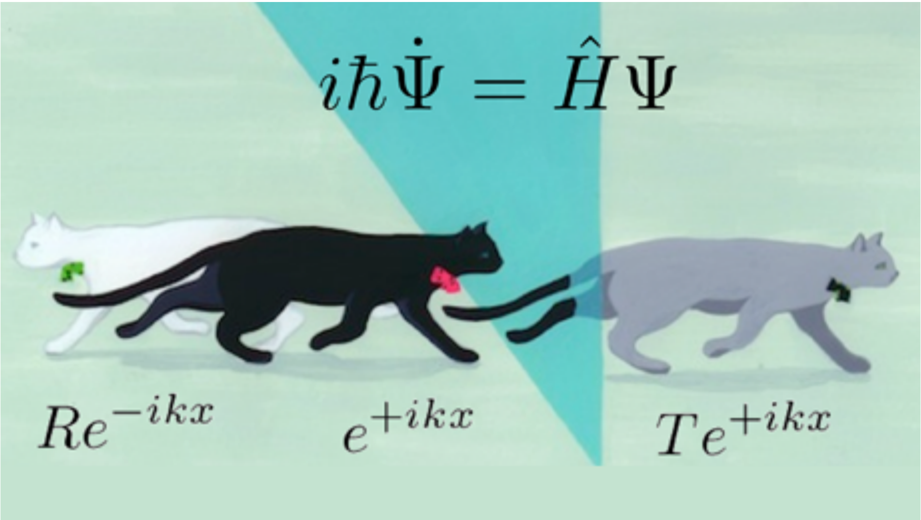
Exploring Quantum Physics

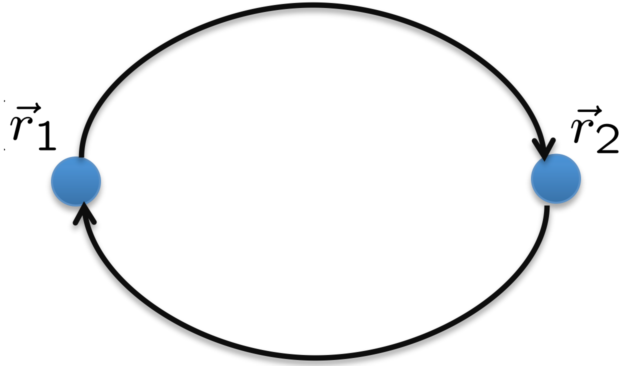
Coursera, Spring 2013 Instructors: Charles W. Clark and Victor Galitski

Cooper pairing in superconductors

Part II: Quantum statistics: bosons and fermions



Particle braiding and spin-statistics theorem



$$\psi(\vec{r}_1, \vec{r}_2) \to e^{i\phi}\psi(\vec{r}_2, \vec{r}_1)$$

What are the possible values of the quantum statistical phase, ϕ?

In his seminal paper, "The Connection Between Spin and Statistics," Phy Rev. **58**, 716722 (1940), Wolfgang Pauli proved that there exists a one-t one correspondence between the internal angular momentum of a partic (spin) and its quantum statistics:

- For particles with integer (including zero) spin, $\phi = 0$ (bosons)
- For particles with half-integer spin, $\phi = \pi$ (fermions).

Pauli exclusion principle for fermions

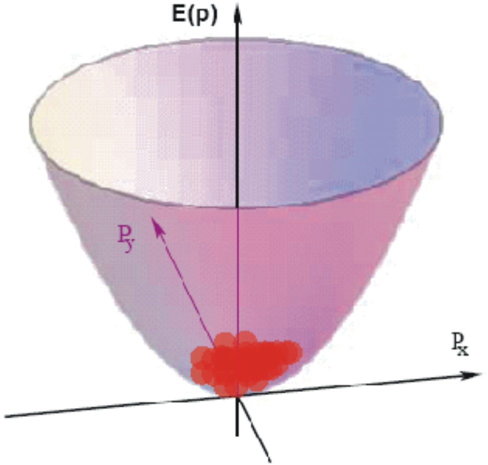
- What are the consequences of $\psi(\vec{r}_1, \vec{r}_2) = e^{i\pi}\psi(\vec{r}_2, \vec{r}_1) = -\psi(\vec{r}_2, \vec{r}_1)$?
- An important one for $\vec{r}_1 = \vec{r}_2$ is $\psi(\vec{r}_1, \vec{r}_1) = -\psi(\vec{r}_1, \vec{r}_1) = 0$!
- It means that the probability of finding two identical fermions in the same point $\propto |\psi(\vec{r}_1, \vec{r}_1)|^2$ is zero.
- More generally, two identical fermions can not occupy the same quantum state. There is no such constraint for bosons.

