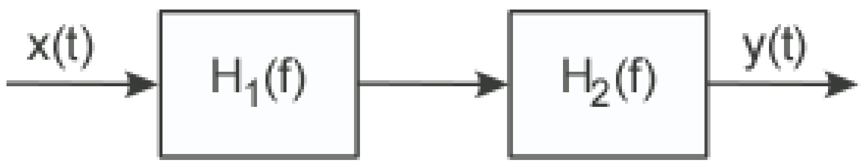
Feedback — Frequency Domain Exercises

You submitted this homework on **Wed 3 Apr 2013 2:08 PM CDT -0500**. You got a score of **0.00** out of **9.00**. You can attempt again, if you'd like.

Question 1

What is the overall transfer function between $x(t)$ and $y(t)$? Here $H_1(f)$ and $H_2(f)$ represent the transfer functions of linear, time-invariant systems in a **cascade** configuration.



Type your answer as an expression, with `H1` representing $H_1(f)$ and `H2` representing $H_2(f)$.

You entered:

Preview Help

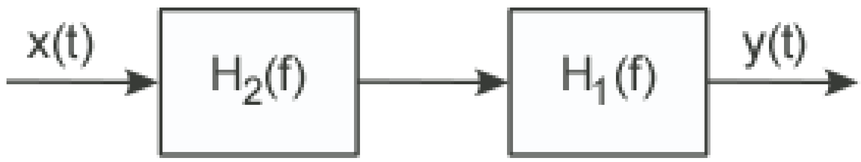
Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

Question Explanation

Call the output of the first system $w(t)$. In terms of Fourier transforms, $W(f) = H_1(f) \cdot X(f)$. Similarly, $Y(f) = H_2(f) \cdot W(f)$. Therefore, $Y(f) = H_1(f)H_2(f)X(f)$, making the overall transfer function $H_1(f)H_2(f)$.

Question 2

What is the overall transfer function between $x(t)$ and $y(t)$? Here $H_1(f)$ and $H_2(f)$ represent the transfer functions of linear, time-invariant systems in a cascade configuration. Note that the systems are in a different order than in the previous problem.



Type your answer as an expression, with `H1` representing $H_1(f)$ and `H2` representing $H_2(f)$.

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

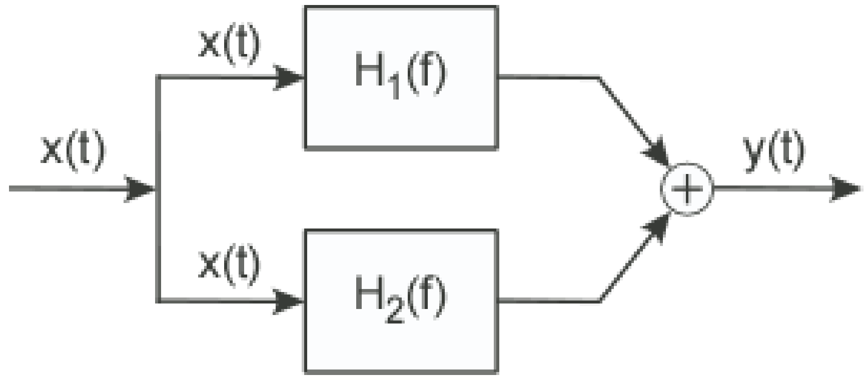
Question Explanation

Using similar reasoning as in the first problem, we find that the transfer function is the *same*. In general, the ordering of linear, time-invariant systems in cascade does not matter in that the transfer function does not change. In the real world, there could be practical and loading reasons for preferring one ordering over another.

Question 3

What is the overall transfer function between $x(t)$ and $y(t)$? Here $H_1(f)$ and $H_2(f)$

represent the transfer functions of linear, time-invariant systems in what is known as a **parallel** configuration.



Type your answer as an expression, with `H1` representing $H_1(f)$ and `H2` representing $H_2(f)$.

