Feedback — Problem Set V

You submitted this homework on **Tue 26 Mar 2013 3:44 PM CDT -0500**. You got a score of **0.00** out of **11.00**. However, you will not get credit for it, since it was submitted past the deadline.

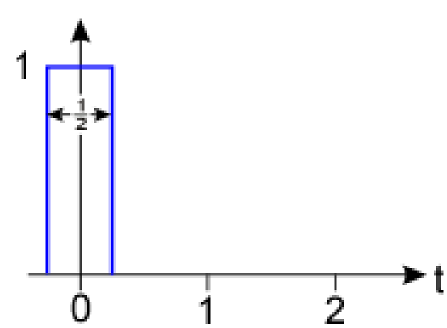
In this problem set, you will be given a total of ten attempts. We will accept late submission until the fifth day after the due date, and late submission will receive half credit. Explanations and answers to the problem set will be available after the due date. Since the homework problems will become gradually more challenging as the course proceeds, we highly recommend you to start the habit of printing out the problems and working on them with paper and pencil. Also, please be sure to read the problem statements carefully and double check your expressions before you submit.

A pdf version of this problem set is available for you to print. Note: all mathematical expressions have to be exact, even when involving constants. Such an expression is required when a function and/or a variable is required in the answer. For example, if the answer is $\sqrt{3}x$, you must type `sqrt(3)*x`, not `1.732*x` for the answer to be graded as being correct.

Question 1

Spectra of pulse sequences. Pulse sequences occur often in digital communication and in other elds as well. Here we investigate their spectral properties.

Calculate the Fourier transform of the single pulse shown below. Please express your answer in terms of f.



$\mathcal{F}\{p(t)\} = P(f) =?$

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Preview Help

Your Answer		Score	Explanation
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Total		0.00 / 1.00	

Question Explanation

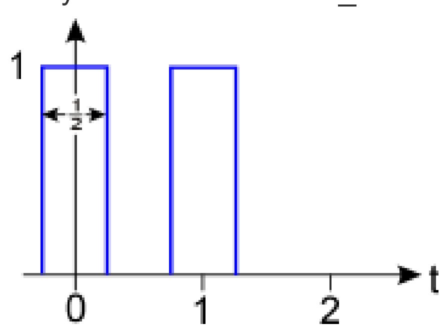
The pulse has a unit amplitude and a width $\Delta = \frac{1}{2}$.

$$P(f) = \frac{\sin \pi f \Delta}{\pi f}, \Delta = \frac{1}{2} \Rightarrow P(f) = \frac{\sin \frac{\pi}{2} f}{\pi f}$$

Question 2

Calculate the Fourier transform of the two-pulse sequence shown below. Please express your answer in terms of the Fourier transform for a single pulse that you calculated in Question 1.

For example, if the answer for the two-pulse sequence was just twice as great as the result for the single pulse you would enter `2*P_f` for the answer.



$\mathcal{F}\{p(t)\} = ?$

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Preview Help

Your Answer	Score	Explanation
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Question Explanation

$$p(t) + p(t-1) \leftrightarrow P(f) + e^{-j\pi f}\, P(f) = P(f)\left[1 + e^{-j2\pi f}\right]$$

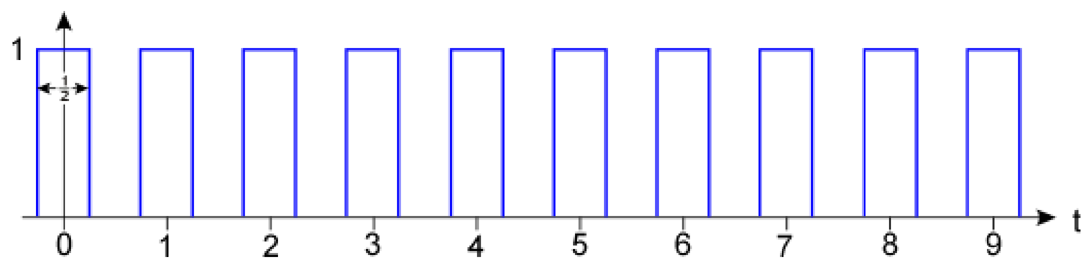
Which simplifies to

$$e^{-j\pi f}\, P(f) 2\cos \pi f.$$

Question 3

Calculate the Fourier transform of the ten-pulse sequence shown below. Please express your answer in terms of the Fourier transform for a single pulse that you calculated in Question 1.

