Fundamentals of Electrical Engineering

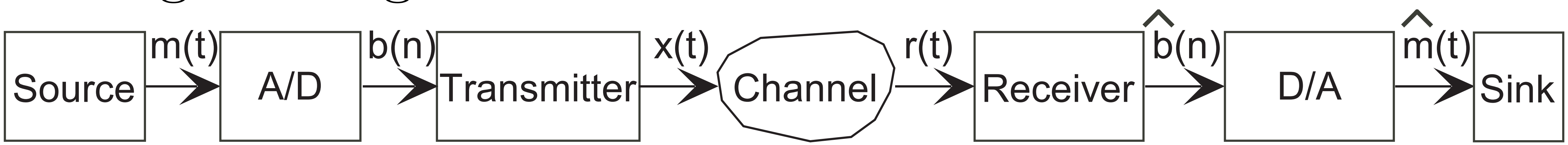
Digital Communication Receivers

- Optimal digital receiver
- Performance comparison for signal set choices

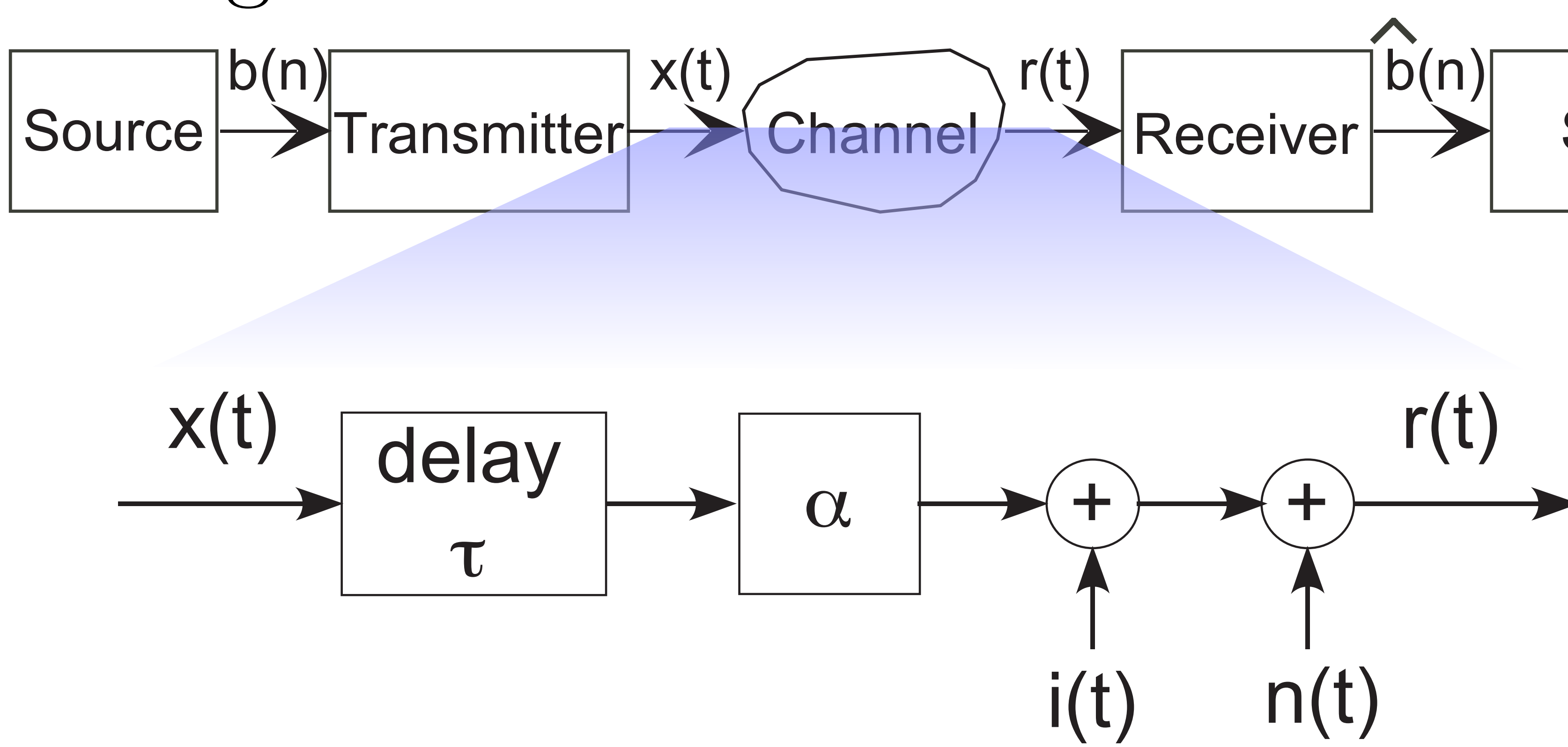
RICE

Digital Communication Model

Analog message

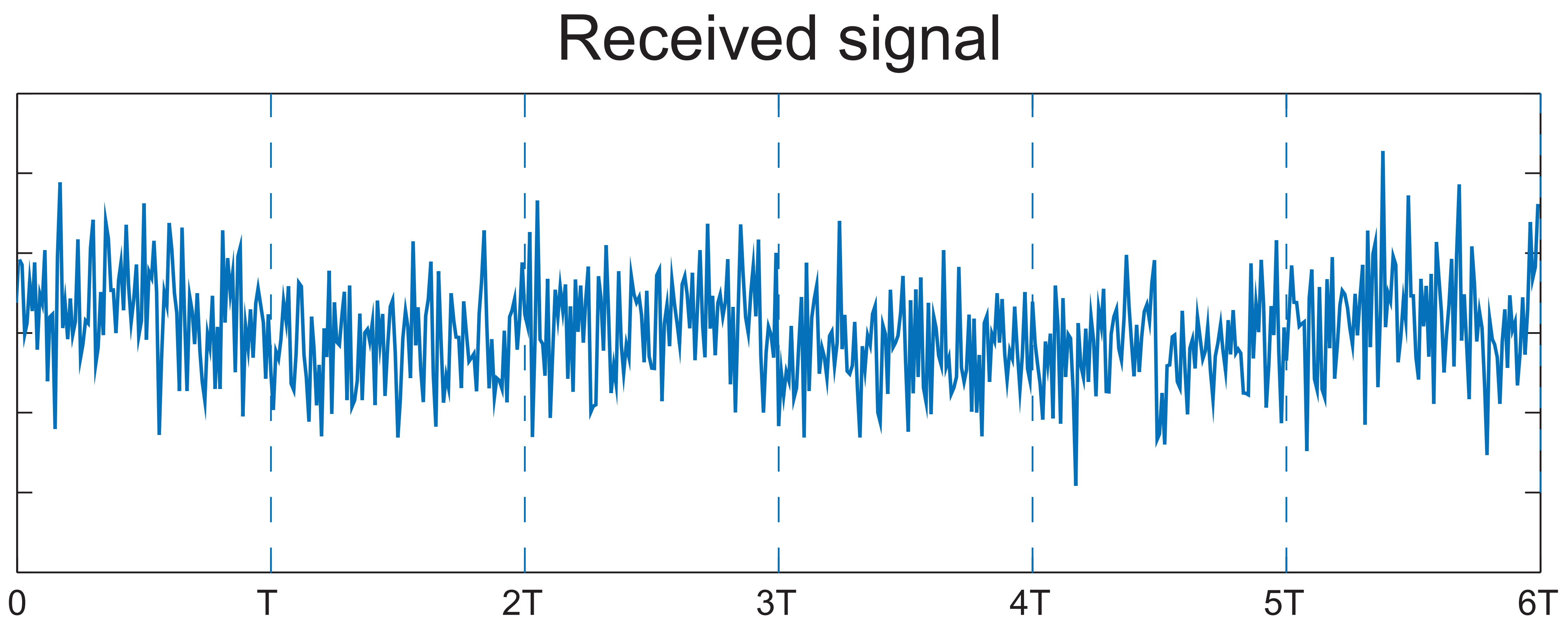


Digital message

