# Feedback — Problem Set IV

You submitted this homework on **Tue 26 Mar 2013 3:43 PM CDT -0500**. You got a score of **0.00** out of **14.00**. However, you will not get credit for it, since it was submitted past the deadline.

In this problem set, you will be given a total of ten attempts. We will accept late submission until the fifth day after the due date, and late submission will receive half credit. Explanations and answers to the problem set will be available after the due date. Since the homework problems will become gradually more challenging as the course proceeds, we highly recommend you to start the habit of printing out the problems and working on them with paper and pencil. Also, please be sure to read the problem statements carefully and double check your expressions before you submit.

A pdf version of this problem set is available for you to print. Note: all mathematical expressions have to be exact, even when involving constants. Such an expression is required when a function and/or a variable is required in the answer. For example, if the answer is $\sqrt{3}x$, you must type `sqrt(3)*x`, not `1.732*x` for the answer to be graded as being correct.

## Question 1

Find the Fourier series coefficients $c_0, c_1, c_2, c_3$ for $\sin t$ *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."

Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$, $c_2 = -j$, $c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0.

**You entered:**

| Your Answer | | Score | Explanation |
|---|---|---|---|
| | ✘ | 0.00 | |
| Total | | 0.00 / 1.00 | |

**Question Explanation**

Because of Euler's formula, $\sin t = \frac{1}{2j}\left(e^{jt} - e^{-jt}\right)$. Therefore $c_1 = \frac{1}{2j} = -\frac{1}{2}j$ and $c_{-1} = \frac{1}{2}j$ (conjugate symmetry). The other coefficients are zero.

# Question 2

Find the Fourier series coefficients $c_0, c_1, c_2, c_3$ for $\sin^2 t$ *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."

Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0,\, c_1 = 1 + j,\, c_2 = -j,\, c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0.

**You entered:**

| Your Answer | | Score | Explanation |
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**Question Explanation**

Because $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$ and using Euler's formula for the cosine—
$\cos 2t = \frac{1}{2}\left(e^{j2t} + e^{-j2t}\right)$, we find that $c_0 = \frac{1}{2}, c_1 = 0, c_2 = -\frac{1}{4}, c_3 = 0$.

# Question 3

Find the Fourier series coefficients $c_0, c_1, c_2, c_3$ for $\cos t + 2\cos 2t$ *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."

Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0,\, c_1 = 1 + j,\, c_2 = -j,\, c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0.

**You entered:**

| Your Answer | | Score | Explanation |
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**Question Explanation**

Using Euler's formula for each term and adding the result, we find that
$c_0 = 0, c_1 = \frac{1}{2}, c_2 = 1, c_3 = 0$.

# Question 4

Find the Fourier series coefficients $c_0, c_1, c_2, c_3$ for $\cos(t)\cos(2t)$ *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."

Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$, $c_2 = -j$, $c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0.

**You entered:**

| Your Answer | | Score | Explanation |
|---|---|---|---|
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**Question Explanation**

Using Euler's formula, we have that $\cos(t)\cos(2t) = \frac{1}{2}\left(e^{jt} + e^{-jt}\right) \cdot \frac{1}{2}\left(e^{j2t} + e^{-j2t}\right) = \frac{1}{4}\left(e^{jt} + e^{j3t} + e^{-jt} + e^{-j3t}\right)$. Therefore, we obtain $c_0 = 0, c_1 = \frac{1}{4}, c_2 = 0, c_3 = \frac{1}{4}$.

# Question 5

Find the Fourier series coefficients $c_0, c_1, c_2, c_3$ for $\cos(2\pi t + \pi/6) \cdot \cos 2\pi t$ *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."

Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$, $c_2 = -j$, $c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0.

Please round all your answers to two decimal places. *Base your answer on a period of 1*.

