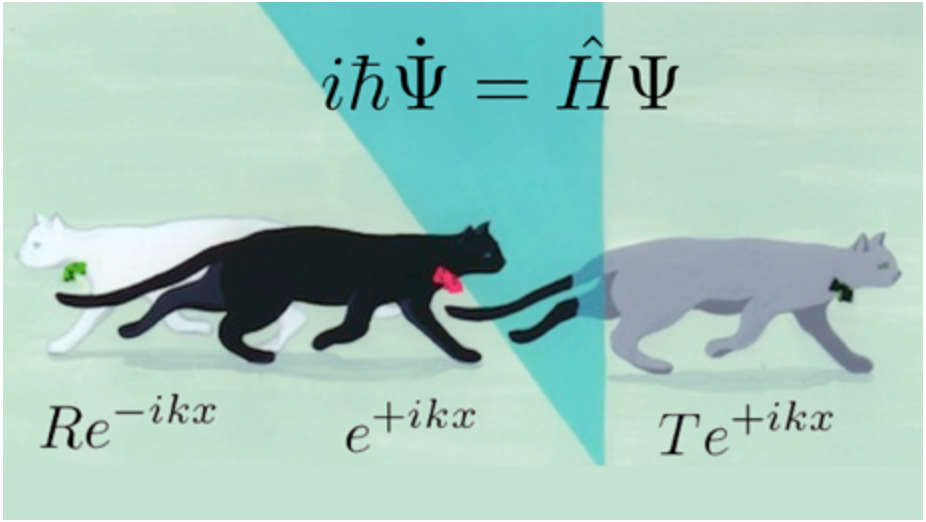
Exploring Quantum Physics

Coursera, Spring 2013 Instructors: Charles W. Clark and Victor Galitski

Cooper pairing in superconductors

Part IV: The Cooper problem



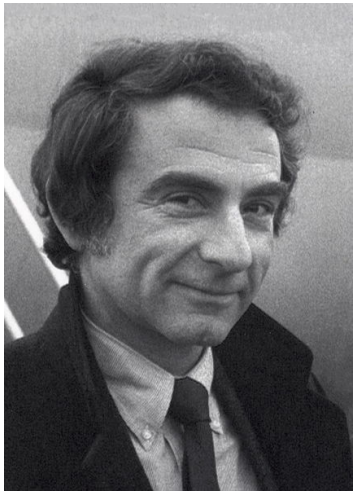
Bardeen-Cooper-Schrieffer theory

Bound Electron Pairs in a Degenerate Fermi Gas*

LEON N. COOPER

Physics Department, University of Illinois, Urbana, Illinois

(Received September 21, 1956)



The Nobel Prize in Physics 1972
John Bardeen, Leon N. Cooper, Robert Schrieffer

The Nobel Prize in Physics 1972 was awarded jointly to John Bardeen, Leon Neil Cooper and John Robert Schrieffer *"for their jointly developed theory of superconductivity, usually called the BCS-theory"*.