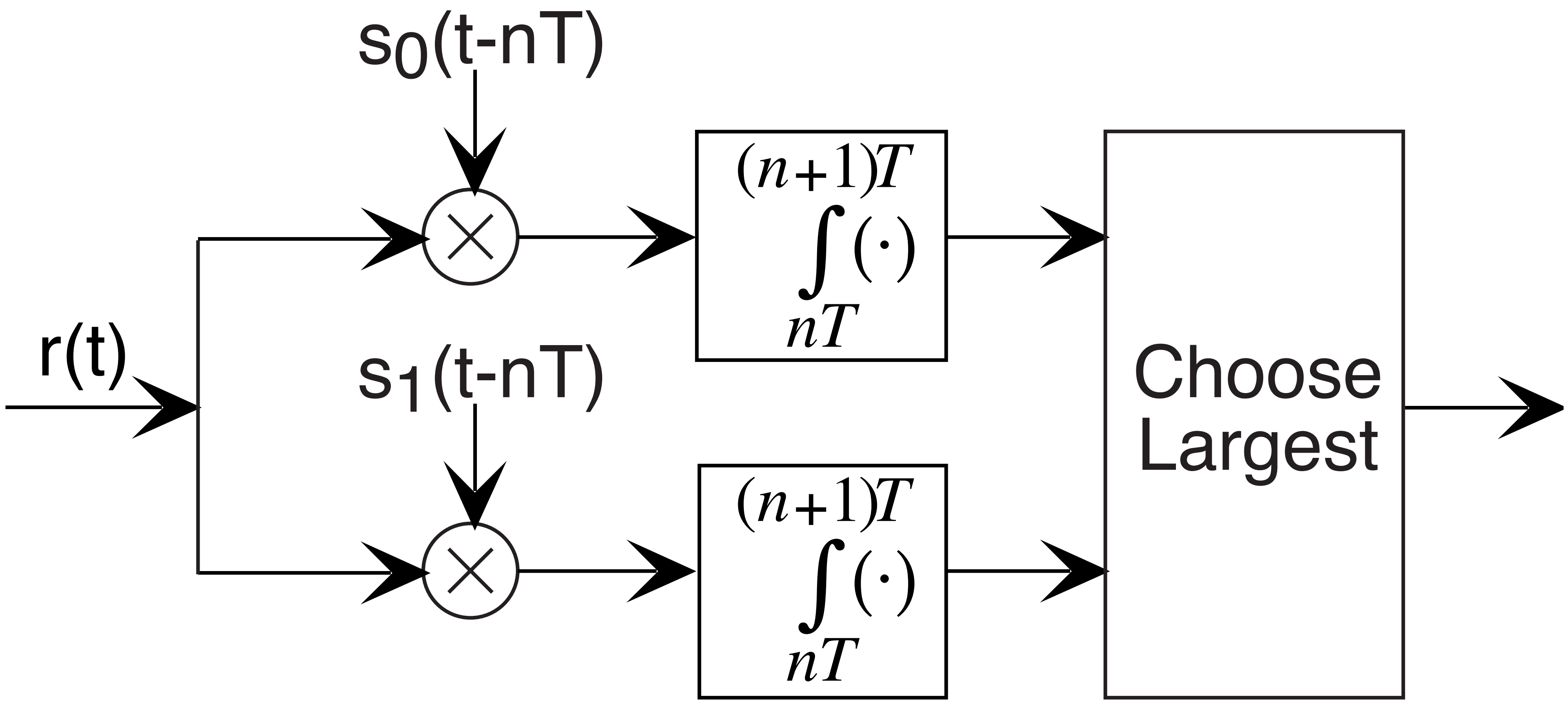


Receiver Front-end

- Because channel delay and when the transmitter started are unknown, the receiver must *synchronize* to bit-interval boundaries