For example, if the answer for the ten-pulse sequence was ten times as great as the result for the single pulse you would enter `10*P_f` for the answer.



$\mathcal{F}\{p(t)\} = ?$

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Preview Help

Your Answer	Score	Explanation
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Question Explanation

$$\sum_{n=0}^{9} p(t-n) \leftrightarrow P(f)\sum_{n=0}^{9} e^{-j2\pi fn} = P(f)\frac{1-e^{-j2\pi f10}}{1-e^{-j2\pi f}}$$
$$= P(f)\cdot\frac{e^{-j2\pi f5}}{e^{-j\pi f}}\frac{\sin 2\pi f5}{\sin \pi f} = P(f)\cdot e^{-j9\pi f}\frac{\sin 10\pi f}{\sin \pi f}.$$

Question 4

Effective Drug Delivery.
In most patients, it takes time for the concentration of an administered drug to achieve a constant level in the blood stream. Typically, if the drug concentration in the patients intravenous line is $C_d\mathrm{u}(t)$, the concentration in the patients blood stream is $C_p(1-e^{-at})\mathrm{u}(t)$.

Assuming the relationship between drug concentration in the patients drug and the delivered concentration can be described as a linear, time-invariant system, what is the transfer function? (When entering your answer, type `C_p` for C_p and `C_d` for C_d.

Note: the transfer function is expressed in the frequency domain.

$H(j2\pi f) = ?$

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Preview Help

Your Answer	Score	Explanation
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Question Explanation

First we notice that $Y(j2\pi f) = H(j2\pi f)X(j2\pi f)$ where $Y(j2\pi f) \& X(j2\pi f)$ are the Fourier transforms of the output and input signals respectively. Also, we use two common Fourier transform pairs

$$e^{-at}u(t) \leftrightarrow \frac{1}{a + j2\pi f} \text{ and } u(t) \leftrightarrow \frac{1}{j2\pi f}.$$

$Y(j2\pi f) = C_p(\frac{1}{j2\pi f} - \frac{1}{a+j2\pi f}) = C_p \frac{a}{j2\pi f(a+j2\pi f)}$ and $X(j2\pi f) = \frac{C_d}{j2\pi f}$.

So by dividing $\frac{Y(j2\pi f)}{X(j2\pi f)}$ we get $H(j2\pi f) = \frac{C_p}{C_d} \frac{a}{a + j2\pi f}$.

Question 5

Sometimes, the drug delivery system goes haywire and delivers drugs with little control. What would the patients drug concentration be if the delivered concentration were a ramp? More precisely, if it were $C_d t\mathrm{u}(t)$? (Remember, to enter $\mathrm{u}(t)$ as part of your solution, please use the `sign(t)` function. If needed, type `C_p` for C_p and `C_d` for C_d.)

$y(t) = ?$

You entered:

Preview Help

Your Answer	Score	Explanation
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Question Explanation

If we think about the relationship between the ramp function and the step function, we realize that the ramp function is the integral of the step function.

Therefore the true input to the patient is the integral of the desired input. Since this operation is linear, and the Fourier transform preserves linear operations, the output is nothing but the integral of the previous output.

$$y(t) = \int_0^t C_p(1 - e^{-a\alpha})u(t)\, d\alpha = \left[C_p t + \frac{C_p}{a}(e^{-at} - 1)\right]u(t).$$

Question 6

A clever doctor wants to have the exibility to slow down or speed up the patients drug concentration. In other words, the concentration is to be $C_p(1 - e^{-bt})\mathrm{u}(t)$, with b bigger or smaller than a. How should the delivered drug concentration signal be changed to achieve this concentration prole? Remember, to enter $\mathrm{u}(t)$ as part of your solution, please use the `sign(t)` function. If needed, type `C_p` for C_p and `C_d` for C_d.

