Feedback — Problem Set IX

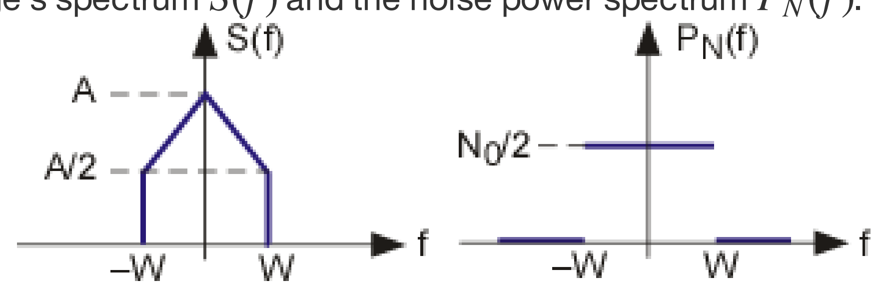
You submitted this homework on **Tue 26 Mar 2013 3:48 PM CDT -0500**. You got a score of **0.00** out of **7.00**. However, you will not get credit for it, since it was submitted past the deadline.

In this problem set, you will be given a total of ten attempts. We will accept late submission until the fifth day after the due date, and late submission will receive half credit. Explanations and answers to the problem set will be available after the due date. Since the homework problems will become gradually more challenging as the course proceeds, we highly recommend you to start the habit of printing out the problems and working on them with paper and pencil. Also, please be sure to read the problem statements carefully and double check your expressions before you submit.

A pdf version of this problem set is available for you to print. Note: all mathematical expressions have to be exact, even when involving constants. Such an expression is required when a function and/or a variable is required in the answer. For example, if the answer is $\sqrt{3}x$, you must type `sqrt(3)*x`, not `1.732*x` for the answer to be graded as being correct.

Question 1

The signal emerging from a communications channel consists of two parts: a message signal $s(t)$ and additive noise $N(t)$. The plot shows the message's spectrum $S(f)$ and the noise power spectrum $P_N(f)$.



The noise power spectrum lies completely within the signal's band and has a constant value there of $\frac{N_0}{2}$.

What is the power in the message signal?

Type the amplitude parameter A as `A` and the bandwidth W as `W`.

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

We must integrate the *square* of the message spectrum.

$$\begin{aligned} \text{power}_s &= 2\int_0^W A^2\left(1-\frac{1}{2}\frac{f}{W}\right)^2 df \\ &= 2A^2\left(-\frac{2W}{3}\right)\left(1-\frac{1}{2}\frac{f}{W}\right)^3\Bigg|_0^W \\ &= 2A^2\cdot\frac{7W}{12} \\ &= \frac{7A^2W}{6} \end{aligned}$$

Question 2

What is the signal-to-noise ratio?

Type the noise spectral height parameter N_0 as `N0` (that's "en-zero"), the amplitude parameter A as `A` and the bandwidth W as `W`.

You entered:

Preview Help

Your Answer	Score	Explanation
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Question Explanation

The noise power is $2 * \frac{N_0}{2} \cdot W = N_0 W$. Since the signal power is $\frac{7A^2 W}{6}$, the signal-to-noise ratio is $\frac{7A^2}{6N_0}$.

Question 3

A clever FEE student suggests pre-filtering the message before transmission so that message signal has a flat spectrum. Her reasoning is that in the high-frequency part of the spectrum, the message has less power while the noise power is constant. She wonders: “Would it be better to have a flat message spectrum in view of the flat noise power spectrum?”. For a fair comparison, the flattened signal's power must equal the power in the unflattened signal. What is the amplitude of the message signal's spectrum after being flattened?
Type the amplitude parameter A as `A` and the bandwidth W as `W`.

You entered:

Preview Help

Your Answer	Score	Explanation
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Question Explanation

Since the noise power is $\frac{7A^2W}{6}$ and the flattened signal's power would be $A'^2 \cdot 2W$, we find that the flattened-signal's spectral amplitude A' equals $A\sqrt{\frac{7}{12}}$.

Question 4

Using the student's idea, the received signal consists of the flattened message signal plus noise, the noise having the same power spectrum as shown above. Because the message signal does not equal the original, the received signal must be filtered so that the signal has the correct spectrum. What is the signal-to-noise ratio of the filter's output?
Type the noise spectral height parameter N_0 as `N0` (that's "en-zero"), the amplitude parameter A as `A` and the bandwidth W as `W`.

