Feedback — Problem Set III

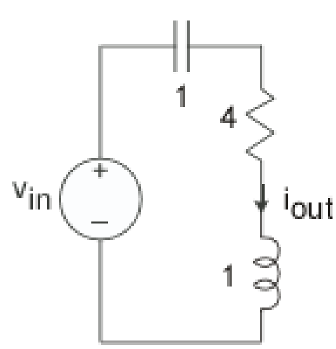
You submitted this homework on **Tue 26 Mar 2013 3:41 PM CDT -0500**. You got a score of **0.00** out of **11.00**. However, you will not get credit for it, since it was submitted past the deadline.

In this problem set, you will only be given a total of three attempts. We will accept late submission until the fifth day after the due date, and late submission will receive half credit. Explanations and answers to the problem set will be available after the due date. Since the homework problems will become gradually more challenging as the course proceeds, we highly recommend you to start the habit of printing out the problems and working on them with paper and pencil. Also, please be sure to read the problem statements carefully and double check your expressions before you submit.

A pdf version of this problem set is available for you to print. Note: all mathematical expressions have to be exact, even when involving constants. Such an expression is required when a function and/or a variable is required in the answer. For example, if the answer is $\sqrt{3}x$, you must type `sqrt(3)*x`, not `1.732*x` for the answer to be graded as being correct.

Question 1

Find the transfer function relating the complex amplitudes of the indicated variable and the source. Use the symbol s to represent $j2\pi f$ and express your answer in terms of s.



You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

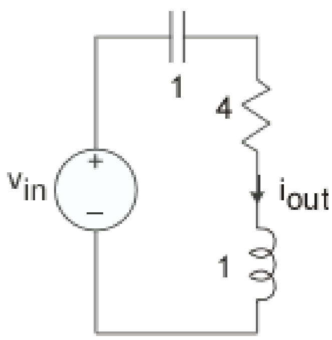
Question Explanation

I_{out} is simply the complex amplitude of the source V_{in} divided by the equivalent impedance of the circuit.

$$\frac{I_{\text{out}}}{V_{\text{in}}} = \frac{1}{\frac{1}{s} + 4 + s} = \frac{s}{s^2 + 4s + 1}$$

Question 2

Re-writing the transfer function you found in the previous question in terms of $j2\pi f$, determine what kind of filter this circuit realizes.

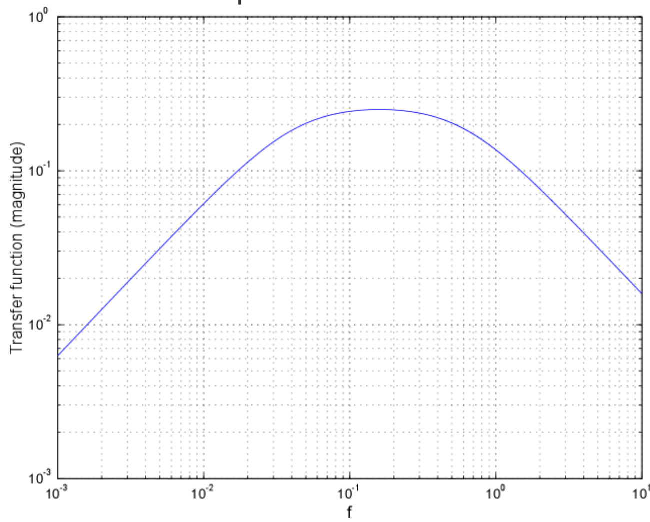


Your Answer	Score	Explanation
Lowpass		
Highpass		
Bandpass		
Something else		

Total	0.00 / 1.00

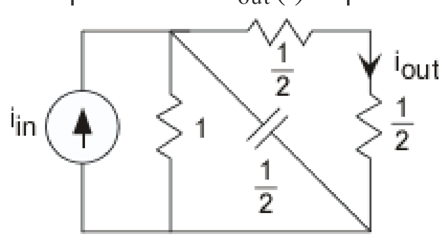
Question Explanation

Either by plotting or by asymptotic evaluation, the transfer function $\left.\frac{s}{s^2+4s+1}\right|_{s=j2\pi f} = \frac{j2\pi f}{1-4\pi^2 f^2 + j8\pi f}$ equals zero at $f=0$ and approaches zero when $f \to \infty$ (as $\frac{1}{f}$). Consequently, this filter cannot be lowpass or highpass. It does serve as a bandpass filter.



Question 3

In the following circuit, the output current $i_{\mathrm{out}}(t)$ equals $\cos(2t)$.



What is the frequency of the source? Write your answer as an expression, not a numeric quantity.

