Introductory Astronomy

Week 6: Relativity and Black Holes

Clip 6: Invariant Interval
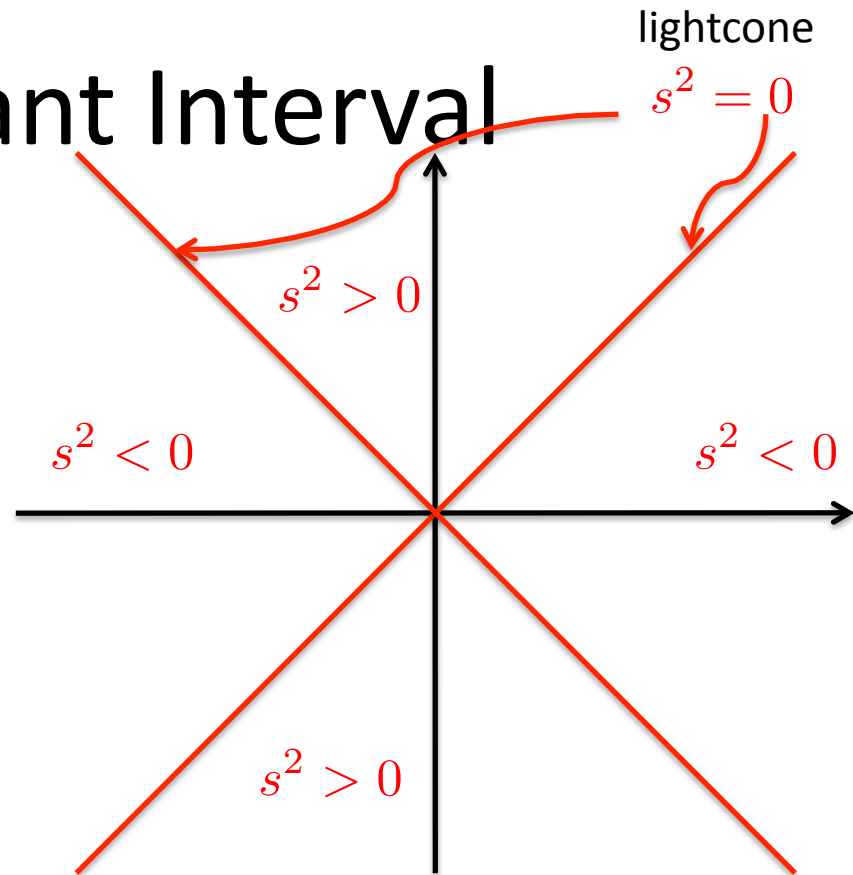
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The Invariant Interval

- Lorentz transformations

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$
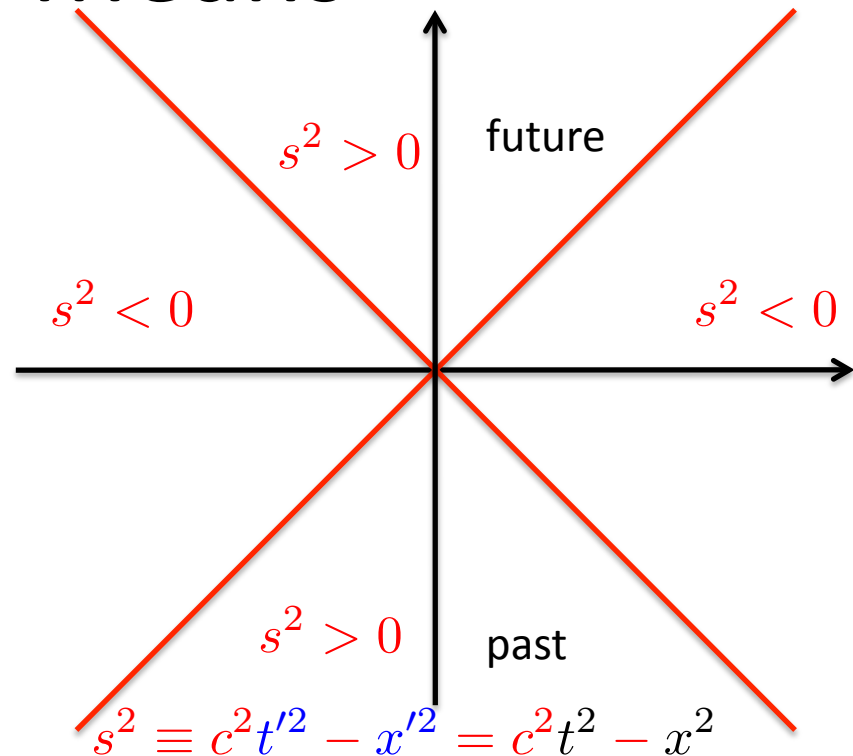
$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

have the property that

$$s^2 \equiv c^2 t'^2 - x'^2 = c^2 t^2 - x^2$$

lightcone

$s^2 = 0$

$s^2 > 0$

$s^2 < 0$

$s^2 < 0$

$s^2 > 0$

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What It Means

- If $s^2 > 0$ $t > 0$ an observer with $v = x/t$ gets to (t, x)

  $$x' = 0 \quad t' = \sqrt{s^2}/c$$

- $\sqrt{s^2}/c$ is proper time

- If $s^2 > 0$ $t < 0$ observer comes from (t, x)

- This is future/past

$s^2 > 0$ future

$s^2 < 0$ $s^2 < 0$

$s^2 > 0$ past

$$s^2 \equiv c^2 t'^2 - x'^2 = c^2 t^2 - x^2$$

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What it Means

- If $s^2 < 0$ observer with

  $v = c^2 t/x$ finds $t' = 0$

  $\sqrt{-s^2} = x'$ is proper distance

- Faster observer has order reversed

- Causal theory means no material particle or information can propagate faster than

$s^2 > 0$ future

$s^2 < 0$ $s^2 < 0$

$s^2 > 0$ past

$$s^2 \equiv c^2 t'^2 - x'^2 = c^2 t^2 - x^2$$

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