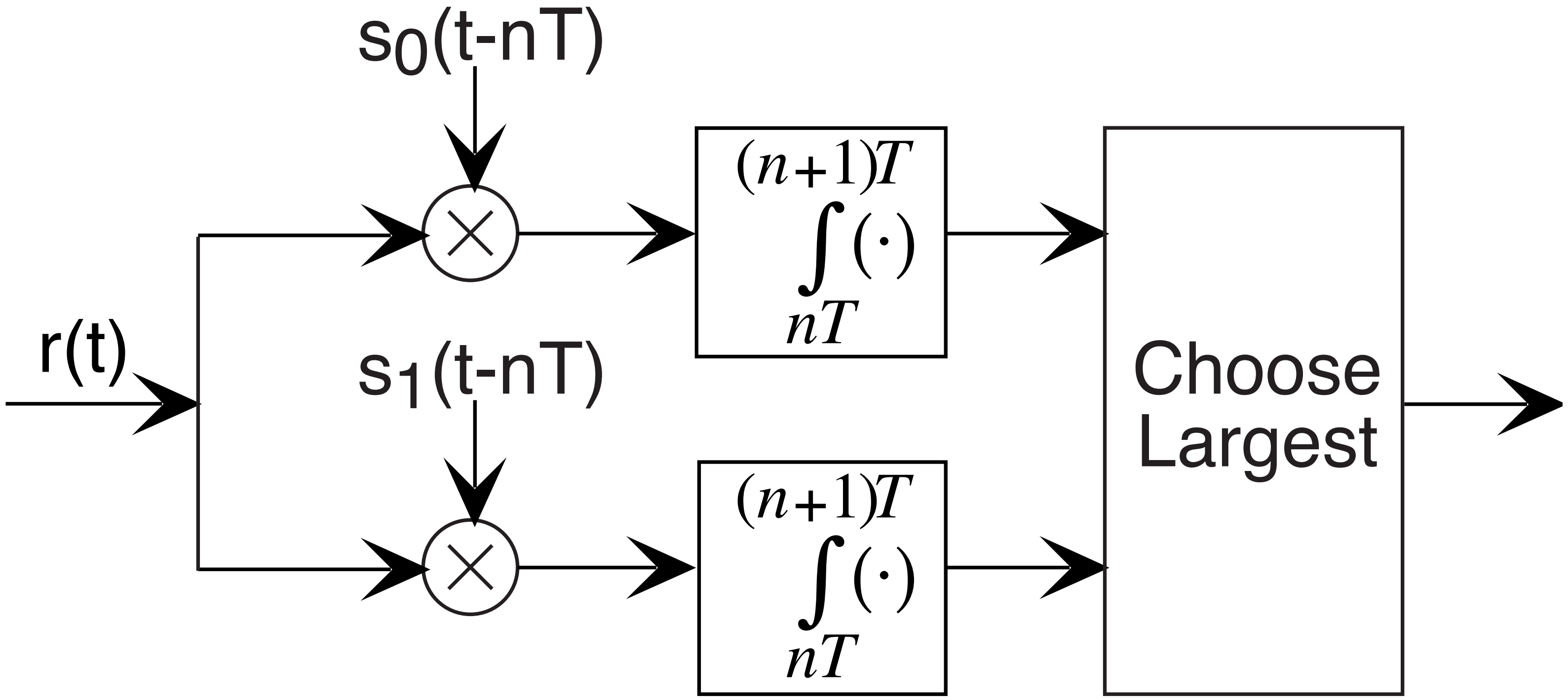


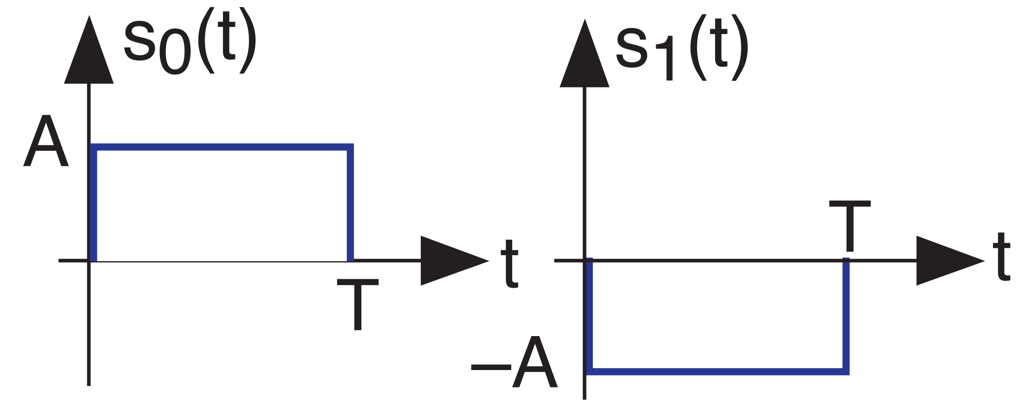
Correlation Receiver



$$\widehat{b}(n) = \arg\max_i \int_{nT}^{(n+1)T} r(t)s_i(t - nT)\,dt$$