**You entered:**

| Your Answer | | Score | Explanation |
|---|---|---|---|
| | ✘ | 0.00 | |
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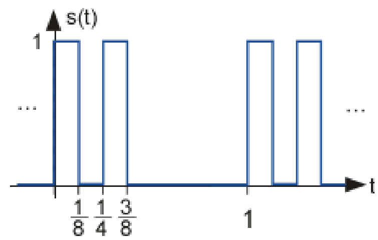
**Question Explanation**

First of all, note that the fundamental frequency is 1 Hz, making the period 1. Euler's formula for $\cos(2\pi t + \pi/6)$ equals $\frac{1}{2}\left(e^{j(2\pi t+\pi/6)} + e^{-j(2\pi t+\pi/6)}\right)$. Multiplying by Euler's formula for $\cos 2\pi t$, we obtain
$\cos(2\pi t + \pi/6)\cos 2\pi t = \frac{1}{4}\left(e^{j\pi/6} + e^{j(4\pi t+\pi/6)} + e^{-j\pi/6} + e^{-j(4\pi t+\pi/6)}\right) = \frac{1}{2}\cos\pi/6 + \frac{1}{4}e^{j\pi/6}e^{j4\pi t} + \frac{1}{4}e^{-j\pi/6}e^{-j4\pi t}$
. We have $c_0 = \frac{1}{2}\cos\pi/6, c_1 = 0, c_2 = \frac{1}{4}e^{j\pi/6}, c_3 = 0.$
Note that $T = 1/2$ could have been chosen as the period. In that case, we have
$c_0 = \frac{1}{2}\cos\pi/6, c_1 = \frac{1}{4}e^{j\pi/6}, c_2 = 0, c_3 = 0.$

# Question 6

Find the Fourier series coefficients $c_0$ and $c_1$ for the depicted signal *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."




Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0, c_1 = 1 + j$ would be typed as 0 0 1 1.

**You entered:**

| Your Answer | | Score | Explanation |
|---|---|---|---|
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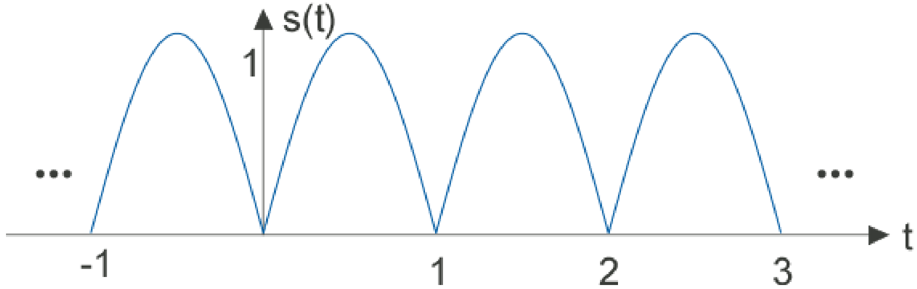
**Question Explanation**

$c_0$ is the average value, which equals $2 \cdot \frac{1}{8} = \frac{1}{4}$. This signal is the superposition of a periodic pulse signal plus the same signal delayed by $\frac{1}{4}$. The general expression for the Fourier series coefficients for the periodic pulse signal is $e^{-j\pi k/8}\dfrac{\sin\frac{\pi k}{8}}{\pi k}$. Therefore, the coefficients for the depicted signal is $\left(1 + e^{-j2\pi k\frac{1}{4}}\right)\cdot e^{-j\pi k/8}\dfrac{\sin\frac{\pi k}{8}}{\pi k} = \left(1 + (-j)^k\right)\cdot e^{-j\pi k/8}\dfrac{\sin\frac{\pi k}{8}}{\pi k}$. Evaluating for $c_1$, we have $(1-j)e^{-j\pi/8}\dfrac{\sin\frac{\pi}{8}}{\pi} = \sqrt{2}e^{-j\pi/4}e^{-j\pi/8}\dfrac{\sin\frac{\pi}{8}}{\pi} = \sqrt{2}e^{-j3\pi/8}\dfrac{\sin\frac{\pi}{8}}{\pi}$. Numeric value of $c_1$ is $0.0659 - 0.1592j$.

