Exploring Quantum Physics

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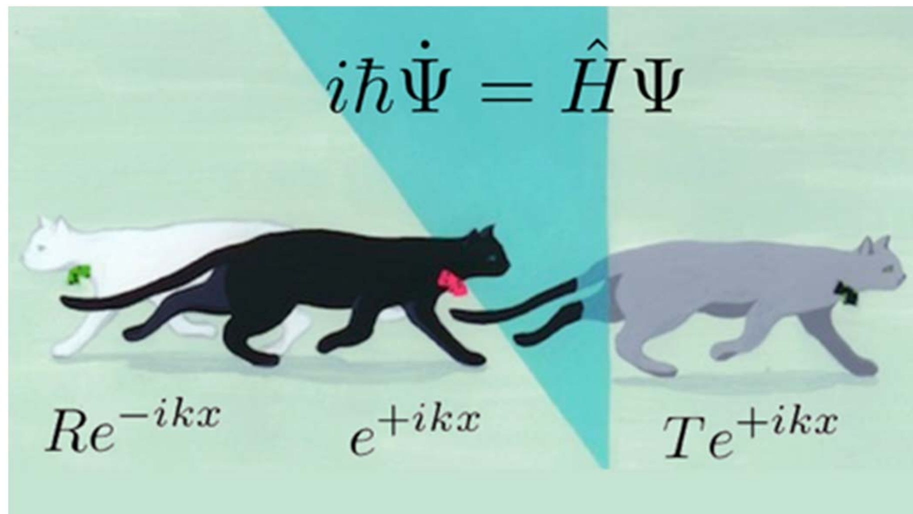
Wave-function and Schrödinger equation

Part IV: "Deriving" the Schrödinger equation



What is more fundamental: particles or waves?

- Particle has well-defined velocity and position at any given time.

- A sinusoidal wave is characterized by

$$u(x,t) = A_0 \sin\left(kx - \omega t + \phi_0\right)$$

or

$$u(x,t) = A_0 \, \mathrm{Im} \, e^{i(kx - \omega t + \phi_0)}$$

It's hard to represent a wave with particles, but we can decompose a localized particle into waves:

Fourier transform: $f(x) = \int dk A_k \, e^{ikx}$

Deriving wave-equation from its solution

- Now we assume that a free electron with momentum, $\vec{p}$, and energy, $\varepsilon(p)$, is described by the "wave function"

$$\Psi_0(\vec{r},t) = Ce^{\frac{i}{\hbar}(\vec{p}\cdot\vec{r}-\varepsilon t)}$$

- Whatever equation Ψ satisfies, it should reproduce the free spectrum

$$\varepsilon(\vec{p}) = \frac{\vec{p}^2}{2m}$$

- So, we "derive" the free Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\vec{r}^2} - i\hbar\frac{\partial}{\partial t}\right]\Psi_0(\vec{r},t) = 0$$