You entered:

Preview Help

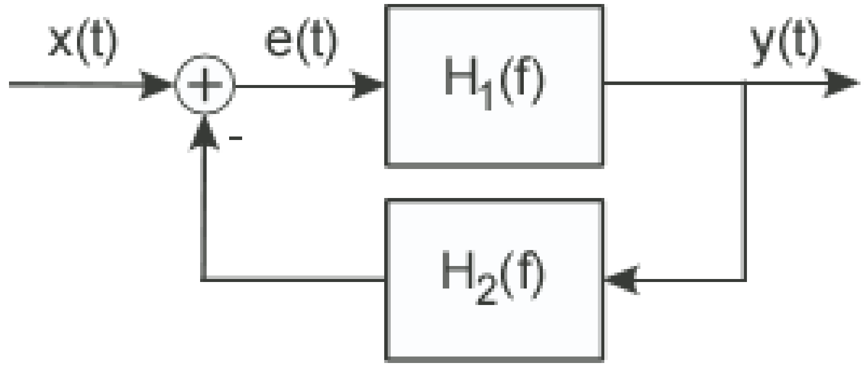
Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

Question Explanation

Define $y_1(t)$ to be the output of the upper system and $y_2(t)$ to be the output of the lower system. We know that $Y_1(f) = H_1(f)X(f)$ and $Y_2(f) = H_2(f)X(f)$. Since $y(t) = y_1(t) + y_2(t)$, $Y(f) = Y_1(f) + Y_2(f)$ because of the linearity of the Fourier transform. Therefore, the overall transfer function is $H_1(f) + H_2(f)$.

Question 4

What is the overall transfer function between the input $x(t)$ and the output $y(t)$? Here $H_1(f)$ and $H_2(f)$ represent the transfer functions of linear, time-invariant systems in what is known as a **feedback** configuration.



Here, the signal $e(t)$ equals the input $x(t)$ minus the output of the lower system. The small minus sign near the lower system's output going into the adder indicates whether that signal is added or subtracted.

Type your answer as an expression, with `H1` representing $H_1(f)$ and `H2` representing $H_2(f)$.

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

Start with the upper system. $Y(f) = H_1(f) \cdot E(f)$. The Fourier transform of the lower system's output is $H_2(f) \cdot Y(f)$. Therefore, $E(f) = X(f) - H_2(f) \cdot Y(f)$. Substituting this into the first equation, we have $Y(f) = H_1(f) \cdot \left(X(f) - H_2(f) \cdot Y(f)\right)$. Solving for $Y(f)$, we have $Y(f) = \dfrac{H_1(f)}{1 + H_1(f)H_2(f)} X(f)$.

Question 5

In some cases, simple mathematical operations on a signal can be implementing by passing the signal through a linear, time-invariant (LTI) system. Suppose we wanted to produce a signal that equalled the *derivative* of some signal. What is the transfer function of a system that accomplishes this operation?

Your Answer	Score	Explanation
$\frac{1}{j2\pi f}$		
Can't be implemented with an LTI system.		
$j2\pi f$		
Total	0.00 / 1.00	

Question Explanation

From the Table, a derivative corresponds to multiplying by $j2\pi f$: $\frac{d\,s(t)}{dt} \leftrightarrow j2\pi f \cdot S(f)$. Hence, the system having transfer function $j2\pi f$ does the job.

Question 6

Suppose we wanted to produce a signal that equalled the *second* derivative of some signal. What is the transfer function of a system that accomplishes this operation?

Your Answer	Score	Explanation
$-4\pi^2 f^2$		
Can't be done.		
$2 \cdot j2\pi f$		
Total	0.00 / 1.00	

Question Explanation

Since $\frac{d\,s(t)}{dt} \leftrightarrow j2\pi f \cdot S(f)$, because of the cascade rule,

$\frac{d^2 s(t)}{dt^2} \leftrightarrow (j2\pi f)^2 \cdot S(f)$. Therefore the transfer function will be $(j2\pi f)^2 = -4\pi^2 f^2$.

Question 7

Reverberation (producing simple echoes) corresponds to adding a signal to its delayed version: $x(t) + x(t - \tau)$.

What is the input-output relationship of a reverberation system? In other words, what is the transfer function, if it exists (reverberation may not correspond to a linear, time-invariant operation)?

Type `tau` to represent τ. If the transfer function does not exist, type `nan`.

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

As $y(t) = x(t) + x(t - \tau)$, in the frequency domain we have $Y(f) = X(f) + e^{-j2\pi f \tau} X(f)$, making the transfer function $1 + e^{-j2\pi f \tau}$.

Question 8

Reverberation corresponds to some kind of filter; what kind?