Correlation Receiver



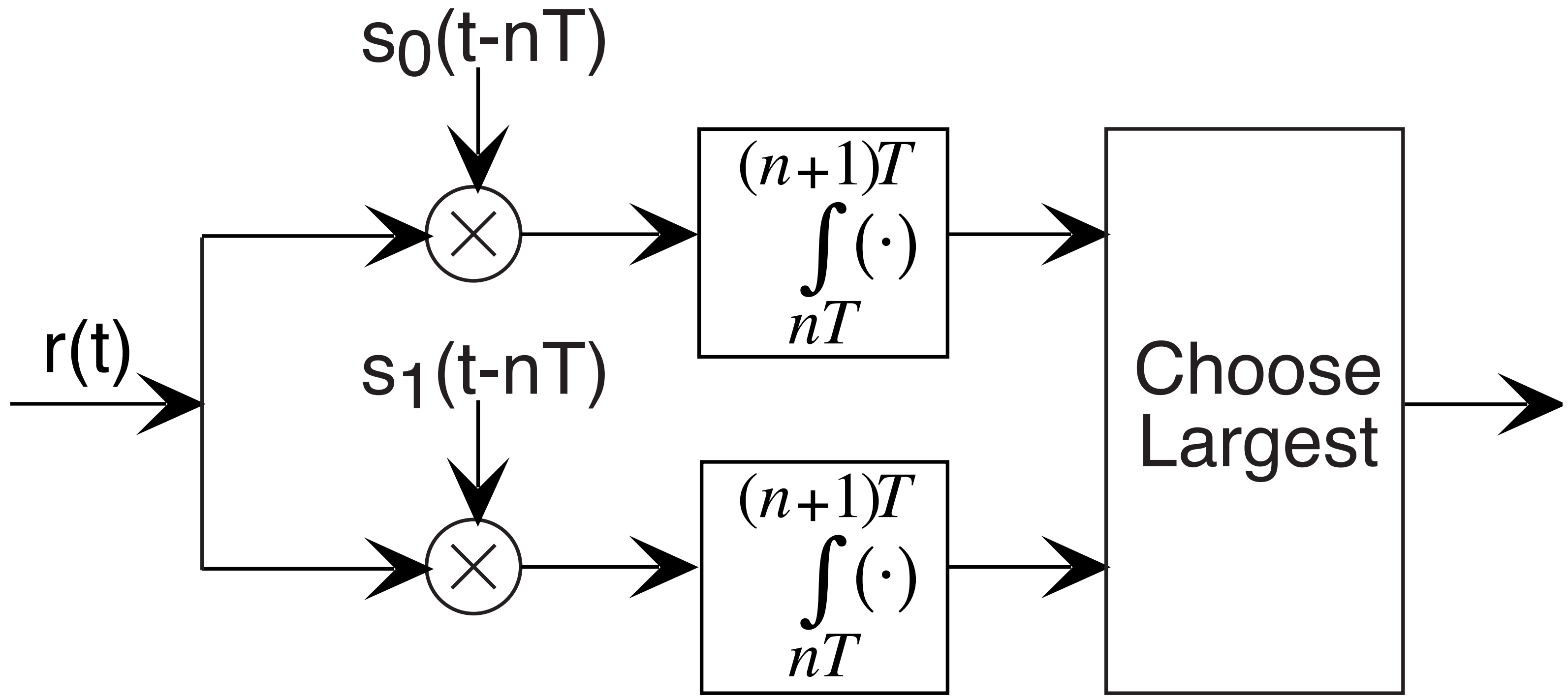


What happens when a "0" is sent and it is received with no noise?

$$\int_{nT}^{(n+1)T} r(t)s_0(t-nT)\,dt = A^2T \qquad \Leftarrow \qquad ✔$$

