Fundamentals of Electrical Engineering

Analog vs. Digital Communication

- Assume we want to communicate an analog signal
- Which communication scheme—analog or digital—when used over the *same* channel results in the largest signal-to-noise ratio?
- Analog communication
  - ✳ Received, demodulated signal will be noisy
- Digital communication
  - ✳ A/D converter introduces unrecoverable error
  - ✳ Channel introduces errors
  - ✳ Received, demodulated signal will be noisy

Analog vs. Digital Communication

Suppose we have a baseband message (bandwidth W of 4 kHz) sampled with an B-bit A/D

- Analog
  - ✶ Transmission bandwidth $= 2W = 8$ kHz
  - ✶ SNR $= \dfrac{A^2 \text{power}[s]}{2N_0 W}$
- Digital communication
  - ✶ Transmission bandwidth $= 3\ F_s B$
  - ✶ $\widehat{s}(nT_s) = s(nT_s) +$ quantization error $+$ communication error

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Quantization error

$$\text{Assume } |s(t)| \le 1 \qquad Q[s(nT_s)] = \frac{\sum_{k=0}^{B-1} b_k 2^k}{2^{B-1}} - 1$$

$$\text{power[error]} = \frac{2^{-2B}}{3}$$

Channel error

$$Q[\widehat{s}(nT_s)] = \frac{\sum_{k=0}^{B-1} \hat{b}_k 2^k}{2^{B-1}} - 1$$

$$\epsilon = \frac{\sum_{k=0}^{B-1} (b_k - \hat{b}_k) 2^k}{2^{B-1}} \qquad \langle \epsilon^2 \rangle = \frac{\sum_{k=0}^{B-1} \left\langle (b_k - \hat{b}_k)^2 \right\rangle 2^{2k}}{2^{2(B-1)}}$$

$$\langle \epsilon^2 \rangle = \frac{\sum_{k=0}^{B-1} p_e 2^{2k}}{2^{2(B-1)}} = p_e \frac{2^{2B} - 1}{3 \cdot 2^{2(B-1)}} \approx \frac{4}{3} p_e$$

$$\boxed{\text{SNR} = \frac{\text{power}[s]}{\frac{2^{-2B}}{3} + \frac{4}{3} p_e}}$$

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To make a fair comparison, must make the transmitter powers the same

Recall that $E_b = A^2 T$ with $T = \dfrac{1}{2W \cdot B}$

$$A^2 = E_b \cdot 2W \cdot B$$

$$\text{SNR}_{\text{analog}} = \frac{A^2}{2N_0 W} = \frac{E_s}{N_0},\ E_s = E_b \cdot B$$

$$\text{SNR}_{\text{digital}} = \frac{1}{\frac{2^{-2B}}{3} + \frac{4}{3}p_e},\ p_e = Q\left(\sqrt{\frac{2E_s}{BN_0}}\right)$$

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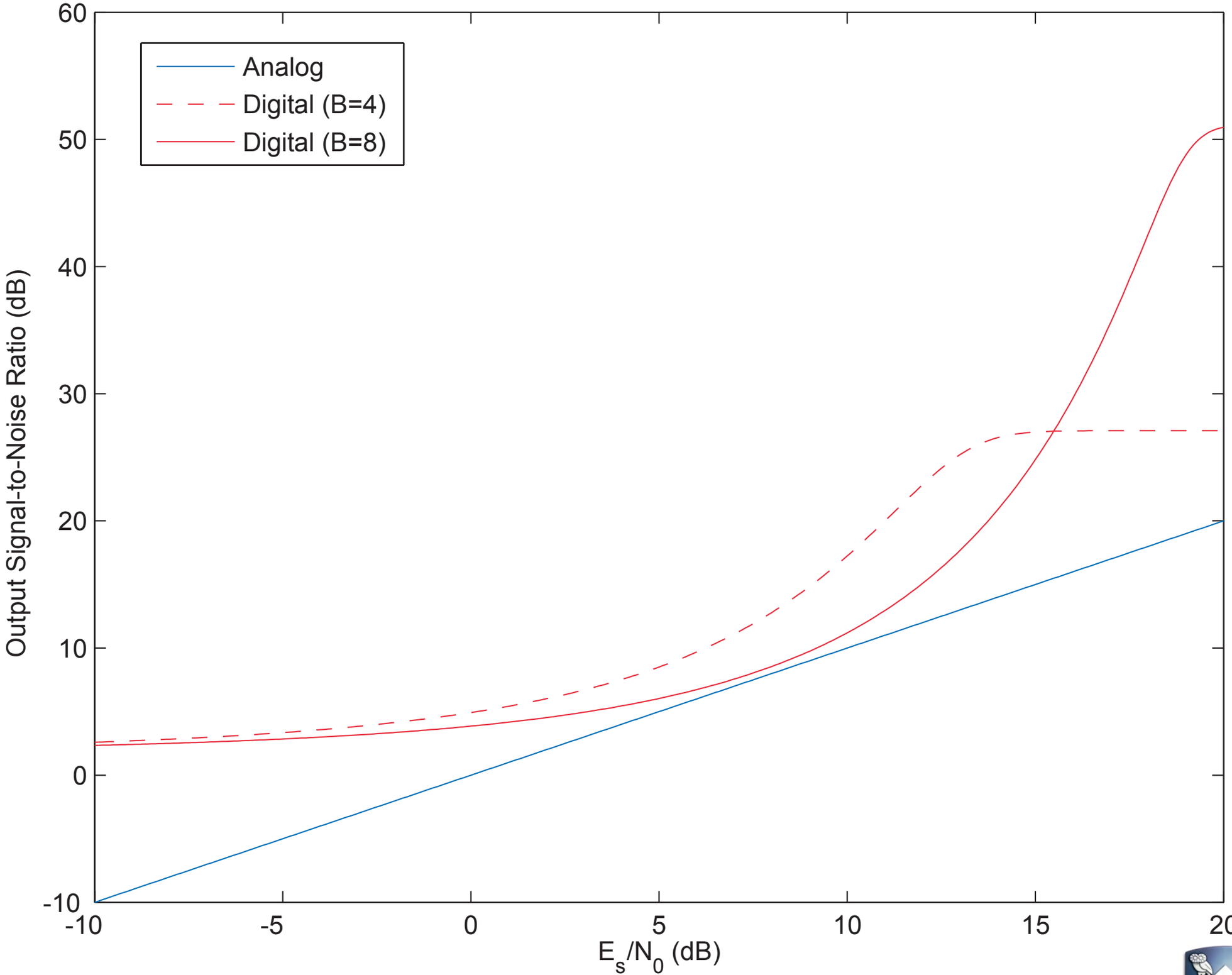

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Analog vs. Digital Communication

- Digital wins!
- However, the channel bandwidth needed for the analog case is much smaller than the digital:
  8 kHz vs. 96 (192) kHz for B=4 (8)
- It can be shown that when digital is constrained to the same bandwidth as analog, a variation of amplitude modulation yields a higher SNR

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