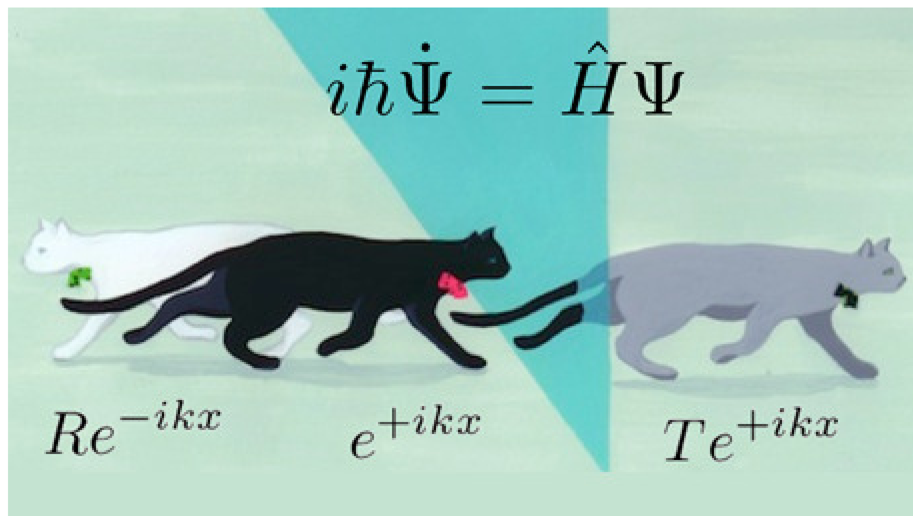
Exploring Quantum Physics

Coursera, Spring 2013 Instructors: Charles W. Clark and Victor Galitski

Guest Lecture: Electron Spin

Part III: Spin Dynamics and LS Coupling



Guest lecturer:
Prof. Ian Appelbaum, U. Maryland Physics Dept.
http://appelbaum.physics.umd.edu

Spin in a magnetic field

$$\vec{S} = S_x\hat{x} + S_y\hat{y} + S_z\hat{z} = \frac{\hbar}{2}(\sigma_x\hat{x} + \sigma_y\hat{y} + \sigma_z\hat{z})$$

σ's are 2x2 Pauli matrices

$$\langle S^2 \rangle = \hbar^2 s(s+1) \qquad s = 1/2$$

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\langle S_z \rangle = m_s\hbar \qquad m_s = \pm 1/2$$

$$H\Psi = E\Psi \qquad H = g\frac{\mu_B}{\hbar}\vec{S}\cdot\vec{B} = \mu_B\sigma_z B_z \qquad \left(\vec{B} = B_z\hat{z}\right)$$

$$= \begin{bmatrix} \mu_B B_z & 0 \\ 0 & -\mu_B B_z \end{bmatrix}$$

Eigenvalues/eigenvectors:

$$E_1 = +\mu_B B_z \qquad E_2 = -\mu_B B_z$$

$\Psi_1 = |\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ m_s=+1/2 "spin up"

$\Psi_2 = |\downarrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ m_s=-1/2 "spin down"

$E = g\mu_B B_z m_s$ (m_s=±½, g=2)

State evolution

Example: An electron spin is prepared in an eigenstate of S_x. It then evolves in a magnetic field along $\hat{z}$. What is the expectation value of the x-component of spin?

$$S_x = \frac{\hbar}{2}\sigma_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Eigenstates:

$$+\frac{\hbar}{2}: \quad |+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$-\frac{\hbar}{2}: \quad |-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Time evolution in B_z:

$$|+\rangle = \frac{1}{\sqrt{2}}\left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{-i\frac{\mu_B B_z}{\hbar}t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{+i\frac{\mu_B B_z}{\hbar}t}\right]$$

Expectation value:

