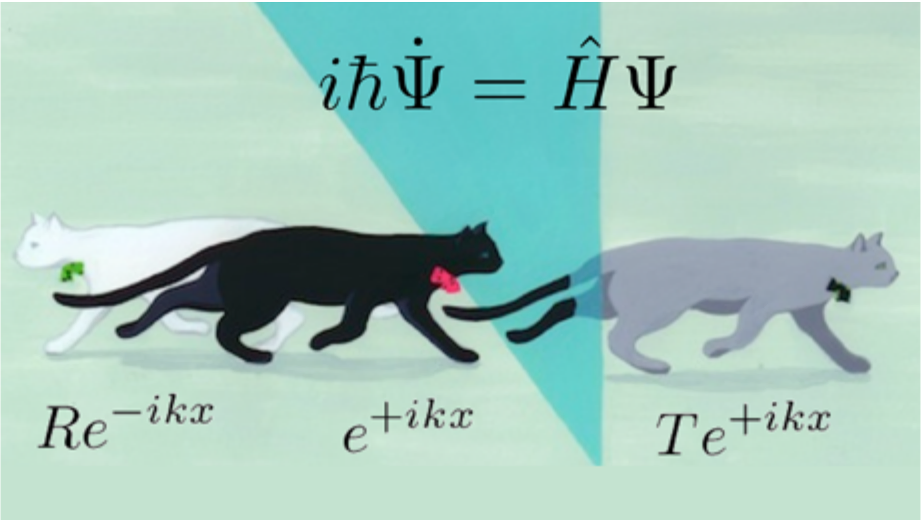
Exploring Quantum Physics

Coursera, Spring 2013 Instructors: Charles W. Clark and Victor Galitski

Atomic Structure and Spectra

Part V. A hidden symmetry



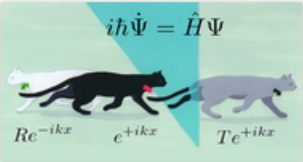
Atomic Structure and Spectra

coursera

UNIVERSITY OF MARYLAND

Exploring Quantum Physics

by Dr. Charles Clark and Dr. Victor Galitski



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- Discussion Forums
- Homeworks and Assignments
- Course Logistics
- Syllabus
- About Us

Additional Materials

Mathematical References

For those who may want to brush up on their mathematics before or during the courses, we present **a short crash course on the math**

Original scientific literature

Some lectures draw on material from original scientific literature, which we provide here for the convenience of students.

- Atomic clocks and quantum computers
- The Bohr model of the atom
- Bose-Einstein condensation
- The discovery of deuterium
- The discovery of deuterium - a simplified account
- The green laser pointer
- The photoelectric effect
- Does quantum mechanics provide a complete description of physical reality?
- Young's double slit experiment and diffraction

Atomic Structure and Spectra

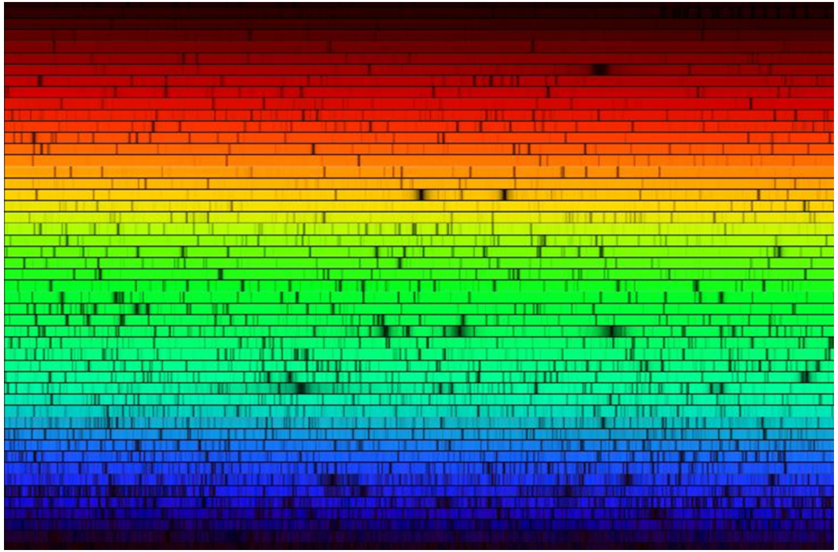
1 $^2S_{1/2}$
H
Hydrogen
1.00794
1s
13.5984

656 nm 410 nm

λ = 650 nm R

λ = 532 nm G

λ = 405 nm V

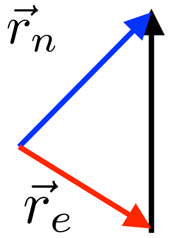


Atomic Structure and Spectra

Classical equations of motion for hydrogen-like systems : Hamiltonian form

$$\dot{\vec{p}} = -Ze^2\vec{r}/r^3$$

$$\dot{\vec{r}} = \frac{1}{\mu}\vec{p}$$



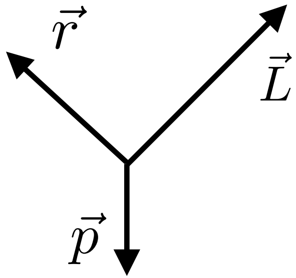
$$\vec{r} = \vec{r}_n - \vec{r}_e$$

$$r = \sqrt{\vec{r}\cdot\vec{r}}$$

$$E = T + V(r) = \frac{p^2}{2\mu} - \frac{Ze^2}{r}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