$x(t) = ?$

You entered:

Preview Help

Your Answer	Score	Explanation
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Question Explanation

Given the desired output and the transfer function, we should be able to find the input that satisfies the doctors demands. It is easiest to work in the frequency domain. From question 4, we know that

$$Y(j2\pi f) = C_p \frac{b}{j2\pi f(b + j2\pi f)} \text{ and } H(2\pi f) = \frac{C_p}{C_d}\frac{a}{a + j2\pi f}.$$

Therefore

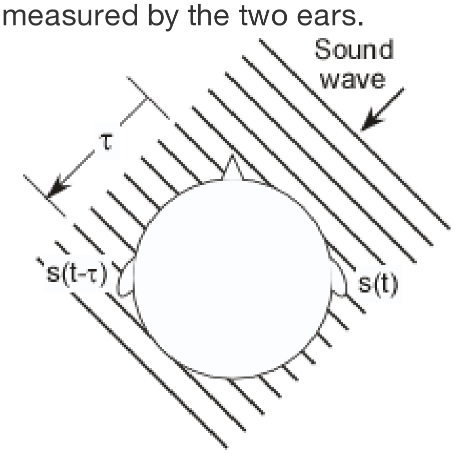
$$X(j2\pi f) = \frac{Y(j2\pi f)}{H(j2\pi f)} = C_d \frac{a + j2\pi f}{a} \frac{b}{j2\pi f(b + j2\pi f)} = C_d \left(1 + \frac{j2\pi f}{a}\right) \frac{b}{j2\pi f(b + j2\pi f)},$$

where $j2\pi f$ is the Fourier transform of the derivative function. So we can find the inverse Fourier transform of $X(j2\pi f)$

$$x(t) = C_d \left(1 - e^{-bt} - \frac{1}{a}\frac{d}{dt}e^{-bt}\right)\mathrm{u}(t) = C_d\left(1 - e^{-bt} + \frac{b}{a}e^{-bt}\right)\mathrm{u}(t)$$
$$= C_d\left(1 + \left(\frac{b}{a} - 1\right)e^{-bt}\right)\mathrm{u}(t).$$

Question 7

Where is that sound coming from?
We determine where sound is coming from because we have two ears and a brain. Sound travels at a relatively slow speed and our brain uses the fact that sound will arrive at one ear before the other. As shown below, a sound coming from the right arrives at the left ear seconds after it arrives at the right ear.
Once the brain finds this propagation delay, it can determine the sound direction. In an attempt to model what the brain might do, clever signal processors want to design an optimal system that delays each ears' signal by some amount then adds them together. τ_L and τ_R are the delays applied to the left and right signals respectively. The idea is to determine the delay values according to some criterion that is based on what is measured by the two ears.



What is the transfer function between the sound signal $s(t)$ and the processor output $y(t)$?

Time delays are represented using the greek letter τ. To enter τ_R type `tau_R`, to enter τ_L type `tau_L`, and to enter τ type `tau`.

$H(f) =?$

You entered:

Preview Help

Your Answer		Score	Explanation
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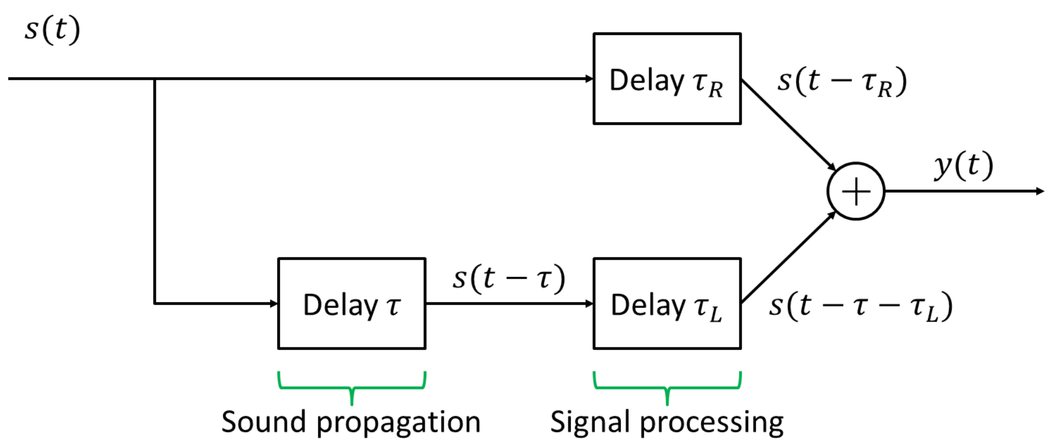
Question Explanation

There are three sources of delay in the system. The input to each ear must be processed which introduces a delay (τ_R & τ_L). Since the sound waves reach the right ear before they reach the left ear this introduces a propagation delay (τ). A block diagram of this system is shown below.



