Exploring Quantum Physics

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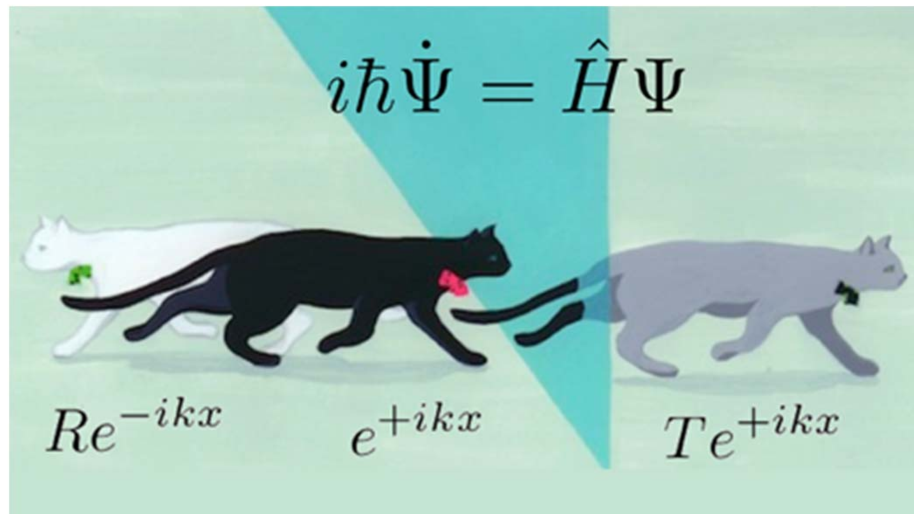
A physical interpretation of quantum theory

Part IV: Superposition principle; Dirac notations; representations



Superposition principle in quantum mechanics

- If $\Psi_1(\vec{r},t)$ and $\Psi_2(\vec{r},t)$ are solutions to the Schrödinger equation,

$$i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + V(\vec{r})\right]\Psi(\vec{r},t),$$

  $\Psi(\vec{r},t) = c_1\Psi_1(\vec{r},t) + c_2\Psi_2(\vec{r},t)$ is also a solution.

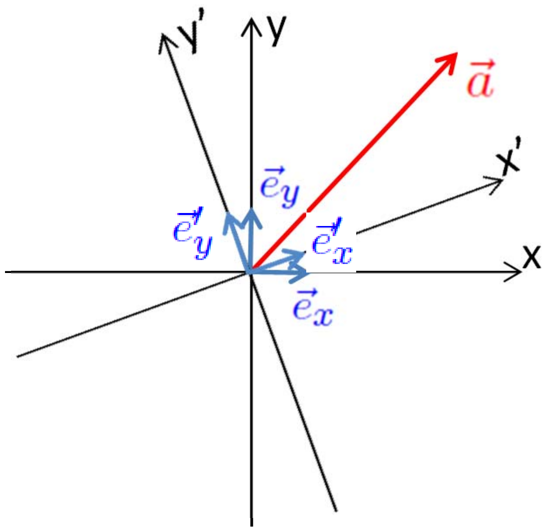
- This motivates the notion of a Hilbert space - a linear vector space, where quantum states live.

- The wave-function, $\Psi(\vec{r})$, is a specific representation of a quantum state (much like coordinates of a vector).

- Dirac suggested notations for the "vectors" of quantum states:

$$\langle\Psi| \text{ and } |\Psi\rangle$$

Simple reminder from linear algebra



$\vec{e}_x$ and $\vec{e}_y$ form a basis.

Written in these specific corrdinates:

$$\vec{a} = a_x \vec{e}_x + a_y \vec{e}_y$$

$$\vec{a} = \binom{a_x}{a_y} \qquad \vec{a}^\dagger = (a_x,\, a_y)$$

$$\vec{e}_x \vec{e}_x^\dagger + \vec{e}_y \vec{e}_y^\dagger = \binom{1}{0}(1,\, 0) + \binom{0}{1}(0,\, 1) = \text{Identity matrix}$$

A quantum state, $|\Psi\rangle$ can be expanded in a basis $\{|q\rangle\}$.

$$\sum_q |q\rangle\langle q| = 1 \text{ or } \int_q |q\rangle\langle q| = 1 \qquad |\Psi\rangle = \sum_q |q\rangle\langle q|\Psi\rangle \text{ or } = \int_q |q\rangle\langle q|\Psi\rangle$$

How to choose a basis/representation

Q: How do we choose a basis for a wave-function?

A: It's a question of convenience, but there are standard choices.

- Physical observables in quantum mechanics are associated with linear, Hermitian operators.
- For a generic operator, $\hat{A}$, the eigenvalue problem

$$\hat{A}\,|a\rangle = a\,|a\rangle$$

  defines eigenvectors that form a basis in the Hilbert space.
- $\Psi(a) = \langle a|\Psi\rangle$ is the wave-function in the a-representation.
- Standard choices: coordinate representation, $\Psi(x) = \langle x|\Psi\rangle$, and momentum representation, $\Psi(p) = \langle p|\Psi\rangle$.

Summary

- Quantum states "live" in a linear vector space - Hilbert space.
- A state-vector $|\Psi\rangle$ (wave-function) correspond to and contains maximum information about a quantum state, but still provides only probabilisic description about experimental outcomes.
- Observables ($\vec{r}$, $\vec{p}$, $\vec{L}$, E, etc) are represented by operators acting on the state-vectors.
- Eigenvectors of an operator, $\hat{A}$ ($\hat{A}\,|a\rangle = a\,|a\rangle$) define a basis and the a-representation of the wave-function, $\psi(a) = \langle a|\psi\rangle$. The coordinate representation, $\psi(\vec{r})$, is the most conventional one.
- $|\Psi(\vec{r},t)|^2\,dV$ gives the probability density of finding the particle in the elementary volume dV at the moment of time t.
- The expectation value of a quantity A in a state $|\psi\rangle$ is given by $\langle\psi|\hat{A}|\psi\rangle$.