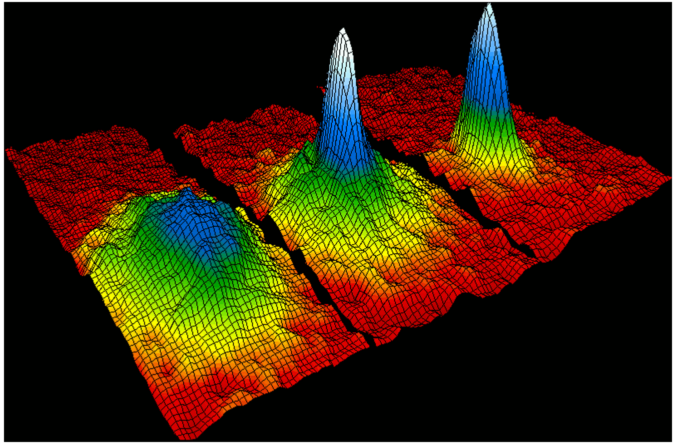
Given a single particle quantum state $|i\rangle$, the possible occupation numbers for fermions are $n_i^{(f)} = 0, 1$, while for bosons $n_i^{(b)} = 0, 1, 2, \ldots \infty$.

Ground state of a many boson system: Bose-Einstein condensate

- What happens if we "pour" identical bosons into a prescribed landscape of quantum states (*i.e.*, single-particle states, $|i\rangle$, with energies, E_i)?

- At low temperatures, they would want to form the lowest-energy state (ground state), which in the absence of interactions corresponds to putting all bosons into a single lowest-energy state.