The transfer function for this system is the summation of the inputs to each ear shifted by an appropriate delay. Using Fourier transform properties, a shift in the time domain is a complex exponential in the frequency domain, therefore

$$H(f) = e^{-j2\pi f\tau_R} + e^{-j2\pi f\tau} e^{-j2\pi f\tau_L} = e^{-j2\pi f\tau_R} + e^{-j2\pi f(\tau+\tau_L)} .$$

Question 8

Using the scenario of the previous question, one way of determining the optimal delay is to choose τ_L and τ_R to maximize the power in $y(t)$. How are these maximum-power processing delays related to τ? In other words, express the delays τ_R and τ_L in relation to the propagation delay τ.

Maximum power occurs when τ=?

You entered:

Preview Help

Your Answer		Score	Explanation
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Question Explanation

From the previous question we have the relationship

$$Y(f) = H(f) \cdot S(f) = \left(e^{-j2\pi f\tau_R} + e^{-j2\pi f(\tau+\tau_L)}\right)S(f).$$

By Parseval's Theorem:

$$\text{Power} = \int |Y(f)|^2 df = \int \left|e^{-j2\pi f\tau_R} + e^{-j2\pi f(\tau+\tau_L)}\right|^2 |S(f)|^2 df$$
$$= \int \left|1 + e^{-j2\pi f(\tau+\tau_L-\tau_R)}\right|^2 |S(f)|^2 df.$$

Only the time-delay difference $\tau_L - \tau_R$ affects power. Let $\Delta = \tau_L - \tau_R$.

$$\left|1 + e^{-j2\pi f(\tau+\Delta)}\right|^2 = \left(1 + \cos 2\pi f(\tau+\Delta)\right)^2 + \sin^2 2\pi f(\tau+\Delta)$$
$$= 2 + 2\cos 2\pi f(\tau+\Delta)$$
$$\frac{d}{d\Delta}\left(2 + 2\cos 2\pi f(\tau+\Delta)\right) = -4\pi f \sin 2\pi f(\tau+\Delta) = 0 \Rightarrow \boxed{\tau = -\Delta}.$$

When $\tau = -(\tau_L - \tau_R) = \tau_R - \tau_L$, we have maximum power. Here, signal processing delay cancels propagation delay.

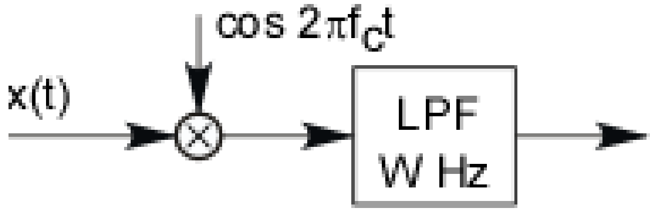
Question 9

Let $m(t)$ denote a message signal bandlimited to W Hz that has been *amplitude modulated* by a radio transmitter.

$$x(t) = A \cdot \left(1 + m(t)\right) \cos 2\pi f_c t$$

Radio stations try to restrict the amplitude of the signal $m(t)$ so that it is less than one in magnitude. The frequency f_c is very large compared to the frequency content of the signal ($f_c \gg W$). What we are concerned about here is not transmission, but reception.

The so-called *coherent demodulator* simply multiplies the signal $x(t)$ by a unit-amplitude sinusoid having the same frequency and phase as the carrier and lowpass filters the result.



Analyze this receiver and determine its the output. Assume the lowpass filter is ideal with unity gain. Use `m_t` to denote the message signal $m(t)$.

demodulator output=?