You entered:

Preview Help

Your Answer	Score	Explanation

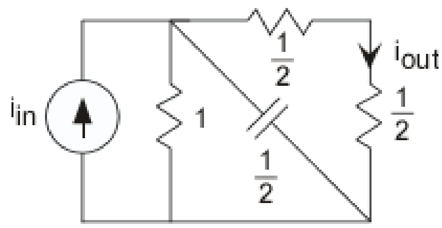
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

A sinusoid has the form $\cos 2\pi f t$. Consequently, $2\pi f = 2 \implies f = \frac{1}{\pi}$.

Question 4

In the following circuit, the output current $i_{\text{out}}(t)$ equals $\cos(2t)$.



What is the transfer function between the source and the output current? Write your answer in terms of s, where $s = j2\pi f$.

You entered:

Preview Help

Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

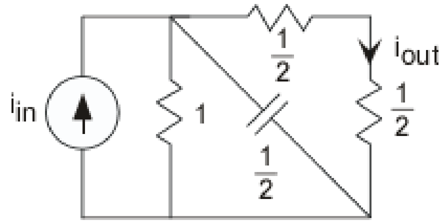
Question Explanation

This circuit's structure is a current source in parallel with three parallel collections of elements: a 1Ω resistor, a $\frac{1}{2}$ F capacitor and a series of $\frac{1}{2}\ \Omega$ resistors (the latter collection functions like a 1Ω resistor). Using current divider, we find that

$$\frac{I_{\text{out}}}{I_{\text{in}}} = \frac{1 \| \frac{1}{\frac{1}{2}s}}{1 \| \frac{1}{\frac{1}{2}s} + 1} = \frac{\frac{2}{s+2}}{\frac{2}{s+2} + 1} = \frac{2}{s+4}$$

Question 5

In the following circuit, the output current $i_{\text{out}}(t)$ equals $\cos(2t)$.



Find the source, expressed as a real-valued signal.

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

Since $\frac{I_{\text{out}}}{I_{\text{in}}} = \frac{2}{s+4}$, when the input is $I_{\text{in}} e^{j2\pi ft}$, the output is $\frac{2I_{\text{in}}}{j2\pi f + 4} e^{j2\pi ft}$. The source frequency is 1π means we have an output equal to $\frac{2I_{\text{in}}}{j2+4} e^{j2t} = \frac{I_{\text{in}}}{j+2} e^{j2t}$. Using the real part to express the output, we have $\cos 2t = \text{Re}\left[\frac{I_{\text{in}}}{j+2} e^{j2t}\right]$. We need only find I_{in} so that $\frac{I_{\text{in}}}{j+2} = 1 \implies I_{\text{in}} = j + 2 = \sqrt{5} e^{j \tan^{-1}(1/2)}$. Finally, we have

$$i_{\text{in}} = \text{Re}\left[I_{\text{in}} e^{j2\pi ft}\right] = \sqrt{5}\cos\left(2t + \tan^{-1}\frac{1}{2}\right)$$

Question 6

In the lab, the open-circuit voltage measured across an unknown circuit's terminals equals $\sin(t)$. When a 1Ω resistor is placed across the terminals, a voltage of $\frac{1}{\sqrt{2}}\sin\left(t+\frac{\pi}{4}\right)$ appears. What is the Thévenin equivalent source voltage? Express your answer as a real-valued signal.

You entered:

Preview Help

Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

Question Explanation

The open-circuit voltage *is* the Thévenin equivalent source voltage. So, the equivalent source voltage is $\sin(t)$.

Question 7

In the lab, the open-circuit voltage measured across an unknown circuit's terminals equals $\sin(t)$. When a 1Ω resistor is placed across the terminals, a voltage of $\frac{1}{\sqrt{2}}\sin\left(t+\frac{\pi}{4}\right)$ appears. What is the Thévenin equivalent impedance? Your answer can be in Cartesian or polar form.

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

The transfer function is $\frac{V_{\text{out}}}{V_{\text{eq}}} = \frac{1}{1+Z_{\text{eq}}}$. Expressing the source as $\text{Im}\left[e^{jt}\right]$, $V_{\text{eq}} = 1$. The output voltage equals $\text{Im}\left[\frac{1}{\sqrt{2}}e^{j(t+\pi/4)}\right]$, which mean $V_{\text{out}} = \frac{e^{j\pi/4}}{\sqrt{2}} = \frac{1+j}{2}$. We can now solve for Z_{eq}:
$\frac{1}{1+Z_{\text{eq}}} = \frac{1+j}{2} \implies Z_{\text{eq}} = \frac{1-j}{1+j} = -j$.

Question 8

In the lab, the open-circuit voltage measured across an unknown circuit's terminals equals $\sin(t)$. When a 1Ω resistor is placed across the terminals, a voltage of $\frac{1}{\sqrt{2}}\sin\left(t+\frac{\pi}{4}\right)$ appears. What voltage will appear if a 1F capacitor replaces the resistor?

