Joint Quantum Institute

Exploring Quantum Physics

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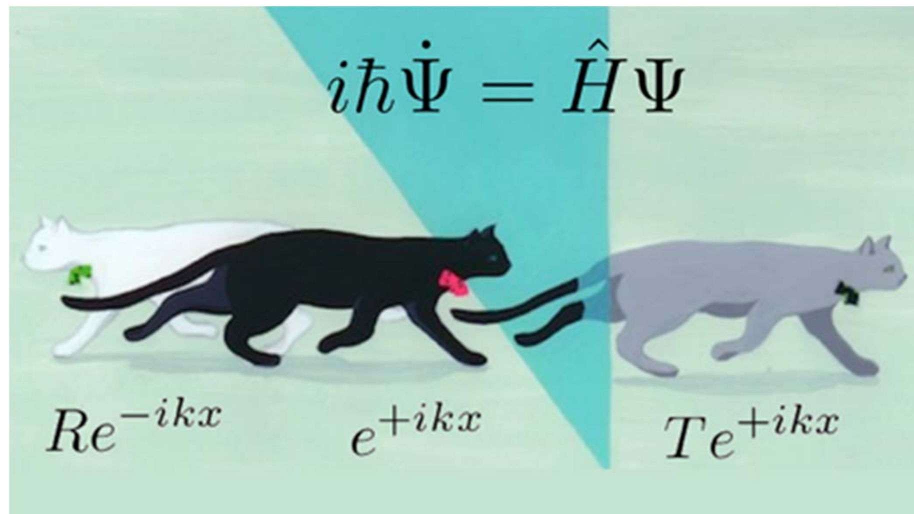
Feynman path integral

Part III: *Deriving the path integral*



Summary of what and how we want to calculate

- Transition amplitude from an initial point, x_i, to a final point, x_f, a.k.a. "propagator" a.k.a. "Green function:"

$$\underbrace{G(x_i, x_f; t) = \langle x_f | e^{-\frac{i}{\hbar}\hat{H}t} | x_i \rangle}$$

(we'll work in 1D for simplicity)

$\hat{u}(t)$

- Useful formulas

$|\psi\rangle$:

$\int |x\rangle\langle x| dx |\psi\rangle = \int \langle x|\psi\rangle |x\rangle dx \equiv |\psi\rangle$

  - Resolution of the identity operator

$$\hat{1} = \int |x\rangle \langle x| \, dx$$

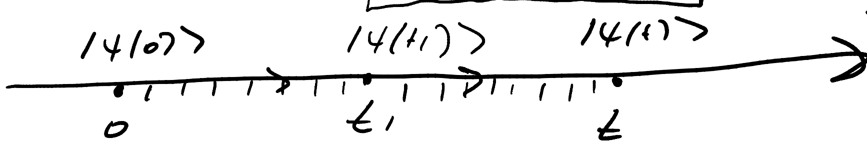
  - Splitting the time interval

$\hat{u}(t)$

$\parallel$

$$|\Psi(t)\rangle = \hat{U}(t)\,|\Psi(0)\rangle = \underbrace{\hat{U}(t-t_1)\,|\Psi(t_1)\rangle} = \hat{U}(t-t_1)\underbrace{\hat{U}(t_1)}\,|\Psi(0)\rangle$$

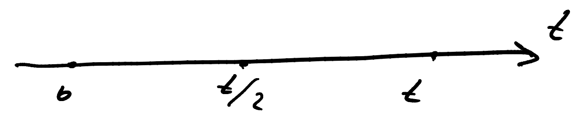
t



Splitting the time interval

- Let's re-write the evolution operator, $\hat{U}(t) = e^{-\frac{i}{\hbar}\hat{H}t}$, using $\frac{1}{2}+\frac{1}{2}=1$:

$$\hat{U}(t) = \hat{U}(t/2)\hat{U}(t/2)$$

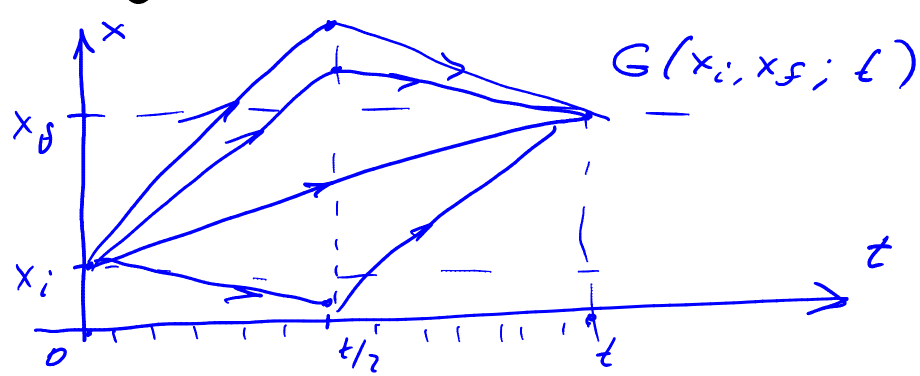


- We can split the propagator in two using $\int dx\,|x\rangle\langle x| = 1$:

$$\langle x_f|\hat{U}(t)|x_i\rangle = \int dx \langle x_f|\hat{U}(t/2)|x\rangle\langle x|\hat{U}(t/2)|x_i\rangle$$

$$\langle x_f| U(\tfrac{t}{2})\ U(\tfrac{t}{2})|x_i\rangle$$

$$\int |x\rangle\langle x|\,dx$$



Getting rid of the operators

- We can continue it further splitting time interval into tiny pieces:

$$\hat{U}(t) = \underbrace{\hat{U}(t/N) \times \hat{U}(t/N) \times \ldots \times \hat{U}(t/N)}_{N \text{ times}} \qquad N \to \infty$$

$\int dx \, |x\rangle\langle x|$

until $\Delta t = t/N$ becomes "really" small and $e^{-\frac{i}{\hbar N}\hat{H}t} \approx 1 - \frac{i}{\hbar}\hat{H}\Delta t$

- So, our propagator becomes a product of matrix elements

$$\langle x_f | \hat{U}(t) | x_i \rangle = \int dx_1 \ldots x_{N-1} \prod_{k=0}^{N} \langle x_{k+1} | 1 - \frac{i}{\hbar N}\hat{H}t | x_k \rangle$$