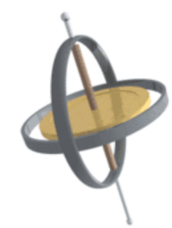
$$\langle +|S_x|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{+i\frac{\mu_B B_z}{\hbar}t} & e^{-i\frac{\mu_B B_z}{\hbar}t} \end{bmatrix} \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} e^{-i\frac{\mu_B B_z}{\hbar}t} \\ e^{+i\frac{\mu_B B_z}{\hbar}t} \end{bmatrix} = \frac{\hbar}{4}\left(e^{+2i\frac{\mu_B B_z}{\hbar}t} + e^{-2i\frac{\mu_B B_z}{\hbar}t}\right)$$

$$= \frac{\hbar}{2}\cos 2\frac{\mu_B B_z}{\hbar}t$$

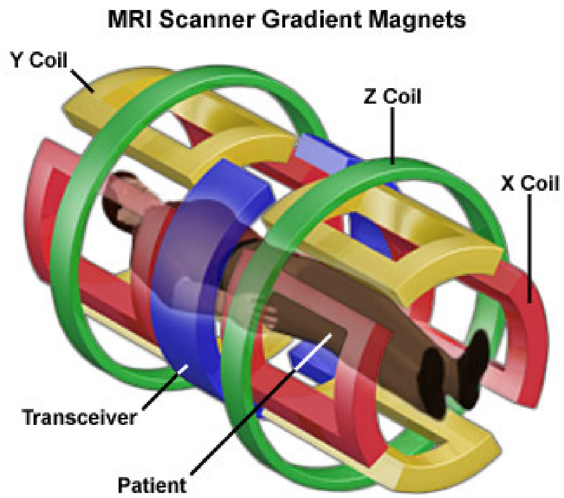
Similarly, $\langle +|S_y|+\rangle = \frac{\hbar}{2}\sin 2\frac{\mu_B B_z}{\hbar}t$

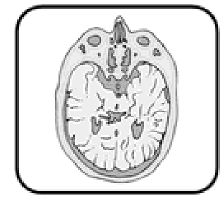
Larmor precession at frequency set by up/down splitting in B_z:

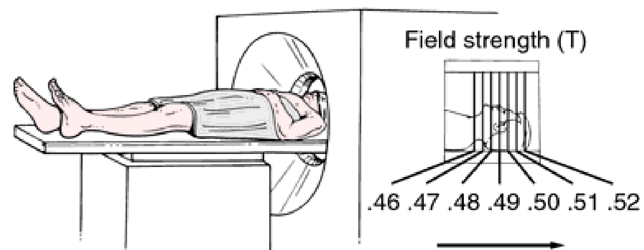
$$\omega = \frac{g\mu_B B_z}{\hbar}$$



Magnetic Resonance Imaging







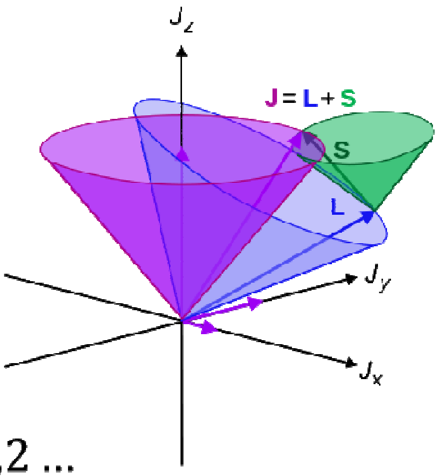
Magnetic moment: $\dfrac{e\hbar}{2m}$

So $\mu_N = \dfrac{m_e}{m_N}\mu_B$

Adding orbital and spin angular momenta

$$H = \frac{\mu_B}{\hbar}\vec{L}\cdot\vec{B} + g\frac{\mu_B}{\hbar}\vec{S}\cdot\vec{B} \quad = \frac{\mu_B}{\hbar}(\vec{L}+2\vec{S})\cdot\vec{B}$$

- Only total angular momentum (J=L+S) is conserved… NOT L+2S!
- Can we still conclude $E = "g"\mu_B B_z m_J$? If so, what is our new "g"?



