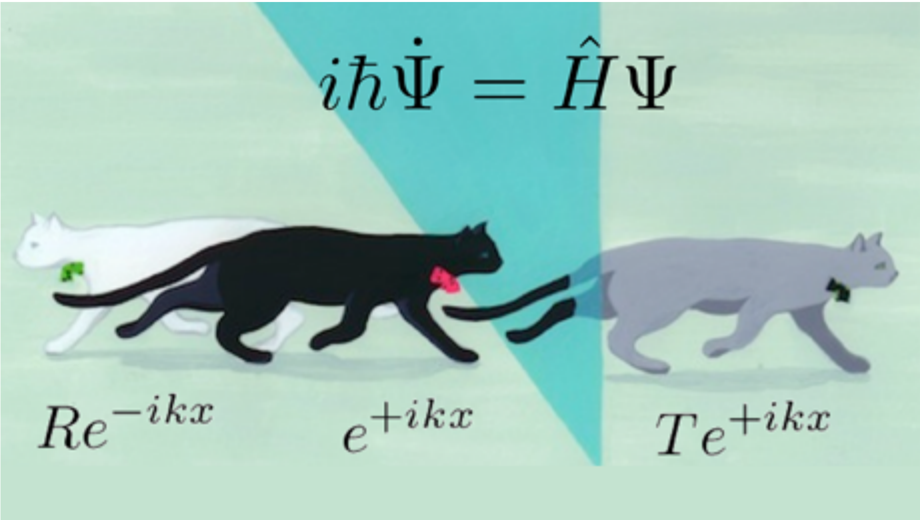
Exploring Quantum Physics

Coursera, Spring 2013 Instructors: Charles W. Clark and Victor Galitski

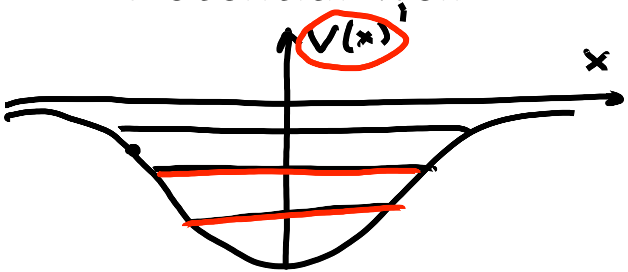
Bound states in quantum potential wells

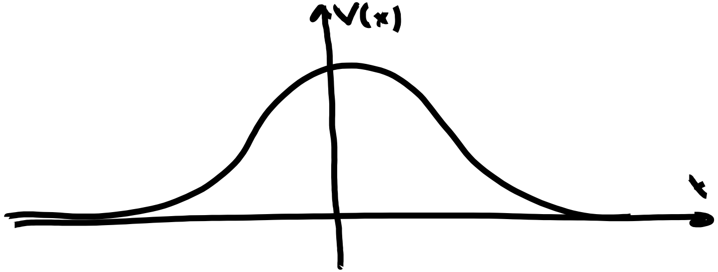
Part I: *Electron in a box*

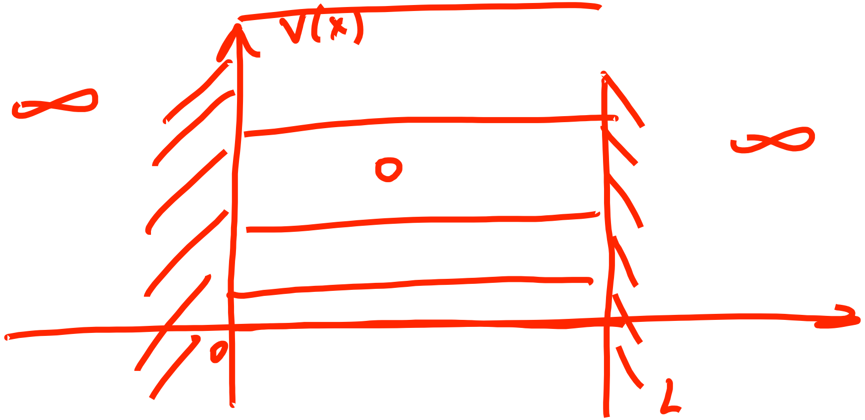


Types of problems for the Schrodinger equation

- Single-particle quantum mechanics, $i\hbar\dot{\Psi} = \left[\frac{\hat{\mathbf{p}}^2}{2m} + V(\mathbf{r})\right]\Psi$
  - Potential well
  - Potential barrier







Quantization in a … guitar string and quantum well

$\omega = v_s \lambda$

- Wavelength "quantization" in a guitar string

$$\boxed{\lambda_n = \frac{2L}{n}, n = 1, 2, \ldots}$$

$$u_n(x) = A_n \sin\left(\frac{2\pi x}{\lambda_n}\right)$$

$\lambda = 2L$

- Wavelength quantization in a quantum well

$$\boxed{\lambda_n = \frac{2L}{n}, n = 1, 2, \ldots}$$

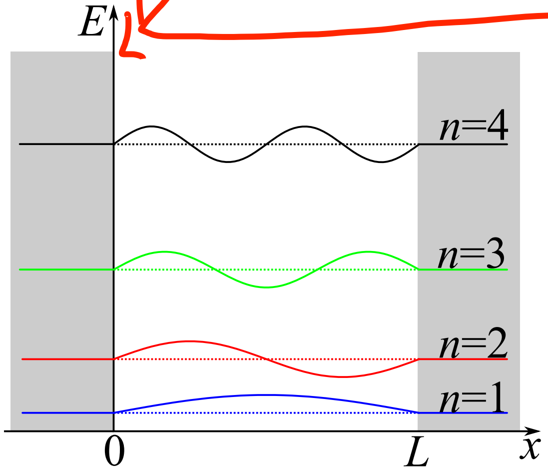
$$\psi_n(x) = A_n \sin\left(\frac{2\pi x}{\lambda_n}\right)$$

$E = \frac{p^2}{2m}$

$p = \frac{2\pi\hbar}{\lambda}$

$E_n = \frac{\pi^2\hbar^2 n^2}{L^2}\frac{1}{2m}$

$n = 1, 2, 3, \ldots$



$|\psi|^2\big|_{x=0,L} = 0$

Formal solution

- The actual equation we have to solve is a free S. Eq.:

$$i\hbar\dot{\Psi}(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}, \quad 0 < x < L, \text{ with } \Psi(0,t) = \Psi(L,t) \equiv 0$$

- Standard Ansatz, $\Psi_{\pm p}(x,t) = e^{\frac{i}{\hbar}(\pm px - Et)}$ is a solution, with $E = \frac{p^2}{2m}$

- The linear combination

$$\Psi(x,t) = \frac{A}{2i}\left(e^{\frac{i}{\hbar}px} - e^{-\frac{i}{\hbar}px}\right)e^{-\frac{i}{\hbar}Et} = A\sin\left(px/\hbar\right)e^{-\frac{i}{\hbar}Et}$$

- The right boundary condition $\Psi(L,t) \equiv 0$ yields:

$$pL/\hbar = \pi, 2\pi, 3\pi, \ldots \to E_n = \frac{\pi^2\hbar^2}{2mL^2}n^2, \; n = 1, 2, 3, \ldots$$