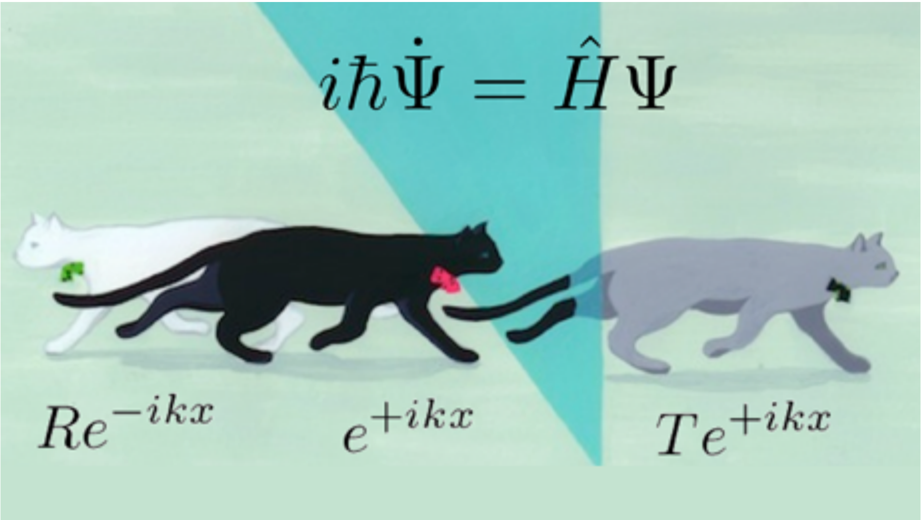
Exploring Quantum Physics

Coursera, Spring 2013 Instructors: Charles W. Clark and Victor Galitski

Atomic Structure and Spectra

Part IV. Still cracking the hydrogen code

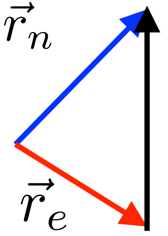


Atomic Structure and Spectra

Classical equations of motion for hydrogen-like systems

Nucleus: position, mass, electric charge $\vec{r}_n; M; +Ze$

Electron: position, mass, electric charge $\vec{r}_e; m; -e$



$$\vec{r} = \vec{r}_n - \vec{r}_e$$

$$r = \sqrt{\vec{r} \cdot \vec{r}}$$

$$M\ddot{\vec{r}}_n = -Ze^2\vec{r}/r^3$$

$$m\ddot{\vec{r}}_e = +Ze^2\vec{r}/r^3$$

$$\ddot{\vec{r}} = -\left[\frac{1}{M} + \frac{1}{m}\right] Ze^2\vec{r}/r^3$$

$$\mu\ddot{\vec{r}} = -Ze^2\vec{r}/r^3$$

$$\frac{1}{\mu} = \left[\frac{1}{M} + \frac{1}{m}\right]$$

$$\vec{p} = \mu\dot{\vec{r}}$$

First-order equations:

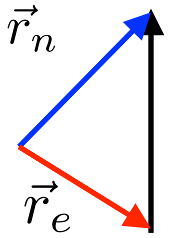
$$\dot{\vec{p}} = -Ze^2\vec{r}/r^3$$

$$\dot{\vec{r}} = \frac{1}{\mu}\vec{p}$$

Atomic Structure and Spectra

Classical equations of motion for hydrogen-like systems : Hamiltonian form

$$\dot{\vec{p}} = -Ze^2\vec{r}/r^3$$
$$\dot{\vec{r}} = \frac{1}{\mu}\vec{p}$$



$$\vec{r} = \vec{r}_n - \vec{r}_e$$
$$r = \sqrt{\vec{r}\cdot\vec{r}}$$

Find constants of motion from products of $\vec{p}$ and $\vec{r}$:

$$T = \frac{\vec{p}\cdot\vec{p}}{2\mu} = \frac{p^2}{2\mu}$$

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Classical equations of motion for hydrogen-like systems : Hamiltonian form

