Introductory Astronomy

Week 6: Relativity and Black Holes

Clip 8: Simultaneity Paradoxes

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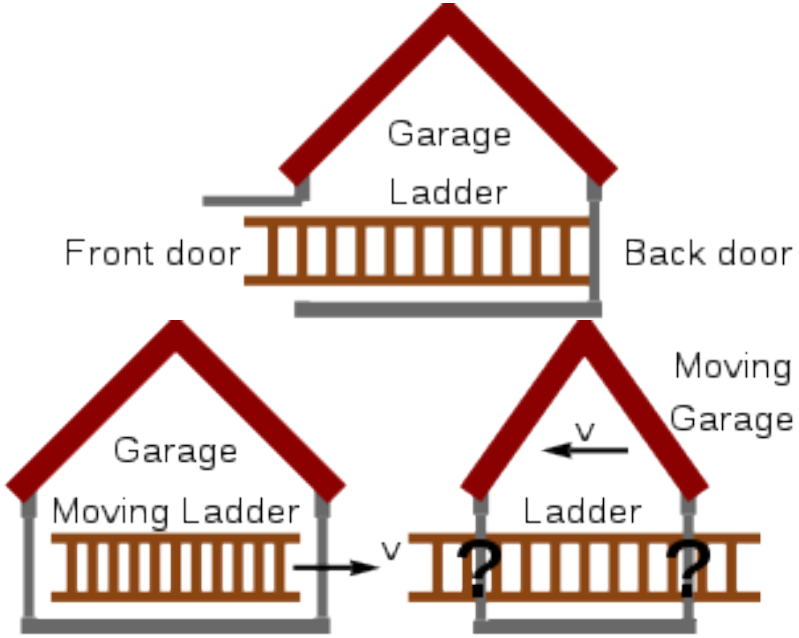
The Ladder Paradox

- Ladder of length L fit in garage of length $G < L$ by moving at v so $L\sqrt{1 - v^2/c^2} < G$
- Problem: as seen by observer moving with ladder, garage contracts
- Does the ladder fit?



Garage Frame Ladder Frame

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Solving with Lorentz

- Take a close look at events as seen by garage (t, x) and by ladder (t', x')
- Coordinates:

  Garage: $x_L = 0 \; x_R = G$

  Ladder: $x'_L = 0 \; x'_R = L$

  $G < L$

- Event 1: $t_1 = t'_1 = 0$ $x_1 = x'_1 = 0$

  When left of ladder at left door of garage

- Event 2: $x_2 = G \; x'_2 = L$

  When right of ladder at right door of garage

- Which came first?

Order of Events

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$t = \frac{t' + vx/c^2}{\sqrt{1 - v^2/c^2}}$$

- Set $x_2 = G \; x'_2 = L$

  $vt_2 = G - L\sqrt{1 - v^2/c^2} > 0$

  Ladder is inside for

  $$0 \leq < t \leq t_2$$

- But

  $vt'_2 = G\sqrt{1 - v^2/c^2} - L < 0$

  Ladder never inside

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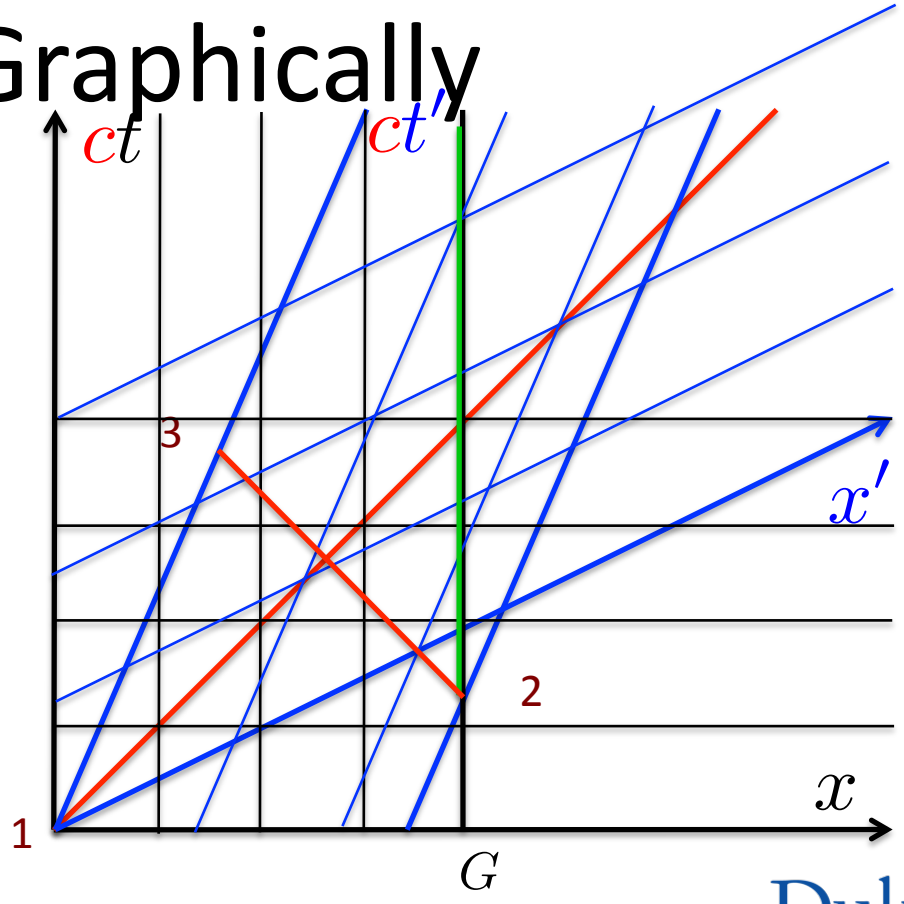
Solving Graphically