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

Once the demodulator multiplies by $\cos 2\pi f_c t$, the resulting signal is $A \cdot \left(1 + m(t)\right) \cos^2 2\pi f_c t = A \cdot \left(1 + m(t)\right) \cdot \frac{1}{2}\left(1 + \cos 2\pi 2 f_c t\right)$. This result means we have two terms: one is related to the unmodulated signal— $\frac{A}{2} \cdot \left(1 + m(t)\right)$ — and one having a spectrum shifted in frequency so that it is centered at $2f_c$—$\frac{A}{2} \cdot \left(1 + m(t)\right) \cos 2\pi 2 f_c t$. The lowpass filter removes this high-frequency component, leaving the output to be $\frac{A}{2} \cdot \left(1 + m(t)\right)$.
Also, the same result obtains if we had assumed that the modulator and demodulator used $\sin 2\pi f_c t$ instead of $\cos 2\pi f_c t$.

Question 10

One issue in coherent reception is the phase of the sinusoid used by the receiver relative to that used by the transmitter. Assuming that the sinusoid of the receiver has a phase ϕ, how does the output depend on ϕ? Type `phi` to represent ϕ.

You entered:

Preview Help

Your Answer		Score	Explanation
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Total		0.00 / 0.75	

Question Explanation

Assume the demodulator multiplies by $\cos(2\pi f_c t + \phi)$, the resulting signal is $A \cdot \big(1 + m(t)\big) \cos 2\pi f_c t \cos(2\pi f_c t + \phi) = A \cdot \big(1 + m(t)\big) \cdot \frac{1}{2} \big(\cos\phi + \cos(2\pi 2 f_c t + \phi)\big)$. Again, we have the original unmodulated signal $\frac{A}{2} \big(1 + m(t)\big) \cos\phi$ and the message signal modulated by a sinusoid at twice the frequency. The lowpass filter removes the high-frequency term, making the output $\frac{A}{2} \big(1 + m(t)\big) \cos\phi$.

Question 11

What is the worst possible value for this phase? Express your answer as a number times π. If your answer is $\pi/4$, you should type `pi/4`.

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Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 0.25	

Question Explanation

The output is multiplied by $\cos\phi$. The worst possible value occurs when $\cos\phi = 0$, which occurs when $\phi = \pm\pi/2$. Phases giving negative values only invert the signal and the message $m(t)$ can still be recovered.

Question 12

A *Fundamentals of Electrical Engineering* student has the bright idea of using a square wave instead of a sinusoid as an AM carrier. The transmitted signal would have the form

$$x(t) = A \cdot \left(1 + m(t)\right)\mathrm{sq}_T(t)$$

where the message signal $m(t)$ is amplitude limited: $$|m(t)|
Assuming the message signal is lowpass and has a bandwidth of W Hz, what values for the square waves period T are feasible. In other words, what combinations of W and T enable reception?

Your Answer	Score	Explanation
All values work.		
$\frac{1}{T} > f_c$		
$\frac{2}{T} > f_c$		
Total	0.00 / 0.25	

Question Explanation

The harmonics of the square wave occur at odd multiples of $\frac{1}{T}$. Consequently, the spectrum of $x(t)$ is nonzero at the frequency intervals $[\pm\frac{1}{T} - f_c, \pm\frac{1}{T} + f_c]$, $[\pm\frac{3}{T} - f_c, \pm\frac{3}{T} + f_c]$,... These must not overlap, else *no* demodulator can recover the message signal. So, $\frac{1}{T} - f_c > 0$ and $\frac{k}{T} + f_c 0$, k odd. These all require $\frac{1}{T} > f_c$.

Question 13

Assuming reception is possible (the conditions of the previous problem apply, if they exist), can standard radios receive this innovative AM transmission? If so, determine the coherent receiver output (adjust the radio's carrier frequency and phase for best reception), expressing $m(t)$ as `m_t`; if not, type your answer as zero. Assume that the message bandwidth $W = 5$ kHz.

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Preview Help

Your Answer		Score	Explanation
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Question Explanation

Since every receiver contains a bandpass filter centered at the carrier frequency f_c with bandwidth $2W$, the higher harmonics of the square wave are filtered out (removed). For all intents and purposes, the receiver thus "sees" a standard amplitude modulated signal and the results is the same as before, save for a change in the amplitude: $\frac{2A}{\pi}\left(1 + m(t)\right)$.