Feedback — Digital Filtering Exercises

You submitted this homework on **Wed 3 Apr 2013 2:11 PM CDT -0500**. You got a score of **0.00** out of **10.00**. You can attempt again, if you'd like.

Question 1

An interesting digital filter is described by the difference equation

$$y(n) = ay(n-1) + ax(n) - x(n-1),\ \ a = \frac{1}{\sqrt{2}}$$

Let's see why it is "interesting."

Find the unit-sample response of this filter. In other words, what is the output $y(n)$ when the input $x(n)$ equals $\delta(n)$? When we have a unit-sample input, the output is frequently called $h(n)$.

Provide **numeric** answers for the first four values of $h(n)$: $h(0)$, $h(1)$, $h(2)$, and $h(3)$ using $a = \frac{1}{\sqrt{2}}$. Separate each value by a space.

You entered:

Your Answer		Score	Explanation
	✘	0.00	
Total		0.00 / 1.00	

Question Explanation

Using a table,

n	$x(n)$	$y(n)$
-1	0	0
0	1	$a = \frac{1}{\sqrt{2}} = 0.707$
1	0	$-1 + a^2 = -1 + \frac{1}{2} = -0.5$
2	0	$a(-1 + a^2) = \frac{1}{\sqrt{2}}\left(-1 + \frac{1}{2}\right) = -0.3535$
3	0	$a^2(-1 + a^2) = \frac{1}{2}\left(-1 + \frac{1}{2}\right) = -0.25$

Question 2

What is this filter's transfer function? Express your answer using the filter parameter a rather than is numeric value.

$H\left(e^{j2\pi f}\right) = ?$

You entered:

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Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

Question Explanation

Evaluating the DTFT of the difference equation, we have $Y\left(e^{j2\pi f}\right) = ae^{-j2\pi f} \cdot Y\left(e^{j2\pi f}\right) + aX\left(e^{j2\pi f}\right) - e^{-j2\pi f} \cdot X\left(e^{j2\pi f}\right)$. The transfer function equals $\frac{Y\left(e^{j2\pi f}\right)}{X\left(e^{j2\pi f}\right)} = \frac{a - e^{-j2\pi f}}{1 - ae^{-j2\pi f}}$.

Question 3

The “interesting” aspect arises when we consider the transfer function's magnitude and phase. What is the magnitude $\left|H\left(e^{j2\pi f}\right)\right|$?

You should answer by starting with an expression that involves the filter's parameter a.

You entered:

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Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

Question Explanation

$$\left|H\left(e^{j2\pi f}\right)\right| = \frac{\left|a - e^{-j2\pi f}\right|}{\left|1 - ae^{-j2\pi f}\right|} = \frac{\sqrt{(a - \cos 2\pi f)^2 + \sin^2 2\pi f}}{\sqrt{(1 - a\cos 2\pi f)^2 + a^2 \sin^2 2\pi f}} = \frac{\sqrt{a^2 - 2a\cos 2\pi f + \cos^2 2\pi f + \sin^2 2\pi f}}{\sqrt{1 - 2a\cos 2\pi f + a^2\cos^2 2\pi f + a^2 \sin 2\pi f}} = \frac{\sqrt{1 - 2a\cos 2\pi f + a^2}}{\sqrt{1 - 2a\cos 2\pi f + a^2}} = 1$$

.

A filter having a constant gain as a function of frequency is known as an *allpass* filter.

Question 4

What is the phase of this transfer function?

Your answer should be an expression that involves the filter's parameter a.

Use the function `atan2` anywhere the arc-tangent function is needed. For example, the phase of $a + jb$ would be typed as `atan2(b,a)` (note the order of `atan2`'s arguments).

You entered:

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Your Answer		Score	Explanation
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Total	0.00 / 1.00

Question Explanation

$\angle H\left(e^{j2\pi f}\right) = \tan^{-1}\dfrac{\sin 2\pi f}{a - \cos 2\pi f} - \tan^{-1}\dfrac{a\sin 2\pi f}{1 - a\cos 2\pi f}$ =`atan2(sin(2*pi*f),a-cos(2*pi*f))-atan2(a*sin(2*pi*f),1-a*cos(2*pi*f))`. If you plot this using Matlab/Octave, this phase is not constant.

Question 5

What is this filter's output to $\sin\left(\frac{\pi}{4}n\right)$ when $a = \dfrac{1}{\sqrt{2}}$?

Type your answer as a expression for the output signal.

You entered:

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Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

Since the input is a sinusoid, we know that the output is a sinusoid. The magnitude of this transfer function is one, so that filter *only* affects the phase. Since the frequency of the input is $f = \frac{1}{8}$, the transfer function at this frequency equals

$\dfrac{\frac{1}{\sqrt{2}} - e^{-j\pi/4}}{1 - \frac{1}{\sqrt{2}}e^{-j\pi/4}} = \dfrac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)} = \dfrac{j\frac{1}{\sqrt{2}}}{1 - \frac{1}{2} + j\frac{1}{2}} = \dfrac{\sqrt{2}e^{j\pi/2}}{\sqrt{2}e^{j\pi/4}} = e^{j\pi/4}$. So, the output is $\sin\left(\frac{\pi}{4}n + \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}(n+1)\right)$.

Question 6

A discrete-time, linear, shift-invariant system has an output $y(n)$ for $n = 0, 1, 2, 3, \ldots$ equal to $1, -1, 0, 0, \ldots$.

