Feedback — Problem Set XII

You submitted this homework on **Tue 16 Apr 2013 5:11 PM CDT -0500**. You got a score of **0.00** out of **11.00**. You can attempt again, if you'd like.

In this problem set, you will be given a total of ten attempts. We will accept late submission until the fifth day after the due date, and late submission will receive half credit. Explanations and answers to the problem set will be available after the due date. Since the homework problems will become gradually more challenging as the course proceeds, we highly recommend you to start the habit of printing out the problems and working on them with paper and pencil. Also, please be sure to read the problem statements carefully and double check your expressions before you submit.

A pdf version of this problem set is available for you to print. Note: all mathematical expressions have to be exact, even when involving constants. Such an expression is required when a function and/or a variable is required in the answer. For example, if the answer is $\sqrt{3}x$, you must type `sqrt(3)*x`, not `1.732*x` for the answer to be graded as being correct.

Question 1

Digital and Analog Speech Communication
Suppose we transmit speech signals over comparable digital and analog channels. We want to compare the resulting quality of the received signals. Assume the transmitters use the same power, and the channels introduce the same attenuation and additive white noise. Assume the speech signal has a 4 kHz bandwidth and, in the digital case, is sampled at an 8 kHz rate with eight-bit A/D conversion. Assume simple binary source coding and a modulated BPSK transmission scheme.

What is the transmission bandwidth of the analog (AM) and digital schemes? Express your answers in kHz separated by spaces. For example, if the AM bandwidth was 1 kHz and if the digital bandwidth was 2 kHz then you would type `1 2`.

You entered:

Your Answer		Score	Explanation
	✘	0.00	
Total		0.00 / 1.00	

Question Explanation

Transmission bandwidth: Analog $= 2W = 8$kHz; Digital $= 3f_sB = 192$ kHz.

Question 2

Assume the speech signal's amplitude has a magnitude less than one. What is maximum amplitude quantization error introduced by the A/D converter?

Type your answer as a decimal number.

You entered:

Your Answer		Score	Explanation
	✘	0.00	
Total		0.00 / 1.00	

Question Explanation

Sample amplitudes in $[-1, 1]$. $2^8 = 256$ quantization levels. Max error $= \frac{1}{2}\frac{2}{2^B} = 2^{-B} = 0.004$

Question 3

In the digital case, each bit in quantized speech sample is received in error with probability p_e that depends on signal-to-noise ratio $\frac{E_b}{N_0}$. However, errors in each bit have a different impact on the error in the reconstructed speech sample. Find the mean-squared error between the transmitted and received amplitude.

MSE $=?$

NOTE: To enter p_e, type `pe`.

You entered:

Preview Help

Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

Question Explanation

Let $s(nT)$ =quantized speech value, and $\hat{s}(nT)$ =received value.

$$s(nT) = \frac{\sum_{k=0}^{B-1} b_k 2^k}{2^{B-1}} - 1$$

$$\hat{s}(nT) = \frac{\sum_{k=0}^{B-1} \hat{b}_k 2^k}{2^{B-1}} - 1$$

$$\text{Error} = \frac{\sum_{k=0}^{B-1} (b_k - \hat{b}_k) 2^k}{2^{B-1}}$$

$$\text{MSE} = \frac{\left\langle \left(\sum_{k=0}^{B-1} (b_k - \hat{b}_k) 2^k \right)^2 \right\rangle}{2^{2(B-1)}}$$

Now

$$\left(\sum_{k=0}^{B-1}(b_k-\hat{b}_k)2^k\right)^2=\sum_{k=0}^{B-1}(b_k-\hat{b}_k)^2 2^{2k}+\sum_{k\neq l}(b_k-\hat{b}_k)(b_l-\hat{b}_l)2^{k+l}$$

Average of first term $\sum_{k=0}^{B-1} p_e 2^{2k}$ because

$$(b_k-\hat{b}_k)^2=\begin{cases}1 & \text{prob } = p_e\\ 0 & \text{prob } = 1-p_e\end{cases}$$

