Introductory Astronomy

Week 6: Relativity and Black Holes

Clip 5: Relativistic Spacetime

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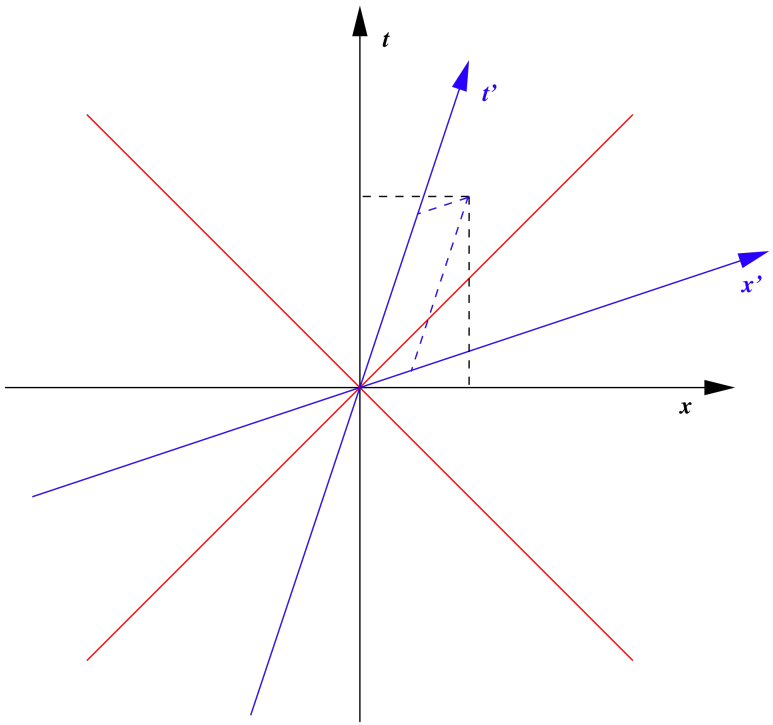
The Answer

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

- Note $v^2 < c^2$ or else this makes no sense



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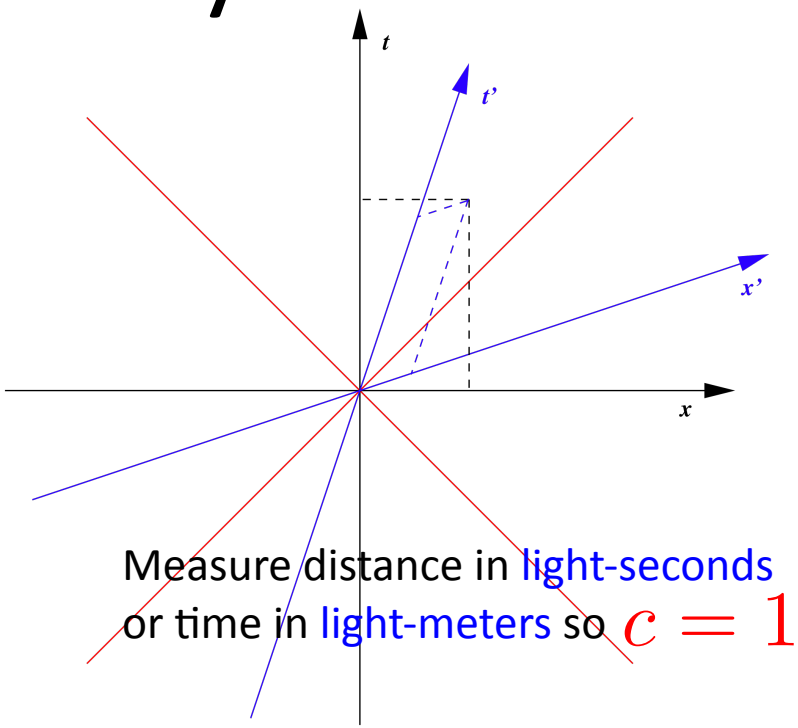
Simultaneity

- Most unintuitive:

  $t' = 0 \rightarrow t = vx/c^2$

  Simultaneity is Relative
- Is causality lost?
- Source of many "paradoxes"



Measure distance in light-seconds or time in light-meters so $c = 1$

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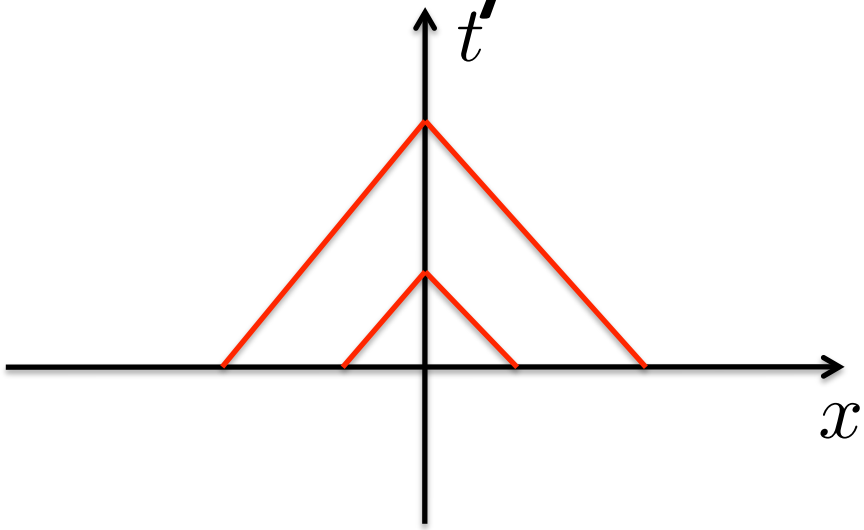
Time and Simultaneity

- Since we know c is constant we can measure time of distant events
- If light from x reaches 0 at time t it was emitted at $t - x/c$
- The present: $t = 0$ Events from which light will reach us at $t = x/c$



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Length Contraction

- A ruler lies at

$$x'_1 = 0 \; x'_2 = L'$$

- This is

$$x_1 = vt$$

$$x_2 = vt + L'\sqrt{1 - v^2/c^2}$$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$L = L'\sqrt{1 - v^2/c^2} \qquad L' = L\sqrt{1 - v^2/c^2}$$

Time Dilation

- Clock at $x' = 0$ ticks at $t'_1 = 0$ $t'_2 = T'$

$$t_1 = 0 \; t_2 = \frac{T'}{\sqrt{1 - v^2/c^2}}$$

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

$$T = \frac{T'}{\sqrt{1 - v^2/c^2}} \qquad T' = \frac{T}{\sqrt{1 - v^2/c^2}}$$

Doppler

- Doppler formula modified by time dilation
  - Transverse Doppler effect

$$\lambda = \frac{\lambda_0}{\sqrt{1-v^2/c^2}}$$

  - Longitudinal Doppler effect

$$\lambda = \lambda_0(1+v/c)\frac{1}{\sqrt{1-v^2/c^2}} = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}}$$

Is This Real?

- We have lots of experimental evidence
- This is real. Any ruler constructed using Lorentz-invariant physics will contract at high speed. Any physical clock will run slow at high speed

Velocity Addition

$$x' = ut'$$

$$x = \frac{(u+v)t'}{\sqrt{1-v^2/c^2}} = ut$$

$$t = \frac{(1+uv/c^2)t'}{\sqrt{1-v^2/c^2}}$$

$$x = \frac{x'+vt'}{\sqrt{1-v^2/c^2}}$$

$$t = \frac{t'+vx'/c^2}{\sqrt{1-v^2/c^2}}$$

$$u = \frac{u'+v}{1+u'v/c^2}$$