Assuming the difference equation is of the form $y(n) = a_1 y(n-1) + a_2 y(n-2) + b_0 x(n) + b_1 x(n-1)$, what are the filter coefficients that correspond to this input-output signal pair?

Your answer should be numeric values for the coefficients, typed in the order a_1 a_2 b_0 b_1 and separated by spaces.

You entered:

Your Answer	Score	Explanation
✘	0.00	
Total	0.00 / 1.00	

Question Explanation

Since the output has a finte duration, we know the filter is in the FIR class. Therefore, $a_1 = a_2 = 0$. That leaves $b_0 = 1, b_1 = -1$.

$$y(n) = x(n) - x(n-1)$$

Question 7

What is this filter's transfer function?

Your answer should be the complex-valued transfer function as a function of frequency f.

You entered:

Preview Help

Your Answer		Score	Explanation
	✘	0.00	Could not parse student submission
Total		0.00 / 1.00	

Question Explanation

Because we have an FIR filter, we can write the transfer function easily: $H\left(e^{j2\pi f}\right) = 1 - e^{-j2\pi f} = 1 - \cos 2\pi f + j \sin 2\pi f$.

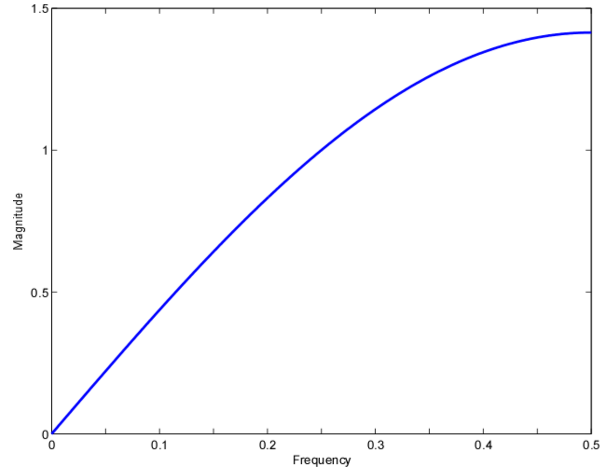
Question 8

What description best fits this filter's action on its inputs?

Your Answer	Score	Explanation
Highpass		
Bandpass		
Allpass		
Lowpass		
Something else.		
Total	0.00 / 1.00	

Question Explanation

The transfer function equals $1 - e^{-j2\pi f}$ that has magnitude $\sqrt{(1 - \cos 2\pi f)^2 + \sin^2 2\pi f} = \sqrt{2 - 2\cos 2\pi f}$. Plotting this expression shows we have a highpass filter.



Note that the plot extends *only* over $[0, .5]$. That is because the transfer functions of *all* discrete-time filters are periodic in frequency with a period of one. Therefore, the frequency range $[.5, 1]$ corresponds to $[-.5, 0]$: the negative frequency range. Since the transfer function of every filter is conjugate symmetric, we have all we need by examining the range $[0, .5]$, the entire frequency range available in discrete-time.

Question 9

Echoes

Echoes occur not only in canyons and deep valleys, but also in auditoriums and telephone systems. In one case in which the signal has been sampled, the input signal $x(n)$ emerges from an echo system along with scaled and delayed copies of itself:
$y(n) = x(n) + a_1 x(n - n_1) + a_2 x(n - n_2)$.

To simulate this echo system the FEE students want to write the most efficient (quickest) program that implements this input/output relationship. Suppose the duration of the input $x(n)$ is 1000 and that $a_1 = \frac{1}{2}$, $n_1 = 10$, $a_2 = \frac{1}{5}$, and $n_2 = 25$. In the Discussion Forum, half the students votes just to program the difference equation and the other half votes to program a frequency-domain approach that exploits the speed of the FFT. Which approach is the most efficient?

Your Answer	Score	Explanation
Time domain.		
Frequency domain.		
Makes no difference.		
Total	0.00 / 1.00	

Question Explanation

The filter's unit-sample response is $h(n) = \delta(n) + \frac{1}{2}\,\delta(n - 10) + \frac{1}{5}\,\delta(n - 25)$, which has a duration of 26. The output's duration determines how long the FFT must be; the output's duration is $1000 + 26 - 1 = 1025$. Thus, the smallest power-of-two length is 2048. The number of computations required by the difference equations is $1000 \cdot (2 * 25 + 1) = 51,000$. The frequency-domain approach requires three Fourier transforms, each of which requires $\frac{3}{2} \cdot 2048 \log_2 2048 = 33,792$ computations. The total number of computations required by using the FFT far exceeds those required by the difference equation. Thus, the difference-equation approach is better. If only we could use a length-1024 FFT...

Question 10

What is the transfer function of the digital filter that removes echoes? In other words, when $y(n)$ is the input, we want the original signal $x(n)$ to be the output.

You entered:

Preview Help

Your Answer	Score	Explanation
✘	0.00	Could not parse student submission
Total	0.00 / 1.00	

Question Explanation

The echo system's transfer function is $H\left(e^{j2\pi f}\right) = 1 + \frac{1}{2}\, e^{-j2\pi 10 f} + \frac{1}{5}\, e^{-j2\pi 25 f}$. To remove the echoes, we want to pass $y(n)$ through what is known as the inverse system, one that cancels the filtering of the echo system. The transfer function we seek is

$$H_{\text{inv}}\left(e^{j2\pi f}\right) = \frac{1}{H\left(e^{j2\pi f}\right)} = \frac{1}{1 + \frac{1}{2}\, e^{-j2\pi 10 f} + \frac{1}{5}\, e^{-j2\pi 25 f}}$$