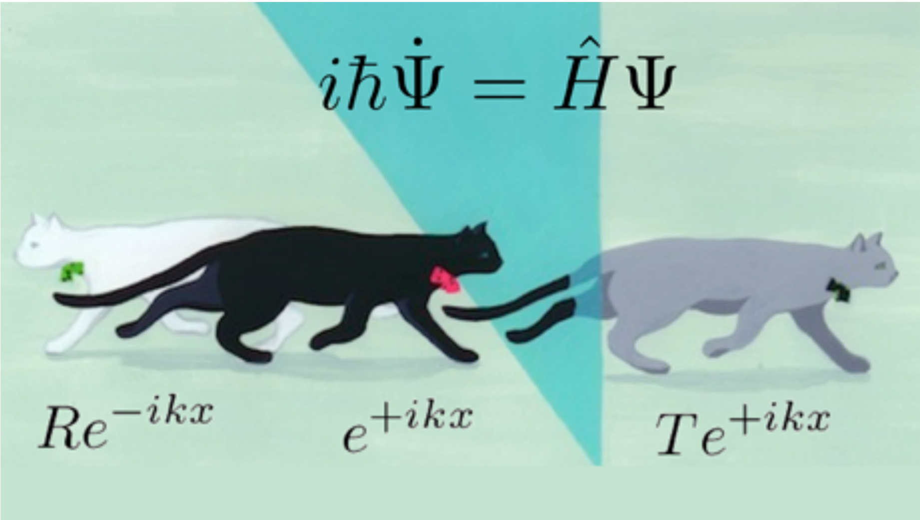
Exploring Quantum Physics

Coursera, Spring 2013 Instructors: Charles W. Clark and Victor Galitski

Quantum harmonic oscillator

Part I: Creation/annihilation operators



Taking the square root of the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{x}^2}{2}$$

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

$$m, \ \omega, \ \hbar$$

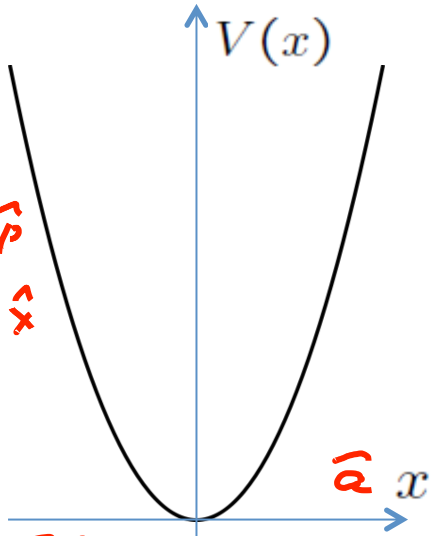
$$\hbar\omega$$

$\hat{p}^\dagger = \hat{p}$

$\hat{x}^\dagger = \hat{x}$

$\hat{a}^\dagger$

$\hat{a}$



$$\frac{\hat{H}}{\hbar\omega} = \frac{m\omega\hat{x}^2}{2\hbar} + \frac{\hat{p}^2}{2m\hbar\omega}$$

$$= \left(\sqrt{\frac{m\omega}{2\hbar}}\,\hat{x}\right)^2 + \left(\frac{\hat{p}}{\sqrt{2m\hbar\omega}}\right)^2 = \left[\sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} - \frac{i\hat{p}}{\sqrt{2m\hbar\omega}}\right]\left[\sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} + \frac{i\hat{p}}{\sqrt{2m\hbar\omega}}\right] + \dots$$

$$= \frac{m\omega\hat{x}^2}{2\hbar} + \frac{\hat{p}^2}{2m\hbar\omega} + \frac{i}{2\hbar}\underbrace{(\hat{x}\hat{p} - \hat{p}\hat{x})}_{\neq 0} + \dots$$

$$\dots = -\frac{i}{2\hbar}[\hat{x}, \hat{p}]$$

Commutation relations