Fundamentals of Electrical Engineering

Noisy Channel Coding Theorem

- Theorem statement
- Channel Capacity
- "Why is everything digital?"

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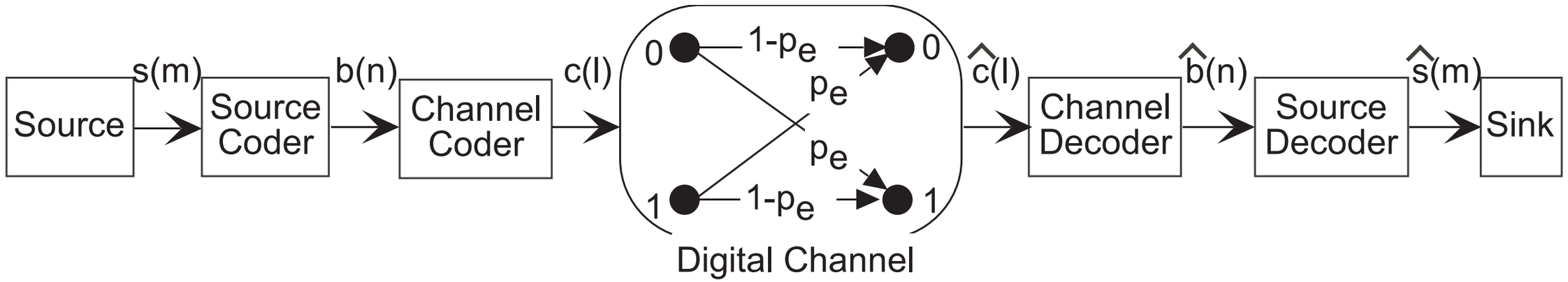
Digital Communication Model



- Can we ~~mitigate~~ channel-induced errors?
  remove

Noisy Channel Coding Theorem

Let E (= K/N) denote the efficiency of a block code. If the efficiency is less than the *capacity* C of the channel, then there exists an error-correcting code that has the property

$$\lim_{N\to\infty} \Pr[\text{block error}] = 0, \quad E < C$$

Furthermore, if $E > C$, all error-correcting code having that efficiency will always incur decoding errors.

