Feedback — Problem Set VI

You submitted this homework on **Tue 26 Mar 2013 3:45 PM CDT -0500**. You got a score of **0.00** out of **9.00**. However, you will not get credit for it, since it was submitted past the deadline.

In this problem set, you will be given a total of ten attempts. We will accept late submission until the fifth day after the due date, and late submission will receive half credit. Explanations and answers to the problem set will be available after the due date. Since the homework problems will become gradually more challenging as the course proceeds, we highly recommend you to start the habit of printing out the problems and working on them with paper and pencil. Also, please be sure to read the problem statements carefully and double check your expressions before you submit.

A pdf version of this problem set is available for you to print. Note: all mathematical expressions have to be exact, even when involving constants. Such an expression is required when a function and/or a variable is required in the answer. For example, if the answer is $\sqrt{3}x$, you must type `sqrt(3)*x`, not `1.732*x` for the answer to be graded as being correct.

Question 1

The signal $s(t)$ is bandlimited to 4 kHz. We want to sample it, but it has been subjected to various signal processing manipulations.

What minimum sampling frequency, F_s, can be used to sample the result of passing $s(t)$ through an RC *highpass* filter with $R = 10\ \text{k}\Omega$ and $C = 8$ nF? If no frequencies work, enter 0.

$F_s\ =?$ Hz. **NOTE:** Answer in Hertz, not kHz.

You entered:

Your Answer	Score	Explanation

✘	0.00	
Total	0.00 / 1.00	

Question Explanation

First of all, the cutoff frequency for this filter is about 2 kHz, within the signal's bandwidth. We can use two aspects of this question to quickly find the required sampling frequency.
1) We are running the signal through a highpass filter, that is, we are *preserving* high frequencies above 2 kHz or so.
2) The sampling theorem tells us that we must sample at least twice as fast as the highest frequency in the signal. Since our input $s(t)$ is bandlimited to 4000 Hz, we need to sample at a minimum of 8000 Hz.

Question 2

What frequency can be used to sample the **derivative** of $s(t)$ mentioned in the previous problem? If no frequencies work, enter 0.

F_s $=?$ Hz. **NOTE:** Answer in Hertz, not kHz.

You entered:

Your Answer	Score	Explanation
✘	0.00	
Total	0.00 / 1.00	

Question Explanation

The derivative of a signal is equivalent to passing the signal through a system having a transfer function $H(f) = j2\pi f$, the resulting signal $s'(t)$ remains bandlimited at 4 kHz so we can still sample at 8000 Hz.

Question 3

The signal $s(t)$ has been modulated by an 8 kHz sinusoid having an unknown phase: the resulting signal is $s(t)\sin(2\pi f_0 t + \phi)$ with $f_0 = 8$ kHz and $\phi = ?$

Can the modulated signal be sampled so that the **original** signal can be recovered from the modulated signal regardless of the phase value ϕ? Enter the smallest sampling rate that allows for full recovery of the original signal. If the original signal can not be recovered enter `0`.

$F_s = ?$ Hz **NOTE:** Answer in Hertz, not kHz.

You entered:

Your Answer		Score	Explanation
	✘	0.00	
Total		0.00 / 1.00	

Question Explanation

Let $x(t) = s(t)\sin(2\pi f_0 t + \phi)$ with $f_0 = 8$ kHz and $\phi = ?$. Modulating the signal does not change the bandwidth of the signal but it does shift the energy to a different location on the frequency axis. Because sampling produces copies of a signal centered at the sampling frequency (F_s), the sampling theorem tells us that we should sample at twice the highest frequency to avoid aliasing. However, in this particular application we know that there is no frequency content in the range $0 - 4$ kHz and that if we do have aliasing in this range there will be no loss of the original signal.