$$\int_{nT}^{(n+1)T} r(t)s_1(t-nT)\,dt = -A^2T$$

Correlation Receiver



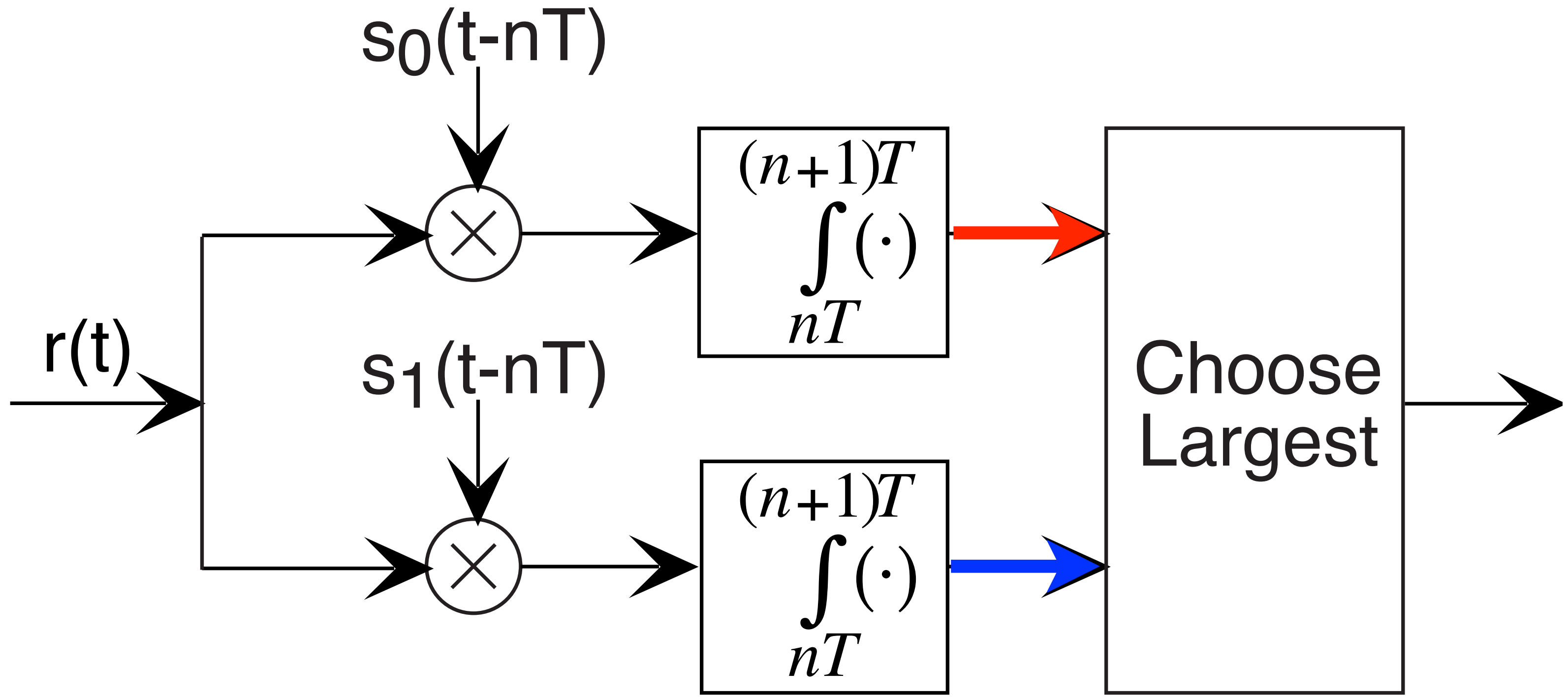
When noise and attenuation are present while a "0" is sent,