You entered:

Preview Help

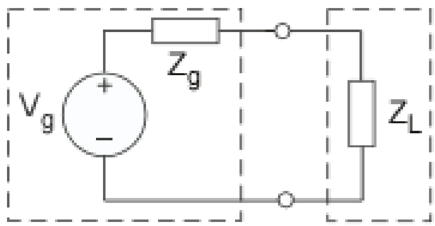
Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

We have found that the source is $\sin t$ and that the impedance at the source frequency ($f = \frac{1}{2\pi}$) is $-j$. This result suggests the impedance can be considered a 1F capacitor. Consequently, using voltage divider, we find that the output voltage is one-half the Thévenin equivalent source voltage: $\frac{1}{2}\sin t$.

Question 9

The following circuit shows a general model for power transmission. The power generator is represented by a Thévinin equivalent and the load by a simple impedance. In most applications, the source components are fixed while there is some latitude in choosing the load.



Suppose we wanted to maximize "voltage transmission": make the voltage across the load as large as possible. In this problem, the source is a sinusoid having some amplitude, frequency and phase. What choice of load impedance creates the largest load voltage? Enter your answer as a mathematical expression as `a+j*b`, entering **exact** values for a and b.

If needed, use `z_r` to represent z_r and `z_i` to represent z_i , the real and imaginary parts of the source impedance $Z_g = z_r + jz_i$. So, if $Z_L = Z_g$ is your answer, enter `z_r+j*z_i`.

You entered:

Preview Help

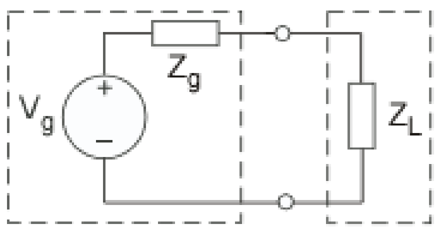
Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

Question Explanation

The transfer function equals $\frac{Z_L}{Z_g + Z_L}$. Let $Z_g = z_r + jz_i$ and $Z_L = a + jb$. To simplify the mathematics, maximize $\left|\frac{Z_L}{Z_g + Z_L}\right|^2 = \frac{a^2 + b^2}{(z_r + a)^2 + (z_i + b)^2}$ with respect to a and b. Clearly, the maximize this ratio, we can choose $b = -z_i$. However, we **cannot** choose $a = -z_r$: **impedances of systems not containing sources cannot have negative real parts**. Considering $\frac{a^2}{(z_r + a)^2}$, it is a monotonically increasing function for $a > 0$. Consequently, the voltage is maximized with an open circuit: $Z_L = \infty$.

Question 10

Suppose we wanted the maximize "current transmission": make the current across the load as large as possible



What choice of load impedance creates the largest load current?

Use `z_r` to represent z_r and `z_i` to represent z_i, the real and imaginary parts of the source impedance: $Z_g = z_r + jz_i$. So, if $Z_L = Z_g$ is your answer, enter `z_r+j*z_i`.

You entered:

Preview Help

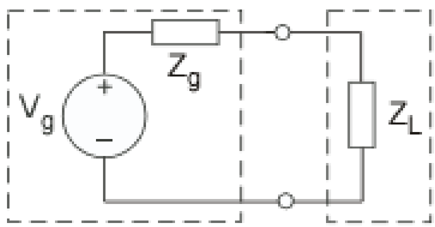
Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

Question Explanation

The transfer function for the current is $\frac{1}{Z_g + Z_L}$. Let $Z_L = a + jb$. Maximize the magnitude-squared: $\frac{1}{(z_r + a)^2 + (z_i + b)^2}$. Again, pick $b = -z_i$. The magnitude-squared becomes $\frac{1}{(g_r + a)^2}$. Since this is a monotonically *decreasing* function of a, it is maximized for $a = 0$. Consequently, $Z_L = -jz_i$.

Question 11

What choice for the load impedance maximizes the average power dissipated in the load?



Use `z_r` to represent z_r and `z_i` to represent z_i, the real and imaginary parts of the source impedance $Z_g = z_r + jz_i$. So, if $Z_L = Z_g$ is your answer, enter `z_r+j*z_i`.

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

The average power is proportional to

$\mathrm{Re}\left[VI^*\right] = \mathrm{Re}\left[\frac{Z_L}{Z_g + Z_L}\frac{1}{(Z_g + Z_l)^*}\right] = \frac{\mathrm{Re}[Z_L]}{|Z_g + Z_L|^2}$. Simplifying by letting $Z_g = z_r + jz_i$ and $Z_L = a + jb$, we have $\frac{a}{(z_r + a)^2 + (z_i + b)^2}$. As before, $b = -z_i$ maximizes the ratio. Taking the derivative of the result of substituting this into our formula, we find $\frac{z_r - a}{(z_r + a)^3} = 0$, which gives $a = z_r$. We obtain the cool result that $Z_L = z_r - jz_i = Z_g^*$ for maximum power transmission. This situation is called **impedance matching**.