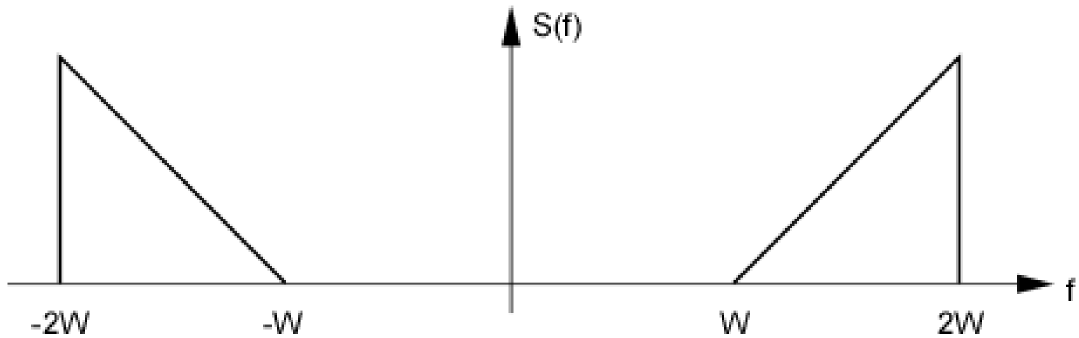
More importantly, in this application we can sample at 8000 **Hz** and two copies of the modulated signal will be centered at the origin. These two copies add together to perfectly reconstruct the original frequency content of $S(f)$. Once we perform the requisite lowpass filtering of the resulting signal we can recover $s(t)$.

$F_s = 8000$ Hz

Question 4

Bandpass Sampling

The signal $s(t)$ has the indicated spectrum.



What is the minimum sampling rate for this signal suggested by the Sampling Theorem? Express your answer as an expression in terms of W (type `W` in your answer).

F_s =? Hz

You entered:

Preview Help

Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

Question Explanation

The signal $s(t)$ has a maximum frequency of $2W$ Hz. The Sampling Theorem suggests that this signal should be sampled at twice this, or $4W$ Hz.

Question 5

Because of the particular structure of this bandpass spectrum, you might wonder whether a lower sampling rate could be used. This is indeed the case, first find the lower sampling rate that can be used to reconstruct $s(t)$ from its samples. Express your answer in terms of W.

F_s =? Hz

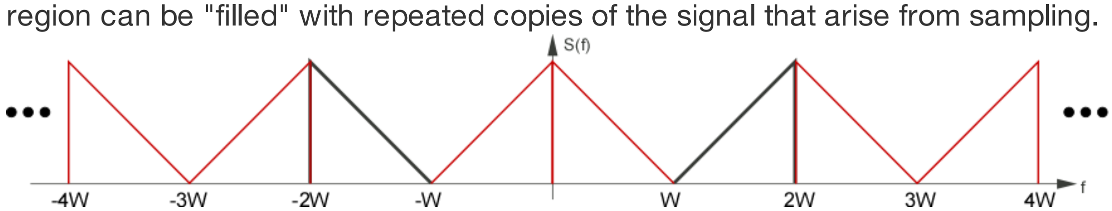
You entered:

Preview Help

Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

Question Explanation

Because the input signal $s(t)$ has no frequency content from $0 - W$ Hz, this region can be "filled" with repeated copies of the signal that arise from sampling.



If we sample $s(t)$ at $F_s = 2W$, then some signal information is aliased into the regions that were previously absent of frequency information. Crucially, all of the aliasing falls in these blank regions, so the signal is not destroyed.

Question 6

After using the lower sampling frequency for the above problem, it is necessary to filter the sampled signal. What filter is required to complete the system?

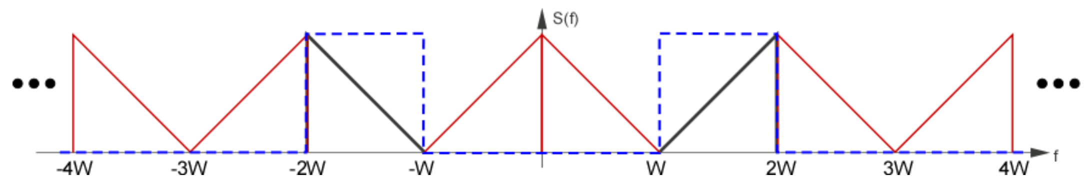
Your Answer	Score	Explanation
Highpass		

- Lowpass to W
- Lowpass to $2W$
- Bandpass

Total	0.00 / 1.00

Question Explanation

Sampling at $2W$ means that the spectrum repeats across the entire frequency axis.