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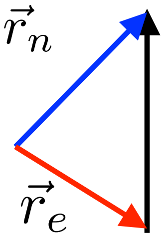
$$\dot{\vec{r}} = \frac{1}{\mu}\vec{p}$$



$$\vec{r} = \vec{r}_n - \vec{r}_e$$
$$r = \sqrt{\vec{r}\cdot\vec{r}}$$

Find constants of motion from products of $\vec{p}$ and $\vec{r}$:

$$T = \frac{\vec{p}\cdot\vec{p}}{2\mu} = \frac{p^2}{2\mu}$$

$$\dot{T} = \frac{\dot{\vec{p}}\cdot\vec{p}}{\mu} = -Ze^2\dot{\vec{r}}\cdot\vec{r}/r^3 = -Ze^2\dot{r}/r^3 = -\frac{\partial}{\partial t}\left(\frac{-Ze^2}{r}\right) = -\dot{V}(r)$$

$$V(r) = -Ze^2/r$$

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Classical equations of motion for hydrogen-like systems : Hamiltonian form

$$\dot{\vec{p}} = -Ze^2\vec{r}/r^3$$

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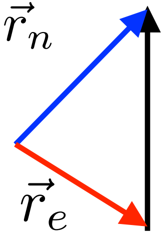

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$$V(r) = -Ze^2/r$$

We have found a constant of motion:

E = T + V(r)

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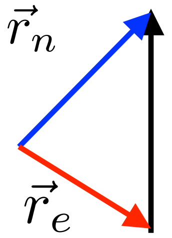
Classical equations of motion for hydrogen-like systems : Hamiltonian form

$$\dot{\vec{p}} = -Ze^2\vec{r}/r^3$$
$$\dot{\vec{r}} = \frac{1}{\mu}\vec{p}$$



$$\vec{r} = \vec{r}_n - \vec{r}_e$$
$$r = \sqrt{\vec{r}\cdot\vec{r}}$$

Find constants of motion from products of $\vec{p}$ and $\vec{r}$:

$T = \frac{\vec{p}\cdot\vec{p}}{2\mu} = \frac{p^2}{2\mu}$ ‖ $V(r) = -Ze^2/r$ ‖ E = T + V(r)

What about a constant of motion involving $\vec{r}\cdot\vec{r}$ or $\vec{r}\cdot\vec{p}$?

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Classical equations of motion for hydrogen-like systems : Hamiltonian form

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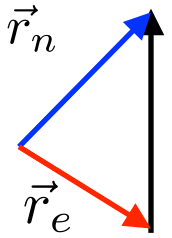

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$T = \frac{\vec{p}\cdot\vec{p}}{2\mu} = \frac{p^2}{2\mu}$ ‖ $V(r) = -Ze^2/r$ ‖ **E = T + V(r)**

What about a constant of motion involving $\vec{r}\cdot\vec{r}$ or $\vec{r}\cdot\vec{p}$?

How about $\vec{L} = \vec{r}\times\vec{p}$?

$$\dot{\vec{L}} = \dot{\vec{r}}\times\vec{p} + \vec{r}\times\dot{\vec{p}}$$

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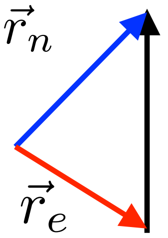
Classical equations of motion for hydrogen-like systems : Hamiltonian form

$$\dot{\vec{p}} = -Ze^2\vec{r}/r^3$$

$$\dot{\vec{r}} = \frac{1}{\mu}\vec{p}$$



$$\vec{r} = \vec{r}_n - \vec{r}_e$$

$$r = \sqrt{\vec{r}\cdot\vec{r}}$$

Find constants of motion from products of $\vec{p}$ and $\vec{r}$:

$T = \frac{\vec{p}\cdot\vec{p}}{2\mu} = \frac{p^2}{2\mu}$ ‖ $V(r) = -Ze^2/r$ ‖ E = T + V(r)

What about a constant of motion involving $\vec{r}\cdot\vec{r}$ or $\vec{r}\cdot\vec{p}$?

How about $\vec{L} = \vec{r}\times\vec{p}$?

$$\dot{\vec{L}} = \dot{\vec{r}}\times\vec{p} + \vec{r}\times\dot{\vec{p}}$$

We have found a constant of motion:

$$\vec{L} = \vec{r}\times\vec{p}$$

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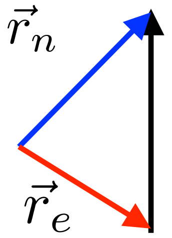
Classical equations of motion for hydrogen-like systems : Hamiltonian form

$$\dot{\vec{p}} = -Ze^2\vec{r}/r^3$$

$$\dot{\vec{r}} = \frac{1}{\mu}\vec{p}$$



$$\vec{r} = \vec{r}_n - \vec{r}_e$$

$$r = \sqrt{\vec{r}\cdot\vec{r}}$$

$$E = T + V(r) = \frac{p^2}{2\mu} - \frac{Ze^2}{r}$$

$$\vec{L} = \vec{r}\times\vec{p}$$

Note that $\vec{r}$ and $\vec{p}$ define a plane that is perpendicular to $\vec{L}$