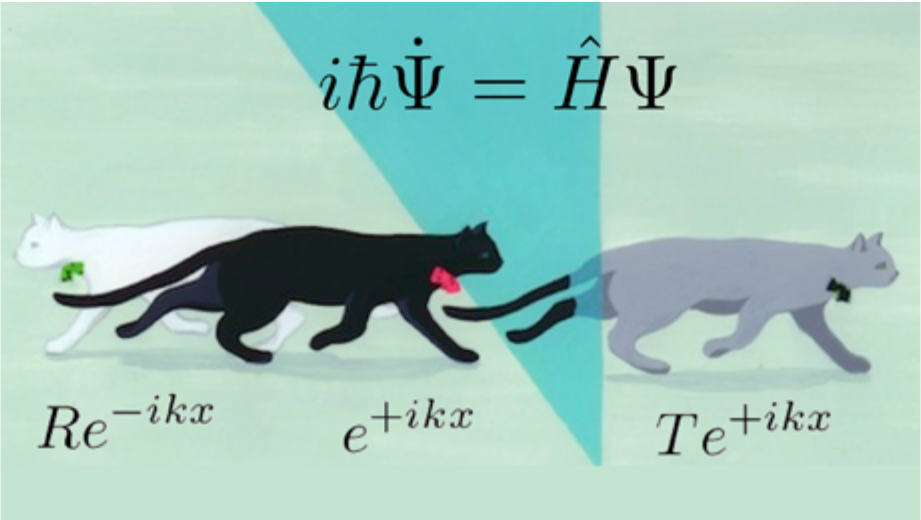
Exploring Quantum Physics

Coursera, Spring 2013 Instructors: Charles W. Clark and Victor Galitski

Symmetry in Quantum Physics

Part IV. Angular momentum: Basic commutation relations



Symmetry in quantum physics

Eugene Paul Wigner
1902-1995
1963 Nobel Prize in Physics
"for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles".

Angular momentum

Now we derive the key commutator identities of the angular momentum operator, using the tools introduced in Part III.

You should work through these yourself. I will outline the steps.

Symmetry in quantum physics

First let's recall the definition of the angular momentum operator

$$\vec{L} = \vec{r} \times \vec{p} = \frac{\hbar}{i}\vec{r} \times \vec{\nabla} = \hat{\mathbf{e}}_i L_i = \epsilon_{uvw}\hat{\mathbf{e}}_u x_v p_w$$

Everything we do in this part requires just the use of the fundamentla "uncertainty principle" commutator

$$[p_j, x_k] = p_j x_k - x_k p_j = \frac{\hbar}{i}\delta_{jk}$$

Try a simple application of this!

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$$[x_a p_j, x_k] = \frac{\hbar}{i}\delta_{jk} x_a$$

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Now let's calculate $[L_a, x_b] = L_a x_b - x_b L_a$

$$[L_a, x_b] = [\epsilon_{acd} x_c p_d, x_b] = \epsilon_{acd}\frac{\hbar}{i} x_c \delta_{bd} = i\hbar\epsilon_{abc} x_c$$

$$[L_a, x_b] = i\hbar\epsilon_{abc} x_c$$

$$\frac{1}{2}\left(\vec{L} \times \vec{x} + \vec{x} \times \vec{L}\right) = i\hbar\vec{x}$$

QUANTUM!

Symmetry in quantum physics

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Now I recommend that you pause and work out the following identities:

$$[L_a, p_b] = i\hbar\epsilon_{abc}p_c$$

$$[L_a, L_b] = i\hbar\epsilon_{abc}L_c$$

$$\vec{L} \times \vec{L} = i\hbar\vec{L}$$ **QUANTUM!**

$$[L_i, L^2] = 0$$ where $L^2 = L_i L_i$