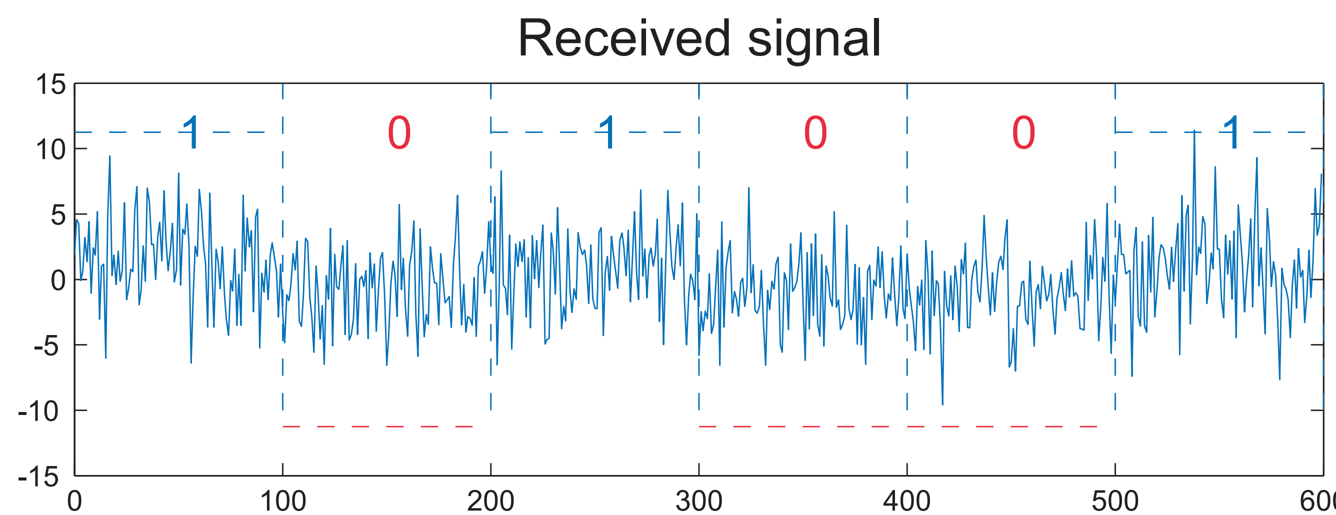
$$\int_{nT}^{(n+1)T} r(t)s_0(t-nT)\,dt = \alpha A^2 T + \int_{nT}^{(n+1)T} n(t)s_0(t-nT)\,dt$$

$$\int_{nT}^{(n+1)T} r(t)s_1(t-nT)\,dt = -\alpha A^2 T + \int_{nT}^{(n+1)T} n(t)s_1(t-nT)\,dt$$

