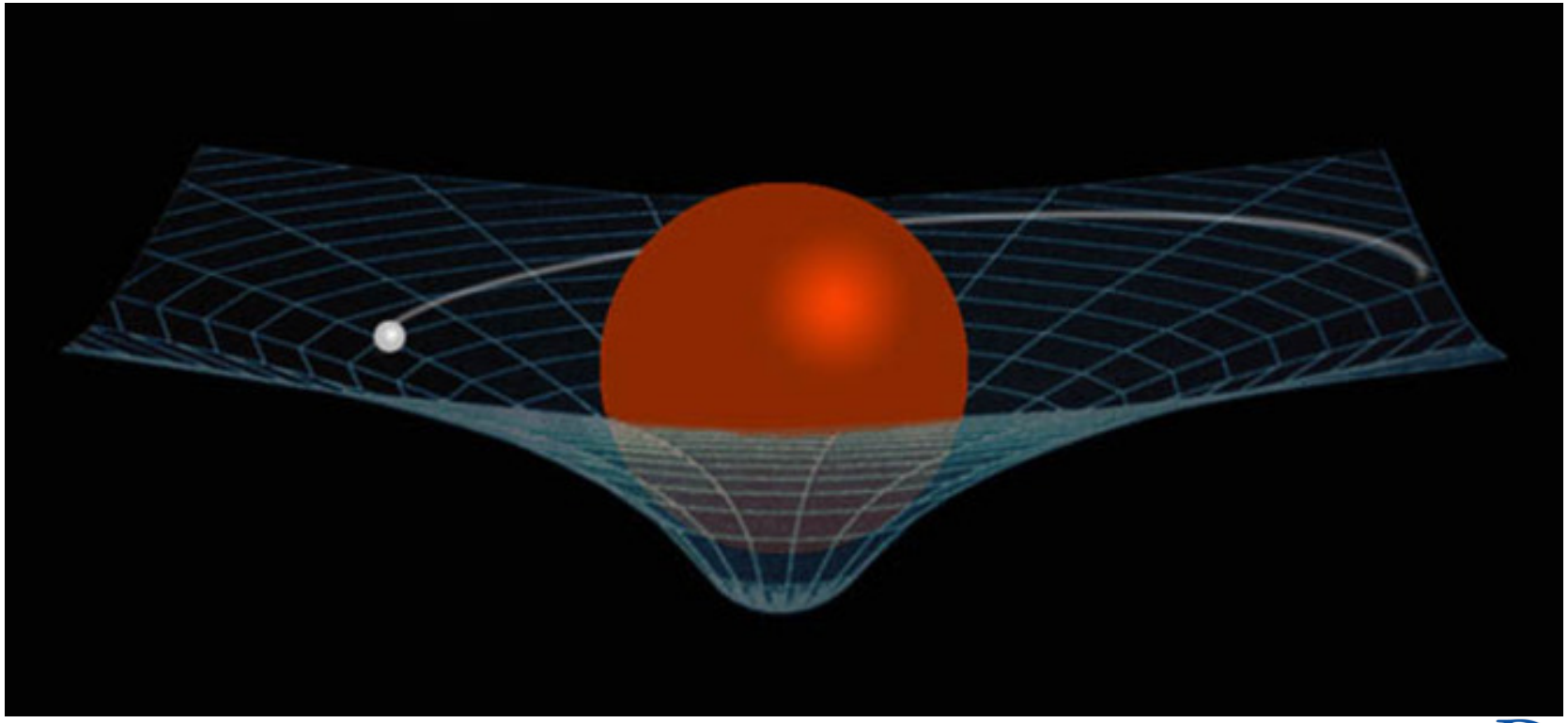
- Gravity

- Geodesics
- Rotations
- Distance

$$r^2 = x^2 + y^2$$

- Curvature

Visualization



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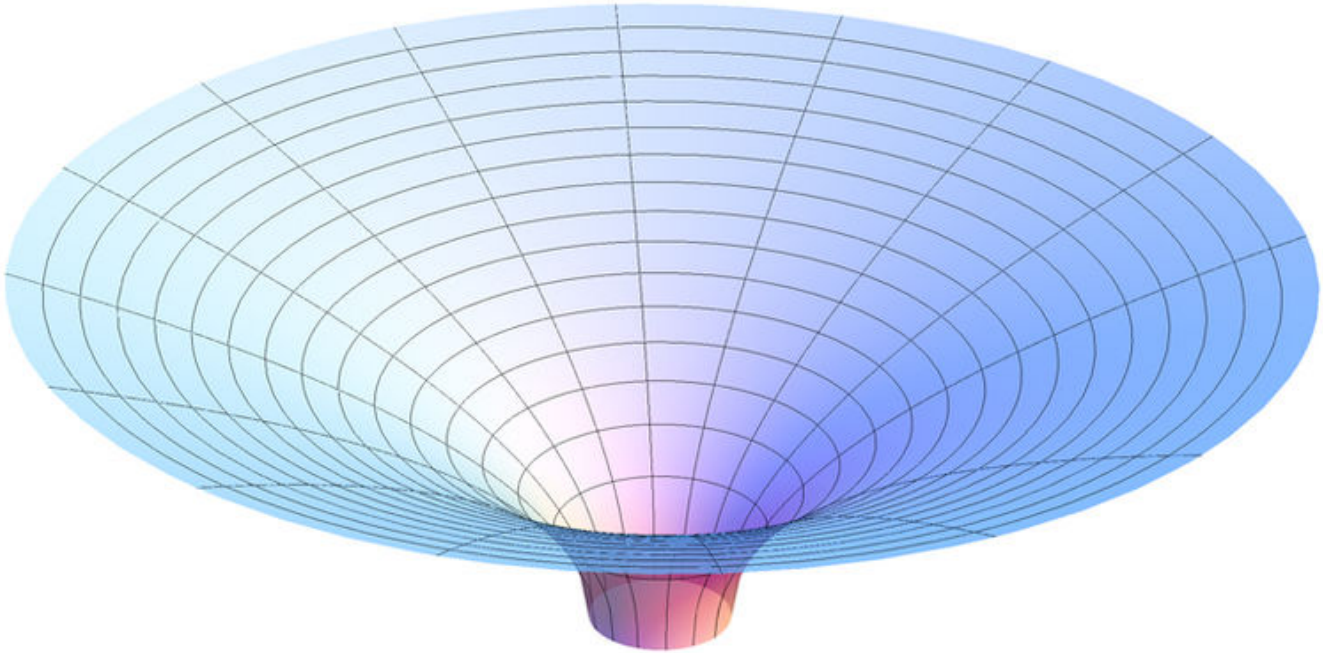
Caveats

- Shape of deformed surface is not related to curved spacetime
- In GR it is spacetime that curves
- In static spacetime space is constant
- Inertial orbits are not geodesics of this space
- Globally useful coordinates do not exist in general

Better Visualization



General Relativity

- Mass (Energy) is a source of spacetime curvature
- Inertial (free-fall) motion is along geodesics
- At small curvature and slow speeds reproduce Newton
- Complication: Gravitational energy is a source. Equations are nonlinear

$$R_{\mu\nu} - g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$\begin{aligned} V_{eff}(r) &\sim -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{mc^2r^3} \\ &\sim -\frac{GMm}{r}(1 - v^2/c^2) + \frac{mv^2}{2} \end{aligned}$$

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Credits

- Astronomy Animations: University of Nebraska-Lincoln Astronomy Education Group http://astro.unl.edu/
- "Hyperboloid Geodesics" from the Wolfram Demonstrations Project (Antonin Slavik, Charles University, Prague) http://demonstrations.wolfram.com/HyperboloidGeodesics/
- Flamm Paraboloid: Wikimedia Commons/AllenMcC http://en.wikipedia.org/wiki/File:Flamm.jpg

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