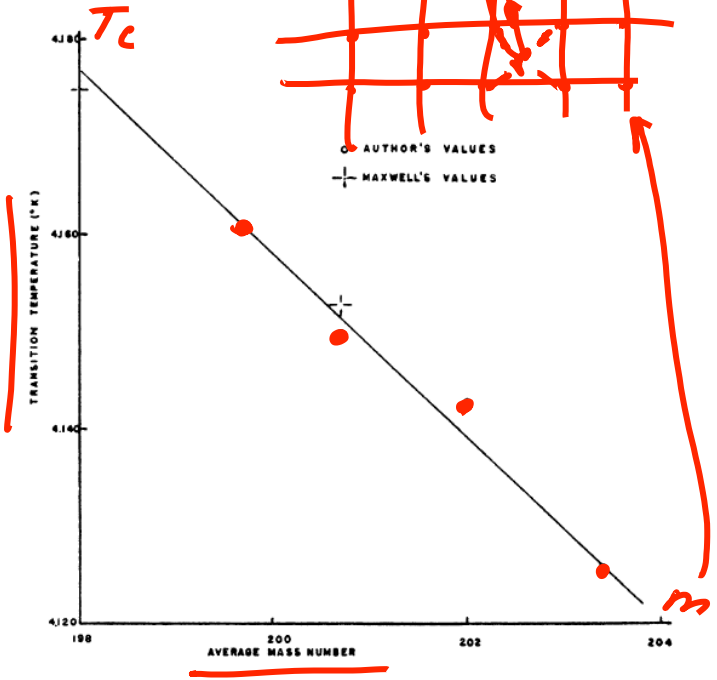
Isotope effect

Superconductivity of Isotopes of Mercury*

C. A. Reynolds, B. Serin, W. H. Wright, and L. B. Nesbitt
Rutgers University, New Brunswick, New Jersey
March 24, 1950

Table I. Transition temperatures.

Sample	Average mass number	T_0(°K)
1	203.4	4.126
2	202.0	4.143
3	200.7	4.150
4	199.7	4.161



This discovery led to the realization that interactions with the crystal lattice play a key role. Electrons exchange phonons (waves running through the lattice), which leads to weak effective phonon-mediated attraction between the electrons.

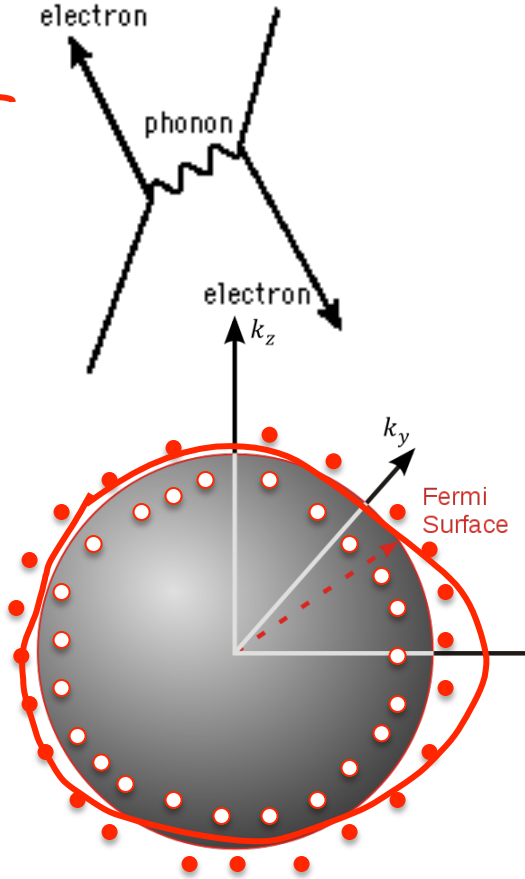
Spherical-cow model of the phonon-mediated attraction

- Electrons interact by exchanging phonons. Energy and momentum must be conserved in this (and any other) process
- The phonon energies available are pretty low compared to the Fermi energy. When converted to temperature:

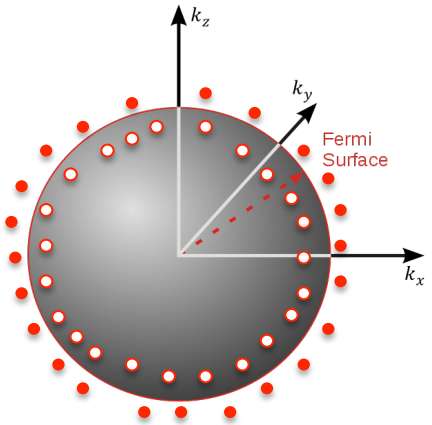
$$T_F \sim 10,000\,K \text{ and } T_D \sim 400\,K$$

- The exact interaction is complicated. We'll use a simple model:

$$V(\vec{p}) = \begin{cases} -V_0, & \text{if } \frac{p^2}{2m} - E_F < \hbar\omega_D \\ 0, & \text{otherwise.} \end{cases}$$



Cooper pairing problem



- The Schrödinger equation (*c.f.*, Lecture 5):

$$\left[\frac{\hbar^2}{m}\vec{p}^2 - V_0\right]\psi(\vec{p}) = E\psi(\vec{p})$$

Keeping in mind that only momenta near the Fermi surface play a role.

- The energy of two electrons: $E = 2E_F + \Delta = \frac{p_F^2}{m} + \Delta$