Average of second term $= 0$ because

$$(b_k-\hat{b}_k)(b_l-\hat{b}_l)=\begin{cases}1 & 2\cdot(\frac{p_e}{2})^2\\ 0 & (1-p_e)^2+2p_e(1-p_e)\\ -1 & 2\cdot(\frac{p_e}{2})^2\end{cases}$$

Therefore

$$\text{MSE } = p_e\frac{\sum_{k=0}^{B-1}2^{2K}}{2^{2(B-1)}}=p_e\frac{\frac{1-2^{2B}}{1-2^2}}{2^{2(B-1)}}=p_e\frac{2^{2B}-1}{3\cdot 2^{2B-2}}\approx\frac{4}{3}p_e$$

Question 4

In the digital case, the recovered speech signal can be considered to have two noise sources added to each sample's true value: One is the A/D amplitude quantization noise and the second is due to channel errors. Because these are separate, the total noise power equals the sum of these two. What is the signal-to-noise ratio of the received speech signal as a function of p_e?

SNR $=?$

NOTE: to enter p_e, type `pe`, and to enter $\text{power}[s]$, type `Ps`

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

$\hat{s}(nT) = s(nT) + \text{quantization noise} + \text{communcation error}$

$$\text{SNR} = \frac{\text{power}[s]}{\frac{2^{-2B}}{3} + \frac{4}{3} p_e}$$

Question 5

Communication System Design

GR8 Communcation Systems has been asked to design a communication system that meets the following requirements.

- The baseband message signal has a bandwidth of 10 kHz.
- The GR8CS engineers find that the entropy H of the sampled message signal depends on how many bits b are used in the A/D converter.

b	H
3	2.19
4	3.25
5	4.28
6	5.35

- The signal is to be sent through a noisy channel having a bandwidth of 25 kHz centered at 2 MHz and a signal-to-noise ratio within that band of 10 dB.
- Once received, the message signal must have a signal-to-noise ratio of at least 20 dB.

Can an analog scheme meet the design requirements?

Your Answer	Score	Explanation

- Yes
- No

Total	0.00 / 1.00

Question Explanation

For the analog scheme, only 20 kHz of the channel bandwidth is needed. The front-end bandpass filter will remove noise outside this band, increasing the signal-to-noise ratio by a factor of $\frac{25}{20} = 1.25$, making the received signal-to-noise ratio 12.5. The analog receiver boosts this SNR by a factor of two, making the message's SNR equal to $25 = 14 \text{ dB}$. Thus, the analog scheme falls far short of the required message SNR.

Question 6

What is the capacity for a digital implementation of the communication system?

$C = ?$ kbps (kilobits per second)

You entered:

Your Answer	Score	Explanation
✘	0.00	
Total	0.00 / 1.00	

Question Explanation

$C = W \log_2(1 + \frac{S}{N}) = 25 \times 10^3 \log_2(1 + 10) = 86.5$ kbps. The system would need to sample at 20 kHz ⇒ $\frac{86.5}{20} = 4.325$ bits/sample on the average.

Question 7

What is the SNR of the digital system?

SNR $=?$ dB (let $B=4$)

You entered:

Your Answer		Score	Explanation
	✘	0.00	
Total		0.00 / 1.00	

Question Explanation

SNR limits placed by quantization $= 3\cdot 2^{2B}$. Let $B=4,\ (\mathrm{SNR})_{\mathrm{Q}} = 3\cdot 2^{8} = 768 = 28.9$ dB.

Question 8

Can a digital system achieve the design specifications?