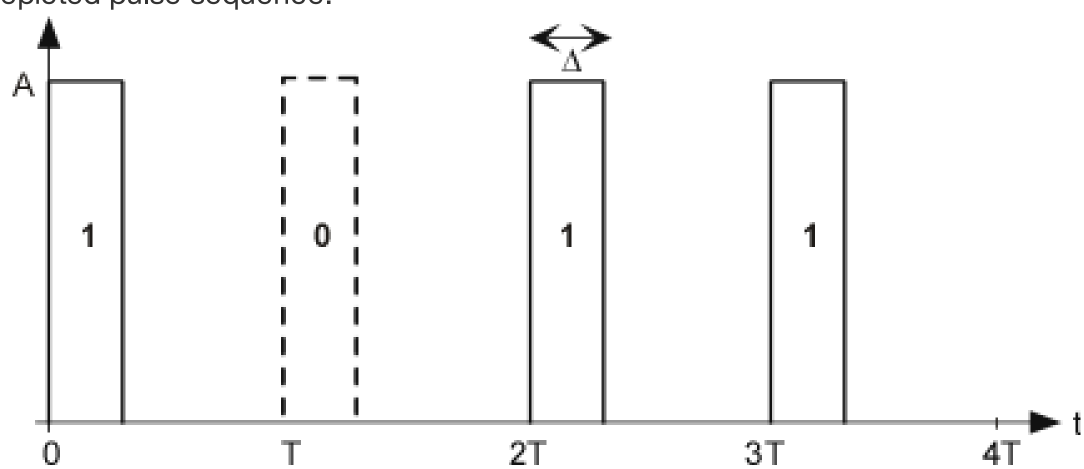


Consequently, we need to **bandpass** filter the sampled signal.

Question 7

Simple D/A Converter

Commercial digital-to-analog converters don't work this way, but a simple circuit illustrates how they work. Let's assume we have a B-bit converter. Thus we want to convert numbers having a B-bit representation into a voltage proportional to that number. The first step taken by our simple converter is to represent the number by a sequence of B pulses occurring at multiples of a time interval T. The presence of a pulse indicates a "1" in the corresponding bit position, and pulse absence means a "0" occurred. For a 4-bit converter, the number 13 has the binary representation 1101 $(13_{10} = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0)$ and would be represented by the depicted pulse sequence.



Note that the pulse sequence is "backwards" from the binary representation. We'll see why that is in the next three questions.

This signal serves as the input to a first-order RC lowpass filter. We want to design the filter and the parameters Δ and T so that the output voltage at time $4T$ (for a 4-bit converter) is proportional to the (decimal) number. This combination of pulse creation and filtering constitutes our simple D/A converter. The requirements are:

- The voltage at time $t = 4T$ should diminish by a factor of 2 the further the pulse occurs from this time. In other words, the voltage due to a pulse at $3T$ should be twice that of a pulse produced at $2T$, which in turn is twice that of a pulse at T.
- The 4-bit D/A converter must support a 10 kHz sampling rate.

What is the response to a pulse when $0 \leq t \leq \Delta$? Express your answer in terms of $A, R, C, t,$ and Δ.

(If necessary, enter Δ by typing `Delta`.)

Output = ?, $0 \leq t \leq \Delta$

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

The voltage corresponding to an input pulse starts at A volts and changes exponentially during the width of the pulse. The rate of the response is controlled by modifying the product RC. Since $t \leq \Delta$ the voltage continues to grow as t increases.

$$A(1 - e^{-t/RC}),\ 0 \leq t \leq \Delta$$

Question 8

What is the response to a single pulse when $t > \Delta$? Express your answer in terms of $A, R, C, t,$ and Δ. (If necessary, enter Δ by typing `Delta`.)

Output = ?, $t > \Delta$

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

After time Δ the input signal goes to zero and the voltage stops increasing. At $t = \Delta$ the voltage starts to fall according to $e^{-t/RC}$.

$$A(1 - e^{-\Delta/RC})e^{-(t-\Delta)/RC}, \quad t > \Delta$$

Question 9

What **numerical** value of the product RC satisfies the design constraints?

RC =? seconds

You entered:

Your Answer	Score	Explanation
✘	0.00	

Total	0.00 / 1.00

Question Explanation

Since the design requires the voltage to decay by a factor of 2 over a T second interval

$$e^{-T/RC} = \frac{1}{2} \Rightarrow RC = \frac{T}{\ln 2}$$

We need to have a 10 kHz sampling rate therefore
$4T = \frac{1}{10{,}000} = 100\mu s \Rightarrow T = 2.5 * 10^{-5}\, s = 25\mu s.$
Using this value of T, the product of RC must be

$$RC = \frac{T}{\ln 2} = \frac{2.5 * 10^{-5}}{0.69} = 3.6 * 10^{-5}\, s = 36\mu s.$$