# Question 7

The depicted signal is known as a *full-wave rectiified sinusoid*, expressed mathematically as the absolute value of $\sin \pi t$: $|\sin \pi t|$.




What is this signal's period?

**You entered:**

| Your Answer | | Score | Explanation |
|---|---|---|---|
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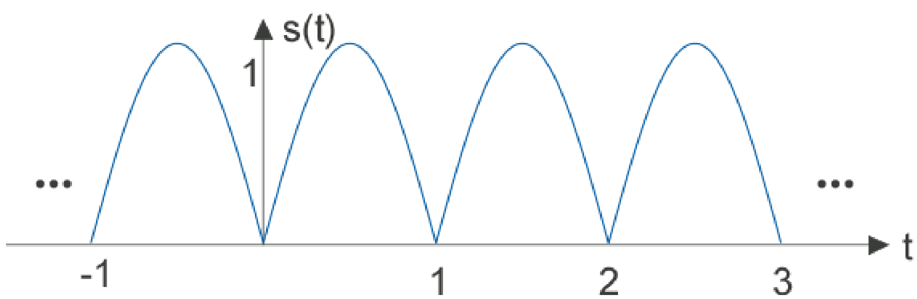
| Total | | 0.00 / 0.25 | |

**Question Explanation**

While the period of the original signal $\sin \pi t$ is 2, once the absolute value is computed the period is 1.

# Question 8

Find the Fourier series coefficients $c_k$ for this *full-wave rectiified sinusoid*, expressed mathematically as $|\sin \pi t|$.




Express your answer as a mathematical expression; if $c_k = \frac{1}{k} - j\frac{1}{2k}$, you would enter 1/k-j/(2*k).

**You entered:**

Preview Help

| Your Answer | Score | Explanation |
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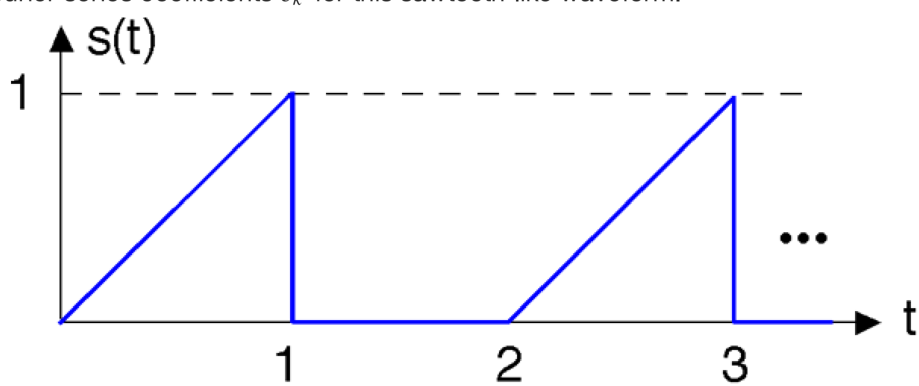
| Total | 0.00 / 0.75 | |

**Question Explanation**

The coefficients are given by $c_k = \int_0^1 \sin \pi t e^{-j2\pi kt}\, dt$. The easiest way to evaluate this integral is to exploit Euler's formula: $c_k = \int_0^1 \frac{1}{2}\left(e^{j\pi t} - e^{-j\pi t}\right)e^{-j2\pi kt}\, dt$. Evaluating this integral gives $\frac{1}{2j}\left[\frac{1}{j\pi(1-2k)}\left(e^{j\pi(1-2k)}\right) + \frac{1}{j\pi(1+2k)}\left(e^{-j\pi(1+2k)}\right)\right]$. Note that $e^{\pm j\pi(1\pm 2k)} = -1$.
Consequently, we obtain $c_k = \frac{2}{\pi} \cdot \frac{1}{1-4k^2}$.
Note that the average value, $c_0$, equals $\frac{2}{\pi} = 0.64$. Furthermore, all the coefficients are real-valued because $|\sin \pi t|$ is an even signal.

# Question 9

Find the Fourier series coefficients $c_k$ for this sawtooth-like waveform.