Your Answer	Score	Explanation
Yes		
No		
Total	0.00 / 1.00	

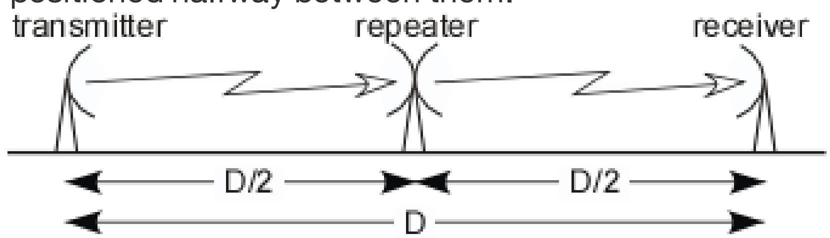
Question Explanation

For the digital scheme, the sampling rate is $20\ \mathrm{kHz}$, which makes the data rate $H\cdot 2\times 10^{4}$ bps. Assuming this data rate is less than capacity, the message signal-to-noise ratio will be $6b+1.76\ \mathrm{dB}$. To achieve the design goal, we must have $6b+1.76\geq 20\Rightarrow b\geq 3.04$. Consequently, we must use 4-bit quantization at least. This choice results in a data rate of

$3.25 \cdot 2 \times 10^4 = 65 \times 10^3$ bps. The capacity of our channel is $C = 25 \times 10^3 \log_2(1 + 10) = 86.5 \times 10^3$bps. Since our required data rate is less than capacity, there is exists an error-correcting code than can correct all errors (as we have assumed). Consequently, only a digital scheme can be used.

Question 9

Because signals attenuate with distance form the transmitter, **repeaters** are frequently employed for both analog and digital communication. For example, let's assume that the transmitter and receiver are D meters apart, and a repeater is positioned halfway between them.



What the repeater does is amplify its received signal to exactly cancel the attenuation encountered along the first portion and to re-transmit the signal to the ultimate receiver. However, the signal the repeater receives contains white noise as well as the transmitted signal. The receiver experiences the same amount of white noise as the repeater.

For digital communication, we must consider the system's capacity.

$$C = W \log_2(1 + \text{SNR})$$

What is the SNR of the system **without using** the repeater? Assume the attenuation factor between the transmitter and the repeater is α. Express the SNR in terms of E_b, the energy-per-bit used by the transmitter, the noise spectral height parameter N_0 and α.

$SNR = ?$

NOTE: to enter E_b type `Eb`; to enter N_0, type `N0`; to enter α, type `a`.

You entered:

Preview Help

Your Answer	Score	Explanation
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Total	0.00 / 1.00	

Question Explanation

The attenuation factor between transmitter and receiver is $\frac{\alpha}{2}$ since the amplitude vary inversely with distance. Therefore, the SNR equals $\frac{(\alpha/2)^2 E_b}{N_0} = \frac{\alpha^2 E_b}{4N_0}$.

Question 10

What is the SNR of the system **using** the repeater? Express the SNR in terms of E_b, the energy-per-bit used by the transmitter, the noise spectral height parameter N_0 and α.

$SNR = ?$

NOTE: to enter E_b type `Eb`; to enter N_0, type `N0`; to enter α, type `a`.

You entered:

Preview Help

Your Answer	Score	Explanation
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Total	0.00 / 1.00	

Question Explanation

The signal experiences an attenuation of α at the receiver. The noise at the receiver comes from two sources: the noise at the receiver and repeater noise transmitted to the receiver. The repeater applies a gain of $\frac{1}{\alpha}$ to its input and the channel between the repeater and the receiver introduces a gain of α. Therefore, the power of the noise from the repeater equals that occurring at the receiver. Since the noise powers add, the signal-to-noise ratio is $\frac{\alpha^2 E_b}{2N_0}$.

Question 11

Assuming the signal-to-noise ratio is large, what is the increase/decrease in capacity when this repeater system is used?

Express the **increase** (express a decrease with a negative value) in terms of E_b, N_0, α and W, the channel's bandwidth.
To enter E_b type `Eb`; to enter N_0, type `N0`; to enter α, type `a`; to enter W, type `W`.

You entered:

Preview Help

Your Answer		Score	Explanation
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Total		0.00 / 1.00	

Question Explanation

For large SNR, $C \approx W \log_2 \text{SNR}$ Using the repeater increases the SNR at the receiver by a factor of two:
$C_{\text{repeater}} = W \log_2 2\text{SNR}_{\text{no repeater}} = W \log_2 \text{SNR}_{\text{no repeater}} + W = C_{\text{no repeater}} + W$
.