Introductory Astronomy

Week 6: Relativity and Black Holes

Clip 4: Lorentz Transformations

Duke UNIVERSITY

Lorentz Transformations

- Send a light pulse from

$$x = t = 0$$
$$x = \pm ct$$

- Seek

$$x' = A(x - Bt)$$
$$t' = Cx + Dt$$

$$x' = 0 \rightarrow x = vt$$
$$B = -Av$$
$$x = 0 \rightarrow x' = -vt'$$
$$B = -Dv$$
$$x = ct \rightarrow x' = ct'$$
$$C = -v/c^2 A$$

Lorentz Transformations

$$x' = A(v)(x - vt)$$

$$t' = A(v)(t - vx/c^2)$$

$$x = \frac{x' + vt'}{A(v)(1 - v^2/c^2)} = A(-v)(x' + vt')$$

$$t = \frac{t' + vx/c^2}{A(v)(1 - v^2/c^2)} = A(-v)(t' + vx'/c^2)$$

$$A(v) = \frac{1}{\sqrt{1 - v^2/c^2}}$$

The Answer

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

- For $v \ll c$ and x not too large, $x' \sim x - vt$

$$t' \sim t$$