$$\lim_{N\to\infty} \Pr[\text{block error}] = 1, \quad E > C$$

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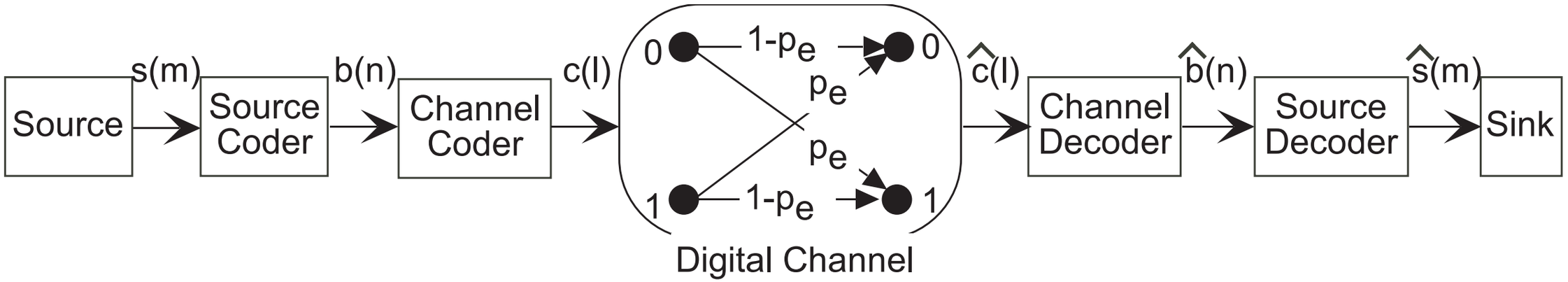
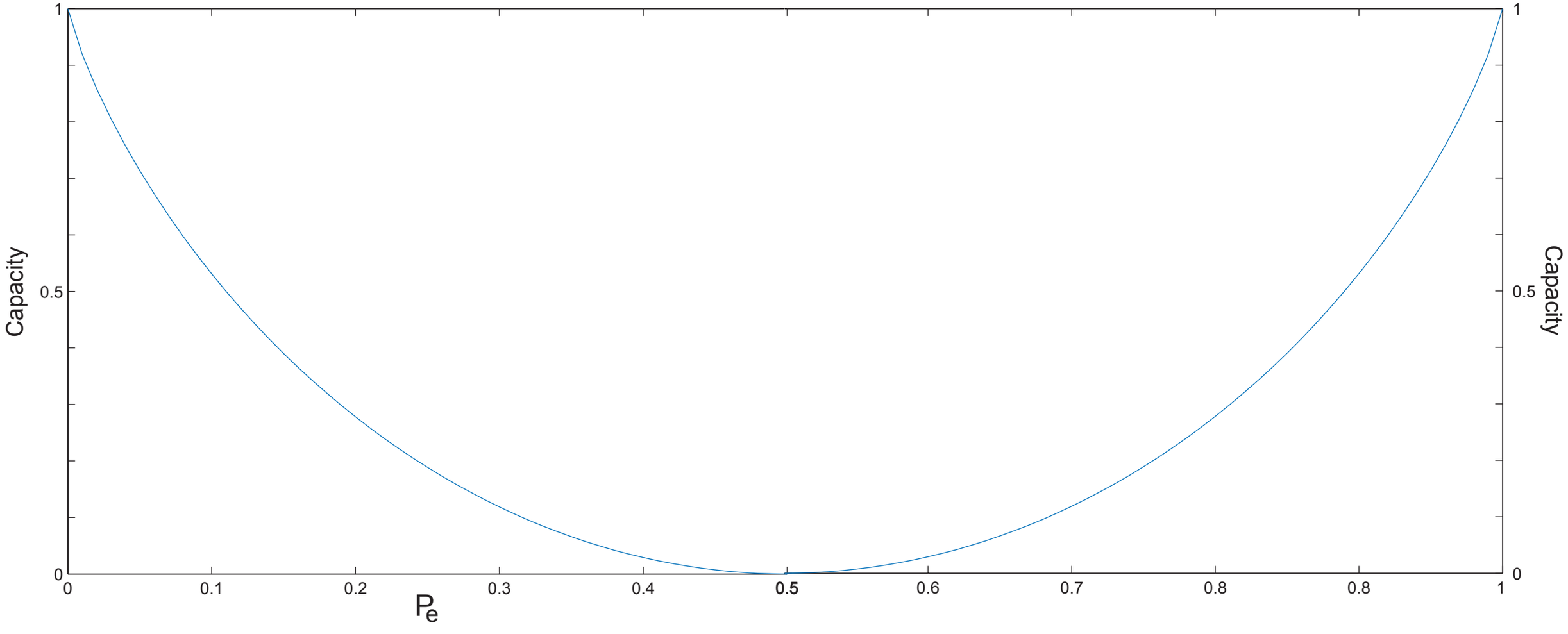
Channel Capacity



$$C = 1 + p_e \log_2 p_e + (1 - p_e) \log_2 (1 - p_e)$$



Noisy Channel Coding Theorem

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Analog Channel Capacity

If the *datarate* R of a discrete source is less than the capacity C of a white-noise channel having bandwidth W, then there exists a signal set and an error correcting code so that as the block length approaches infinity, reliable communication (no errors) is possible

$$R < C,\ C = W \log_2(1 + \mathrm{SNR})$$

If $R > C$, reliable communication is impossible.