Express your answer as a mathematical expression; if $c_k = \frac{1}{k} - j\frac{1}{2k}$, you would enter 1/k-j/(2*k).

**You entered:**

Preview Help

| Your Answer | Score | Explanation |
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| ✘ | 0.00 | Could not parse student submission |
| Total | 0.00 / 1.00 | |

**Question Explanation**

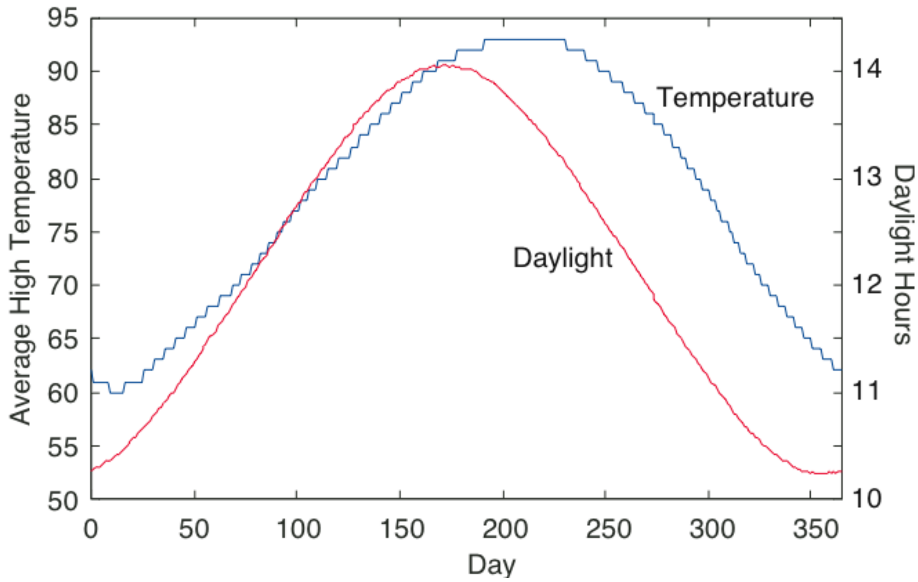
$c_k = \frac{1}{2}\int_0^1 te^{-j\pi kt}\,dt$, evaluated by integrating by parts.

$$u = t \quad dv = e^{-j\pi kt} \implies du = dt \quad v = -\frac{1}{j\pi k}e^{-j\pi kt}$$

We have $c_k = -\frac{t}{2j\pi k}e^{-j\pi kt}\Big|_0^1 + \frac{1}{2j\pi k}\int_0^1 e^{-2j\pi kt}\,dt = j\frac{(-1)^k}{2\pi k} + \left((-1)^k - 1\right)\cdot\frac{1}{2\pi^2k^2}$.

# Question 10

The daily temperature is a consequence of several effects, one of them being the sun's heating. If this were the dominant effect, then daily temperatures would be proportional to the number of daylight hours. The plot shows that the average daily high temperature does **not** behave that way.




In the next four problems, we want to understand the temperature component of our environment using Fourier series and linear system theory. The file `temperature.mat` (`temperature.data` in ascii) contains these data (daylight hours in the first row, corresponding average daily highs in the second) for Houston, Texas. Once you download this data file, load it into Matlab or Octave using the `load` command. You will find two signals (vectors): `high` and `daylight`.

Let the length of day serve as the sole input to a system having an output equal to the average daily temperature. Examining the plots of input and output, would you say that the system is linear or not?

| Your Answer | Score | Explanation |
|---|---|---|
| Yes | | |
| No | | |
| Total | 0.00 / 1.00 | |

**Question Explanation**

It appears to be a sinusoid in, a sinusoid out, which is indeed consistent with a linear system.

## Question 11

Using Matlab or Octave, find the first four Fourier series coefficients $c_0, c_1, c_2, c_3$ for the `high` dataset. Approximate the integral as a sum:

$$c_k \approx \frac{1}{366} \sum_{n=0}^{365} \mathtt{high}(n) e^{-j2\pi kn/366}$$

Express your answer numerically, typing the real and imaginary parts for each answer *to two decimal places* separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$, $c_2 = -j\sqrt{3}$, $c_3 = 0$ would be typed as 0 0 1 1 0 -1.73 0 0. *note*: in Matlab you can simply use `j` to denote $j$.