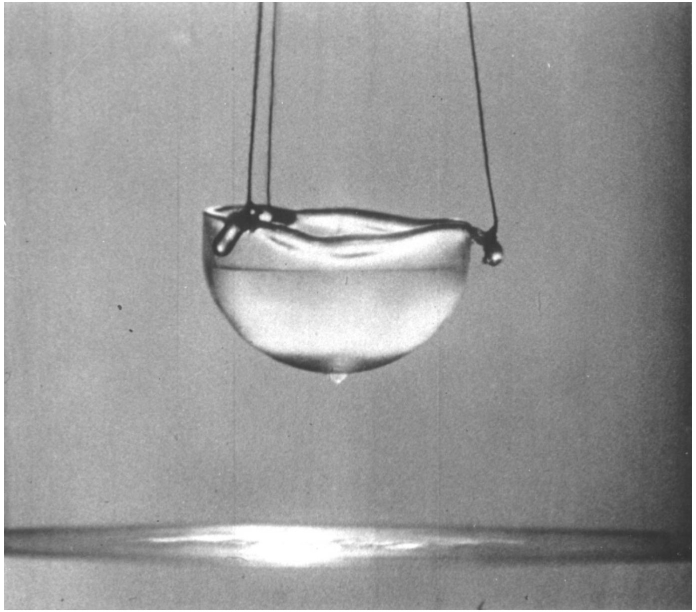




From Science **269**, 198 (1995) - 2001 Nobel Prize in Physics

Superfluidity in a BEC

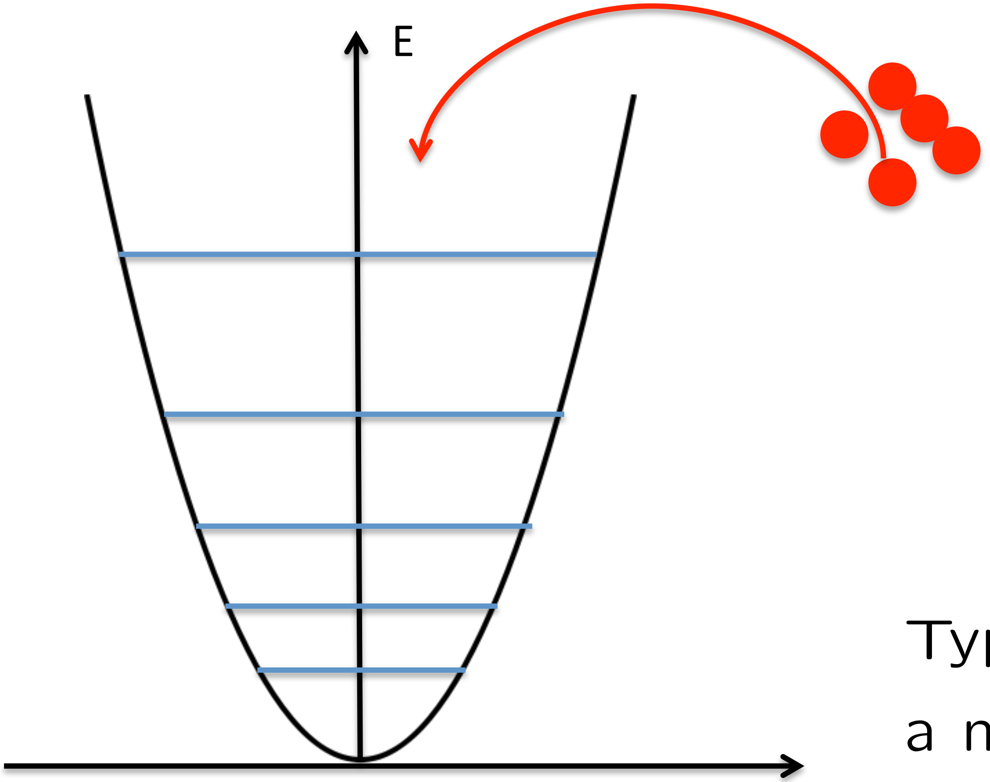
Closely related to Bose-Einstein condensation is the phenomenon of superfluidity, where a condensate of bosons forms a quantum liquid, which has zero viscocity and flows without resistance.

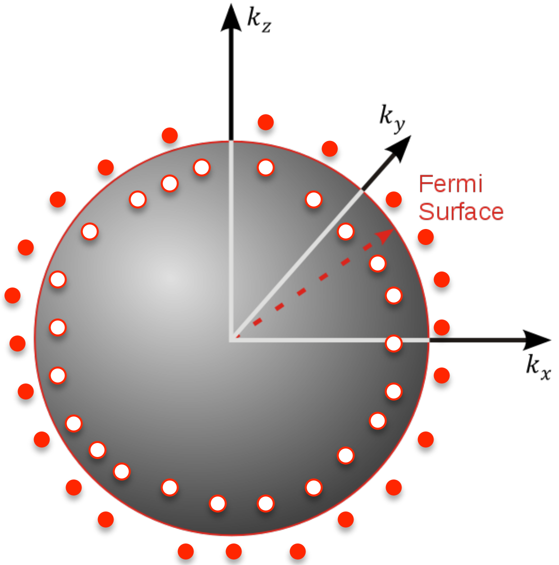


A cup containing superfluid Helium

- Superfluidity was discovered experimentally in Helium by Pyotr Kapitsa 1937 (Nobel prize, 1978)
- Mathematical theory of superfluidity was put together by Lev Landau (Nobel prize, 1962)

Ground state of a many-fermion system: Fermi gas





Typical Fermi temperature in a metal, $T_F = \frac{1}{k_B}\frac{p_F^2}{2m} \sim 10,000\,\mathrm{K}$

A key to solving superconductivity turned out to be figuring out how to convert electron liquid into a bosonic superfluid.