You entered:

Preview Help

Your Answer		Score	Explanation
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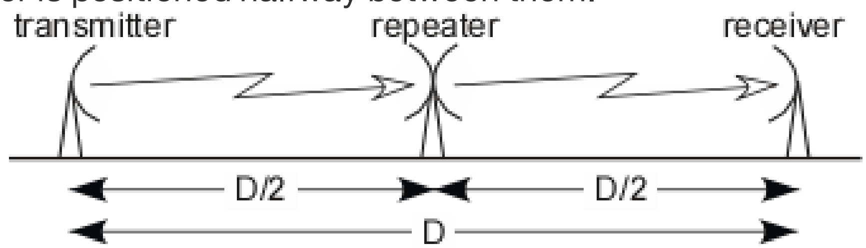
Question Explanation

The received signal must be filtered by a lowpass filter having a frequency response that equals the shape (gain is arbitrary) of the original message spectrum. Form the calculations we have already performed, the message signal's power after filtering equals $A'^2 \cdot \frac{7W}{6} = \frac{49}{72} A^2 W$. The noise is also filtered and the power in the result has the same mathematical form: $\frac{N_0}{2} \cdot \frac{7W}{6}$. Consequently, the signal-to-noise ratio equals $\frac{49A^2 W/72}{N_0/2 \cdot 7W/6} = \frac{7A^2}{6N_0}$. ***No** change in SNR!*

Question 5

Because signals attenuate with distance from the transmitter, **repeaters** are frequently employed in both analog and digital communication systems. As shown below, the transmitter and receiver are D m apart and a repeater is positioned halfway between them.



The repeater amplifies its input to exactly compensate for the signal attenuation along the first leg and re-transmits the amplified input to the receiver. Both the repeater and receiver experience the same amount of additive white noise.

Assuming we are using a wireless channel, what is the signal-to-noise ratio of the repeater's output?

Assume the signal power at the transmitter is P_s over a bandwidth of W and that the spectral height of the noise is $\frac{N_0}{2}$. For simplicity, set the constant in the attenuation formula to one.

Type P_s as `Ps`, W as `W`, D as `D` and N_0 as `N0` (that's "en-zero").

You entered:

Preview Help

Your Answer	Score	Explanation

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Question Explanation

The repeater's received signal power is $P_s/(D/2)^2 = 4P_s/D^2$. The noise power at the repeater's input is $N_0 W$. Amplification does *not* change the signal-to-noise ratio.

$$\text{SNR} = \frac{4P_s}{N_0 W D^2}$$

Question 6

How does this signal-to-noise ratio change if a repeater was used on a wireline channel?

Assume the signal power at the transmitter is P_s over a bandwidth of W and that the spectral height of the noise is $\frac{N_0}{2}$. The attenuation constant of the coaxial cable is a.

Type P_s as `Ps`, W as `W`, a as `a`, D as `D` and N_0 as `N0` (that's "en-zero").

You entered:

Preview Help

Your Answer		Score	Explanation
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Question Explanation

For the wireline channel, the signal power at the repeater's input is

$P_s e^{-2a(D/2)} = P_s e^{-aD}$. The noise power at the repeater's input is $N_0 W$. Again, amplification does *not* change the signal-to-noise ratio.

$$\text{SNR} = \frac{P_s e^{-aD}}{N_0 W}$$

Question 7

Does using a repeater result in a bigger signal-to-noise ratio at the receiver's input than not using it? Calculate the signal-to-noise ratio in each case and provide the ratio of the signal-to-noise ratios: $\frac{\text{SNR}_{\text{with repeater}}}{\text{SNR}_{\text{no repeater}}}$. If better with the repeater, this ratio will be bigger than one. Use the wireless case for this comparison.

Assume the signal power at the transmitter is P_s over a bandwidth of W and that the spectral height of the noise is $\frac{N_0}{2}$.

Type P_s as `Ps`, W as `W`, D as `D` and N_0 as `N0` (that's "en-zero").

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
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Question Explanation

First, the no-repeater case. The repeater's received signal power is P_s/D^2. The noise power at the repeater's input is $N_0 W$.

$$\text{SNR}_{\text{no repeater}} = \frac{P_s}{N_0 W D^2}$$

With the repeater, the receiver signal power is the same as at the repeater, $P_s/(D/2)^2 = 4P_s/D^2$, since the repeater provides an amplification of $D/\sqrt{2}$, making the signal power at the repeater's output P_s. The noise power at the receiver's input consists of two components. First is the noise sent by the repeater, which has power at the receiver of $N_0 W \cdot \frac{(D/2)^2}{(D/2)^2}$, the numerator factor $(D/2)^2$ being the power gain of the repeater, which is canceled by the attenuation along the second leg that appears in the denominator. Second is the power experienced only by the receiver, which has power $N_0 W$. The total noise power is the sum of the powers.

$$\text{SNR}_{\text{with repeater}} = \frac{4P_s/D^2}{N_0 W + N_0 W} = \frac{2P_s}{N_0 W D^2}$$

Therefore

$$\frac{\text{SNR}_{\text{with repeater}}}{\text{SNR}_{\text{no repeater}}} = 2$$

Note that a repeater is *uniformly* better: improvement does not depend on system parameters.