Joint Quantum Institute

Exploring Quantum Physics

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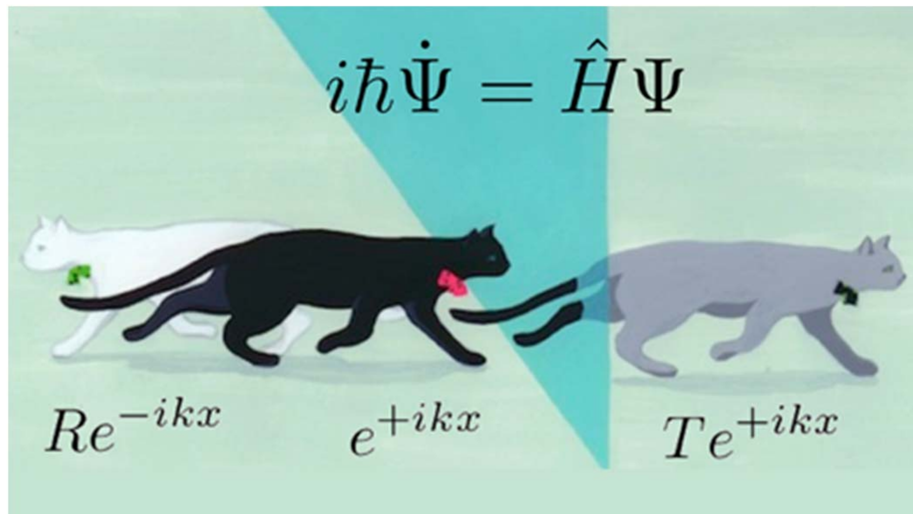
The meaning of the wave function

Part III: Operators



A reminder on how the Schrödinger equation was “derived”

Experimental fact: quantum electrons may exhibit wave-like properties

→ Assume that they may be described by a plane-wave function:

$$\Psi(\vec{r},t) \propto e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{r}-\varepsilon t)}$$

→ We have to reconcile it with

$$\varepsilon(\vec{p}) = \frac{\vec{p}^2}{2m}$$

→ Hence the free Schrodinger equation

$$\left[i\hbar\partial_t + \frac{\hbar^2\nabla^2}{2m}\right]\Psi = 0$$

→ We can write it as $i\hbar\partial_t\Psi = \hat{H}\Psi$

if we identify, $\hat{H} = \frac{\hat{\vec{p}}^2}{2m}$ & $\hat{\vec{p}} = -i\hbar\nabla$

→ Generalize it to $i\hbar\partial_t\Psi = \hat{H}\Psi$

with $\hat{H} \to \hat{K} + V(\vec{r})$

- Classical properties (energy, momentum, coordinate, etc), become operators acting on the wave-function

momentum $\to \hat{\vec{p}} = -i\hbar\nabla$

coordinate $\to \hat{\vec{r}} \equiv \vec{r}$ (multiplication operator)

kinetic energy $\to \hat{K} = -\frac{\hbar^2\nabla^2}{2m}$

...

Expectation values

- The Born rule tells us that $|\Psi(\vec{r})|^2$ is the probability distribution function of different positions of our quantum particle.

- Hence, the mean value of the position is

$$\langle \vec{r} \rangle = \int dV\, \Psi^*(\vec{r})\vec{r}\Psi(\vec{r})$$

- Educated guess: assume that for a generic QM observable, X (with the corresponding operator, $\hat{X}$), the expectation value of X is

$$\boxed{\langle X \rangle = \int dV\, \Psi^*(\vec{r})\hat{X}\Psi(\vec{r})}$$

- This conjecture indeed works out and becomes a basic principle of QM: physical observables are associated with (self-adjoint) operators acting on the wave-function with the expectation values as defined above.