**You entered:**

| Your Answer | | Score | Explanation |
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| | ✘ | 0.00 | |
| Total | | 0.00 / 1.00 | |

**Question Explanation**

**Sample program:**

```
c = [];
for k = 0:3
  c(k+1) = sum(exp(-j*2*pi*k*(n-1)/366).*high(n))/366;
end
>> c
c =
78.6066 -7.3551 + 2.5788i -0.8418 - 0.0073i -0.2413 + 0.0426i
```

## Question 12

Using Matlab or Octave, find the first four Fourier series coefficients $c_0, c_1, c_2, c_3$ for the `daylight` dataset. Approximate the integral as a sum:

$$c_k \approx \frac{1}{366} \sum_{n=0}^{365} \mathtt{daylight}(n) e^{-j2\pi kn/366}$$

Express your answer numerically, typing the real and imaginary parts *to two decimal places* for each answer separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$, $c_2 = -j\sqrt{3}$, $c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0.

**You entered:**

| Your Answer | | Score | Explanation |
|---|---|---|---|
| | ✘ | 0.00 | |
| Total | | 0.00 / 1.00 | |

**Question Explanation**

**Sample Program**

```
c = [];
for k = 0:3
  c(k+1) = sum(exp(-j*2*pi*k*(n-1)/366).*daylight(n))/366;
end
>> c
c =
12.1656 -0.9166 - 0.1695i -0.0071 - 0.0019i -0.0200 - 0.0139i
```

# Question 13

What is the harmonic distortion for the two datasets? Enter your numeric answers as "daylight_harmonic_distortion high_harmonic_distortion" (note the space between the two values). Provide as many decimal places that are needed to reach the first non-zero digit. For example, a distortion of 0.0012 would be entered as `0.001`.

**You entered:**

| Your Answer | | Score | Explanation |
|---|---|---|---|
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**Question Explanation**

The total signal power *after subtracting average values of 12.17 and 78.61 respectively* is 123.2 for `daylight` and 1.74 for `high`. To find the power contained in the second and higher harmonics, subtract $2|c_1|^2$, which gives 0.0014 for `daylight` and 1.71 for `high`. The harmonic distortion is the ratio of the two: 0.00078 for `daylight` and 0.014 for `high`.

# Question 14

Because the harmonic distortion is small, let's concentrate only on the first harmonic. What is the phase shift between input (`daylight`) and output (`high`) signals? This phase shift is the key quantity that will allow us to model the earth's temperature environment as a linear system. Express your answer in radians.

**You entered:**

| Your Answer | | Score | Explanation |
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**Question Explanation**

The angle of the `daylight` signal's first harmonic is -2.96 radians and that for the `high` signal is 2.80. Since the `daylight` signal's Fourier coefficient lies in the third quadrant and the `high` signal's in in the second quadrant. Consequently, the phase shift is negative and less than $\pi/2$. Subtracting the phase for `daylight` from that for `high`, we obtain 5.76 radians. To obtain the correct result, we must subtract $2\pi$, obtaining -0.52 radians (-29.8 degrees).

# Question 15

It is an old axiom that the simplest model is usually the best. We are assuming that the model is a linear filter of some kind. *Because of the phase shift*, which models the response of the earth to the sun's heating process?

| Your Answer | Score | Explanation |
|---|---|---|
| First-order (RC) lowpass filter | | |
| First-order (RC) highpass filter | | |
| Simple gain (amplifier) | | |
| Something more complicated | | |
| Total | 0.00 / 1.00 | |

**Question Explanation**

Because the phase shift is negative, a lowpass filter is simplest. Furthermore, since the phase shift is less than $\pi/4$, the frequency equivalent to a period of one year is less than the cutoff frequency of the filter.