$$H = \frac{\mu_B}{\hbar}\frac{(\vec{L}+2\vec{S})\cdot\vec{J}}{|J|}\cdot\frac{\vec{B}\cdot\vec{J}}{|J|} \qquad = \frac{\mu_B}{\hbar}\frac{(\vec{L}+2\vec{S})\cdot(\vec{L}+\vec{S})J_z B_z}{|J|^2}$$

$$= \frac{\mu_B}{\hbar}\frac{(|L|^2+2|S|^2+3\vec{S}\cdot\vec{L})\,J_z B_z}{|J|^2}$$

$$\langle S^2\rangle = \hbar^2 s(s+1) = \frac{3\hbar^2}{4}$$

$$\langle L^2\rangle = \hbar^2 l(l+1) \qquad l = 0,1,2\ldots$$

$$\langle J^2\rangle = \hbar^2 j(j+1) \qquad j = l-s, l+s$$

But what is S · L?

$$J^2 = (\vec{L}+\vec{S})^2 = |L|^2+|S|^2+2\vec{S}\cdot\vec{L}$$

$$\vec{S}\cdot\vec{L} = \frac{J^2-L^2-S^2}{2}$$

Landé g-factor

$$E = \frac{\mu_B}{\hbar}\frac{(L^2 + 2S^2 + \frac{3}{2}(J^2 - L^2 - S^2))}{J^2}\hbar m_J B_z \qquad = \mu_B\left[1 + \frac{J^2 - L^2 + S^2}{2J^2}\right] m_J B_z$$

$$= g_L \mu_B m_J B_z$$ (m_J=-j,-j+1, … , j-1,+j)

Alfred Landé

g_L: "Landé g-factor" $=1 + \frac{j(j+1)-l(l+1)+s(s+1)}{2j(j+1)}$

Examples:

$l = 0 \quad (j = s = ½)$ $\quad g_L = 1 + \frac{½(½+1)-0(0+1)+½(½+1)}{2½(½+1)}=2$

$l = 1 \quad (j = l \pm s = {}^3/_2 \; or \; {}^1/_2)$

$g_L = 1 + \frac{3/2(3/2+1)-1(1+1)+½(½+1)}{2\cdot 3/2(3/2+1)}=4/3 \quad (j = {}^3/_2)$

$g_L = 1 + \frac{½(½+1)-1(1+1)+½(½+1)}{2½(½+1)}=2/3 \quad (j = {}^1/_2)$

Anomalous Zeeman effect explained

$l = 1$:

j	m_J	g_L	$g_L m_J$
$3/2$	$+3/2$	$4/3$	2
$3/2$	$+1/2$	$4/3$	2/3
$1/2$	$+1/2$	$2/3$	1/3
$1/2$	$-1/2$	$2/3$	-1/3
$3/2$	$-1/2$	$4/3$	-2/3
$3/2$	$-3/2$	$4/3$	-2

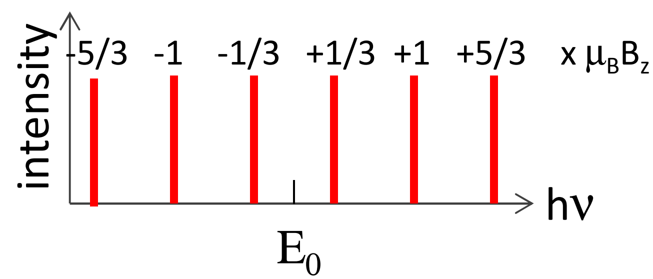
Selection rules:
Δj=+1 or -1 and
Δm_J=0,+1, or -1.

$l = 0$:

j	m_J	g_L	$g_L m_J$
$1/2$	$+1/2$	2	1
$1/2$	$-1/2$	2	-1

$$\Delta E \propto (g_L m_J)^i - (g_L m_J)^f$$

2-1= +1
2/3-1= -1/3
2/3-(-1)= +5/3
-2/3-1= -5/3
-2/3-(-1)= +1/3
-2-(-1)= -1



Note SIX spectral lines, and ALL shift with B_z… because of different g-factors!