- Important: Distances on this plot mean nothing!

$$x^2 + c^2t^2$$

- Right door closed? Front stops at 2, rear can learn of this at 3 or later



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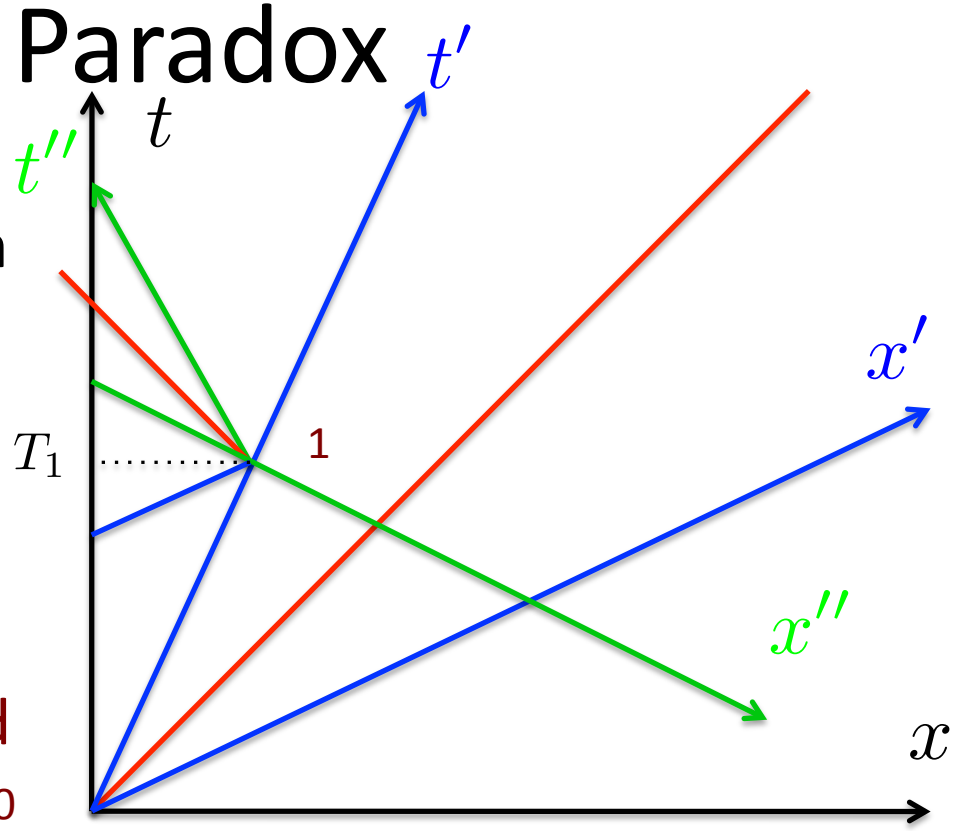
Twin Paradox

- Twin 1 stays here
- Twin 2 travels away, then returns
- Who's younger?

$$T_1 = \frac{T_1'}{\sqrt{1 - v^2/c^2}}$$

- The one who accelerated



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