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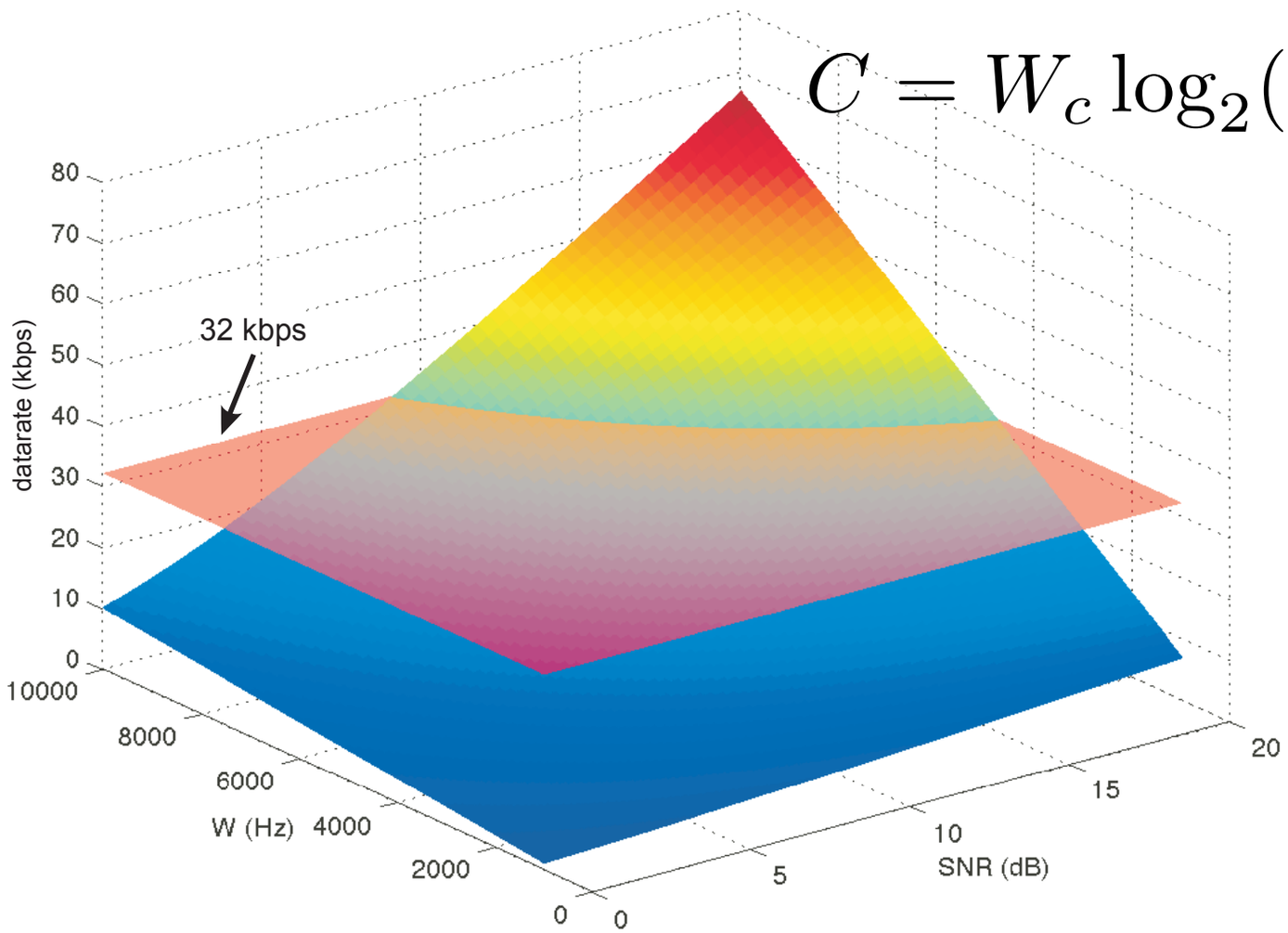
Capacity Calculation