Your Answer	Score	Explanation
Something else		

Bandpass

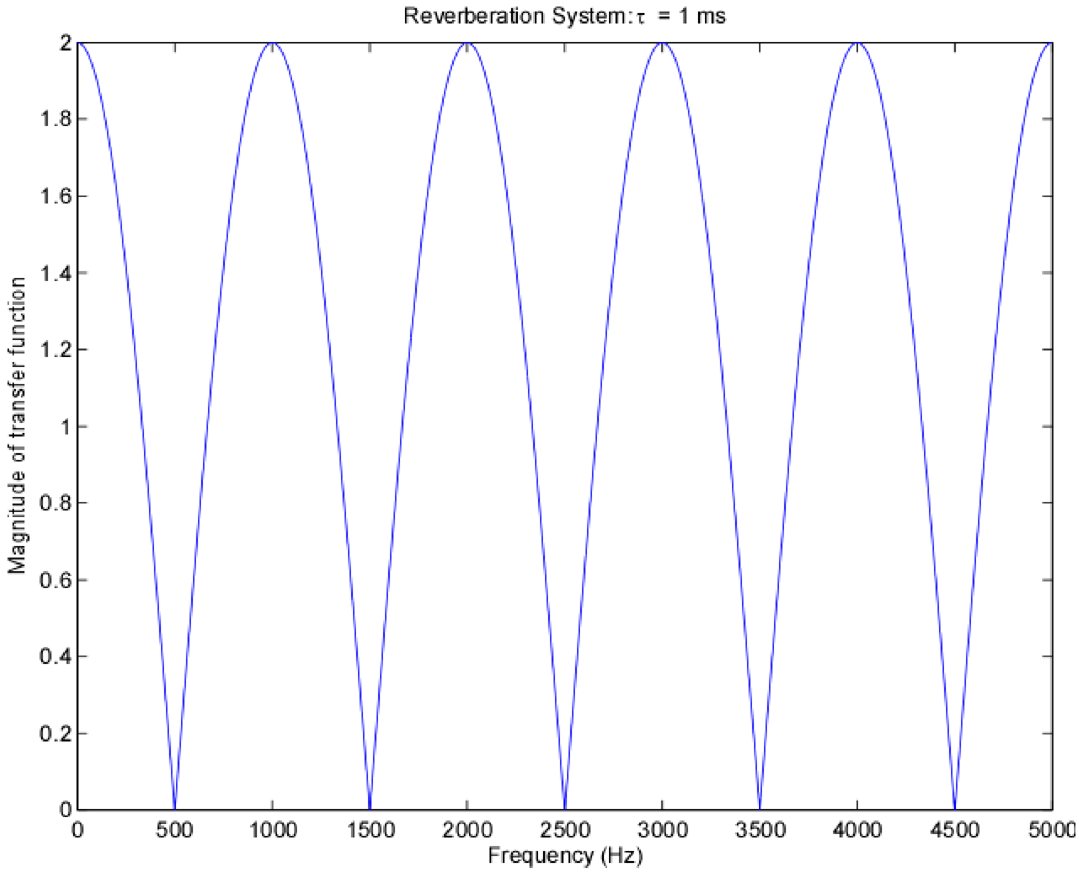
Highpass

Lowpass

Total	0.00 / 1.00

Question Explanation

Since $H(f) = 1 + e^{-j2\pi f \tau}$,
$\left|H(f)\right| = \sqrt{\left(1+\cos(2\pi f \tau)\right)^2 + \sin^2(2\pi f \tau)} = \sqrt{2 + 2\cos 2\pi f \tau}$. Thus, this filter accentuates some frequencies and attenuates others, what is known as a **multi-band** filter.



Question 9

The music group FEE is having trouble selling its recordings. The record company's engineer gets the idea of applying different delays to low and high frequencies, then adding the results to create a novel listening experience that will sell **lots** of records. Thus, FEE's audio would be separated into two parts, lowpass filtered to f_0 with an LTI system having transfer function $H(f)$ and highpass filtered to f_0 with a system having transfer function $\left(1 - H(f)\right)$. The lowpass part would then be delayed by τ_l, and the highpass part delayed by τ_h. What is the transfer function of this music system?

Denote $H(f)$ by `H`, τ_l by `tau_l`, and τ_h by tau_h.

You entered:

Preview Help

Your Answer	Score	Explanation
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Total	0.00 / 1.00	

Question Explanation

The overall transfer function is $H(f)e^{-j2\pi f\tau_l} + \left(1 - H(f)\right)e^{-j2\pi f\tau_h} = H(f)\left(e^{-j2\pi f\tau_l} - e^{-j2\pi f\tau_h}\right) + e^{-j2\pi f\tau_h}$. Factoring out $e^{-j2\pi f\tau_l}$, we have $e^{-j2\pi f\tau_l} \cdot \left[H(f)\left(1 - e^{-j2\pi f(\tau_h - \tau_l)}\right) + e^{-j2\pi f(\tau_h - \tau_l)}\right]$. An overall delay of τ_l does not affect the overall quality (or lack of it). So this system applies a reverberation of $\tau_h - \tau_l$ to the low frequencies, which is added to a a delayed version of the input.