Note that $\vec{r}$ and $\vec{p}$ define a plane that is perpendicular to $\vec{L}$



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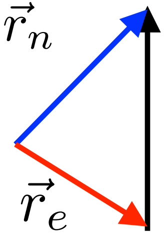

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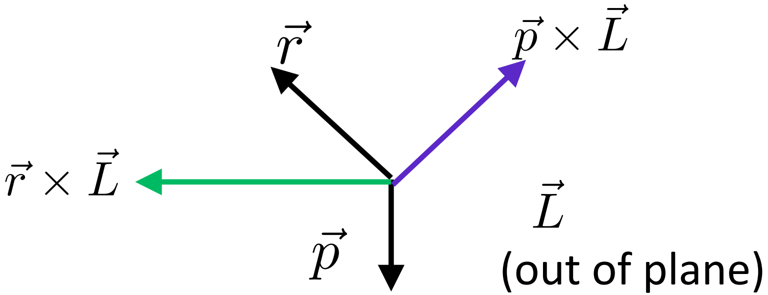

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$$\vec{r} \times \vec{L} = \vec{r} \times (\vec{r} \times \vec{p}) = \mu\vec{r} \times (\vec{r} \times \dot{\vec{r}})$$

BAC-CAB rule:

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A}\vec{B})$$

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$$\vec{r} \times (\vec{r} \times \dot{\vec{r}}) = \vec{r}(\vec{r} \cdot \dot{\vec{r}}) - \dot{\vec{r}}(\vec{r} \cdot \vec{r})$$
$$= \vec{r}r\dot{r} - \dot{\vec{r}}r^2 = -r^3\dot{\hat{r}}$$

$\hat{r} = \vec{r}/r$ the unit vector of $\vec{r}$

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The Runge – Lenz vector

$$\vec{A} = \vec{p} \times \vec{L} - Ze^2\mu\hat{r}$$

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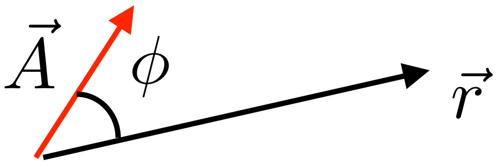
The Runge-Lenz vector defines an orbit:



$$rA\cos\phi = \vec{r}\cdot(\vec{p} \times \vec{L}) - rZe^2\mu$$

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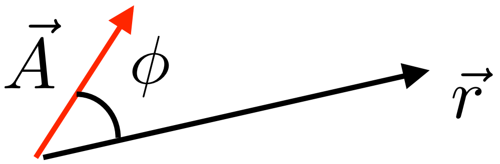

$$r A \cos\phi = \dot{\vec{r}}(\vec{p} \times \vec{L}) - rZe^2\mu$$
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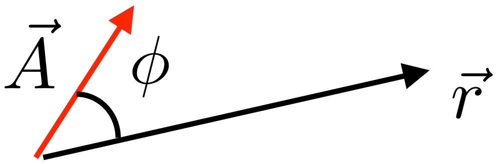

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$$r = \frac{L^2}{Ze^2\mu + A\cos\phi}$$

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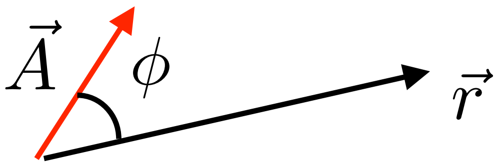
One final calculation:

$$A^2 = \vec{A}\cdot\vec{A} = 2\mu E L^2 + Z^2e^4\mu^2$$



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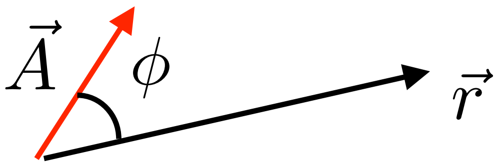

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$$E = \frac{p^2}{2\mu} - \frac{Ze^2}{r}$$

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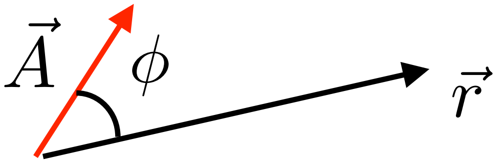

Electron – nucleus system completely described by constants of motion up to choice of initial time.

All orbits are ellipses for E < 0 (like planets)
These correspond to the bound states of atoms and molecules that produce sharp emission or absorption lines

All orbit are hyperbolas for E > 0
These correspond to the states produced in the photoelectric effect and the ionization of gases by ultraviolet radiation, as discussed by Einstein, and in Rutherford's scattering experiment.