Recall our example of sending an analog signal having a bandwidth of 4 kHz

$$R = F_s \cdot B = 32 \text{ kbps for } B = 4$$

$$C = W_c \log_2(1 + \text{SNR})$$



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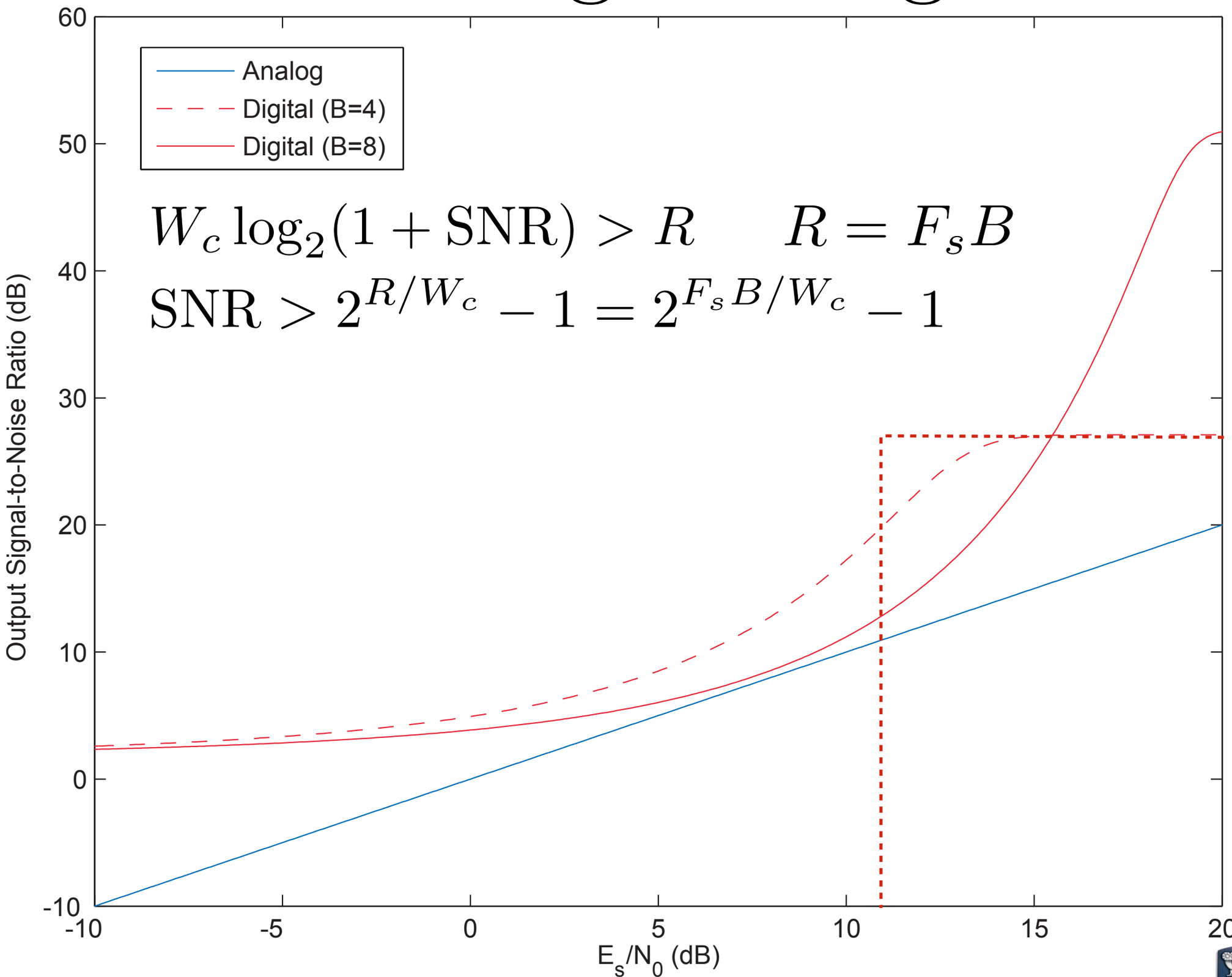
Analog vs. Digital



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Why is "Everything" Digital?

- Sources can be compressed (lossy or lossless)
- Digitization of analog signals *always* introduces error, but…
- Digitized signals and inherently digital signals (text) can be transmitted with digital systems, without incurring any error for well-designed schemes
- Leads to computer networks

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