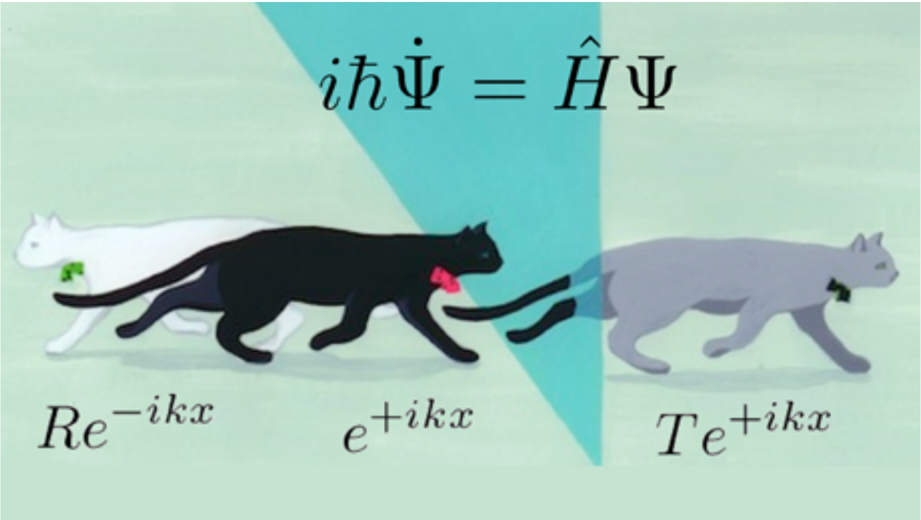
Exploring Quantum Physics

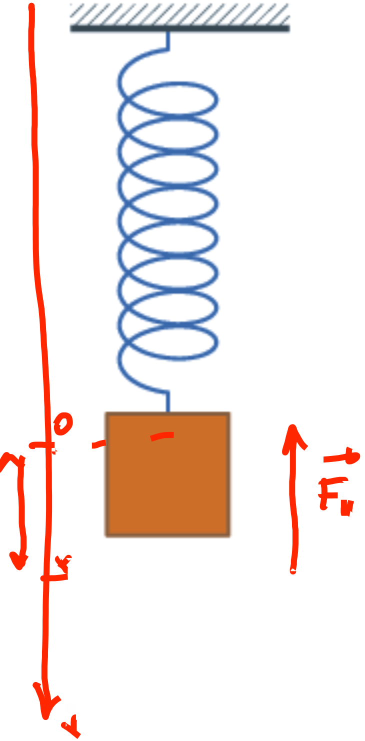
Coursera, Spring 2013 Instructors: Charles W. Clark and Victor Galitski

Quantum harmonic oscillator

Part I: Reminder from classical physics



Classical harmonic oscillator



- Hook's force, $F = -kx = -\frac{d}{dx}\underbrace{\left(\frac{kx^2}{2}\right)}_{V(x)}$

- The 2nd Newton law: $\ddot{x} + \omega^2 x = 0$

$$m\frac{d^2x}{dt^2} = F = -kx \quad \omega = \sqrt{\frac{k}{m}} \quad A\sin(\omega t + \varphi_0)$$

- Solution:

$$\boxed{x = x_0 \sin\left(\omega t + \phi_0\right),} \text{ with } \omega = \sqrt{\frac{k}{m}}$$

Phase portrait of a harmonic oscillator

- Let us choose the initial condition, so that $\phi_0 = 0$

$$x(t) = x_0 \sin(\omega t)$$

- Velocity

$$v(t) = \frac{dx}{dt} = x_0 \omega \cos(\omega t)$$

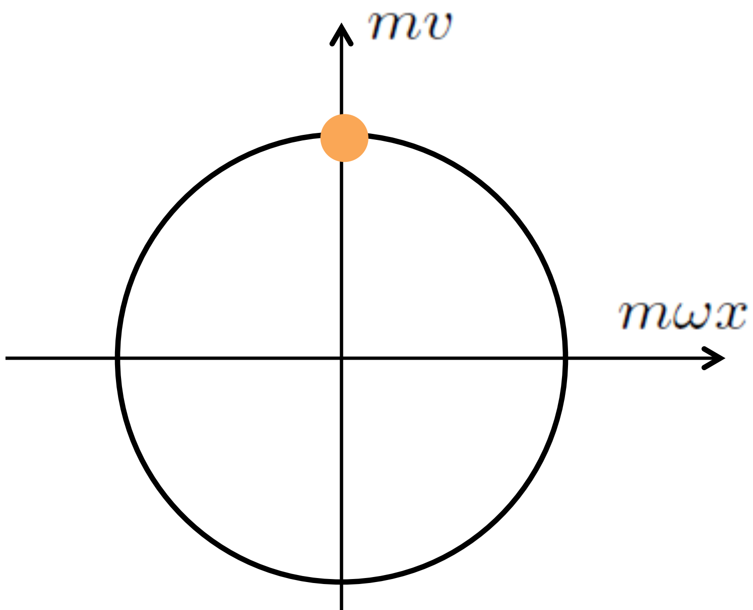
$V(x) = \frac{1}{2} k x^2$

- Energy

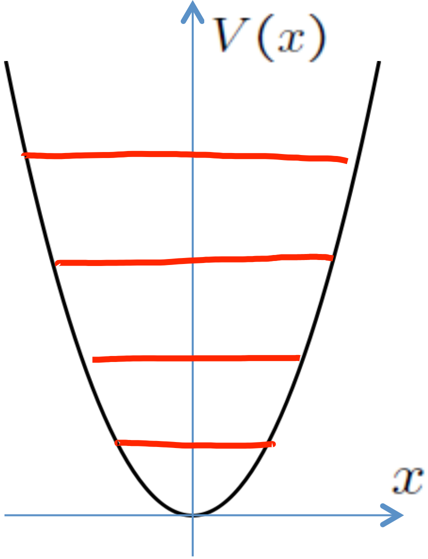
$$E = \frac{m v^2(t)}{2} + \frac{1}{2} \underbrace{m\omega^2}_{k} x^2(t)$$

$E = \frac{m}{2} x_0^2 \omega^2 \equiv const$

$\cos^2\alpha + \sin^2\alpha = 1$



Making the oscillator quantum



- Classical energy

$$E = \frac{p^2}{2m} + \underbrace{\frac{m\omega^2 x^2}{2}}_{V(x)}$$

- Usual prescirption:

$$E \longrightarrow \hat{H}, \; p \longrightarrow \hat{p} = -i\hbar\frac{d}{dx}, \; x \longrightarrow \hat{x} = x$$

- Quantum Hamiltonian

$$\boxed{\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 \hat{x}^2}{2}}$$

$\hat{H}|\psi\rangle = E|\psi\rangle$