Non-zero probability that the bit choice is wrong

Correlation Receiver





✔ ✔ ✔ ✔ ✔ ✔

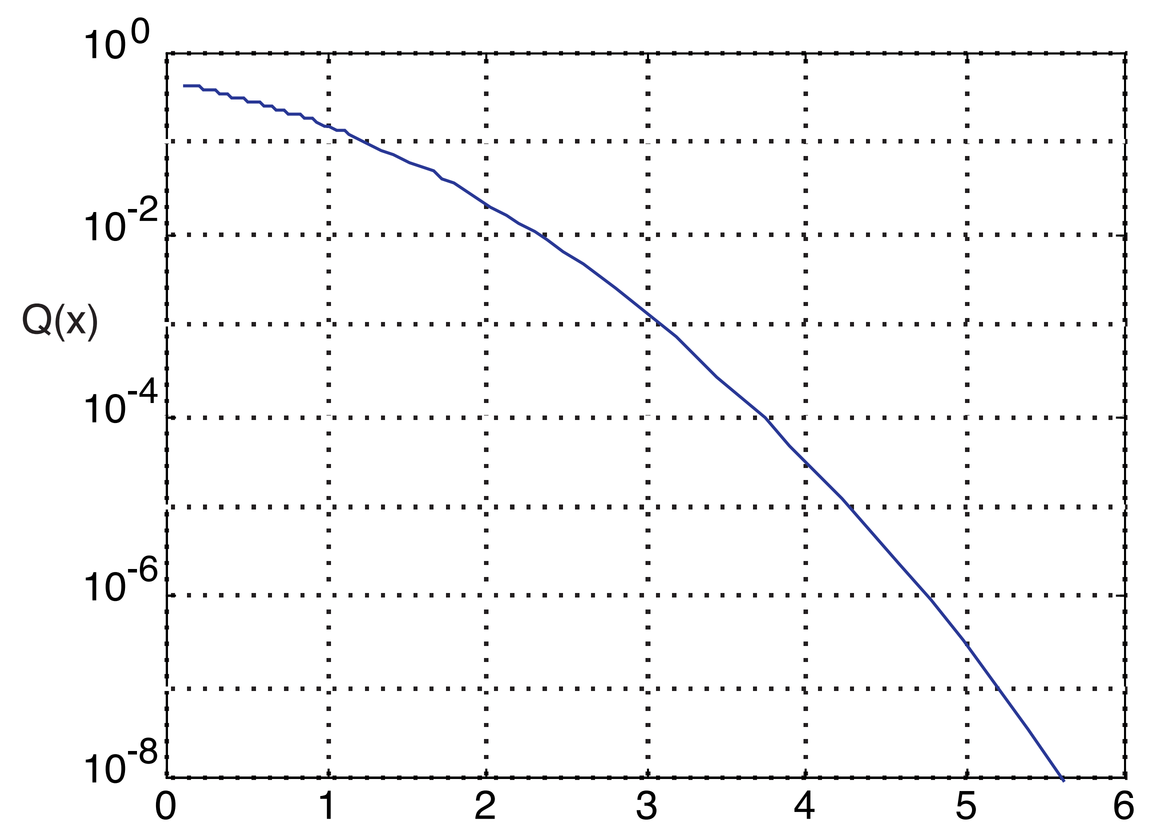
Correlation Receiver Performance

Receiver probability of error affected by…

- Signal set choice $\int_0^T \left[s_0(t) - s_1(t)\right]^2 dt$
- Channel attenuation
- Variability of the noise (N_0)
- Probability distribution of the noise

$$P_e = Q\left(\sqrt{\frac{\int_0^T \alpha^2 \left[s_0(t) - s_1(t)\right]^2 dt}{2N_0}}\right)$$

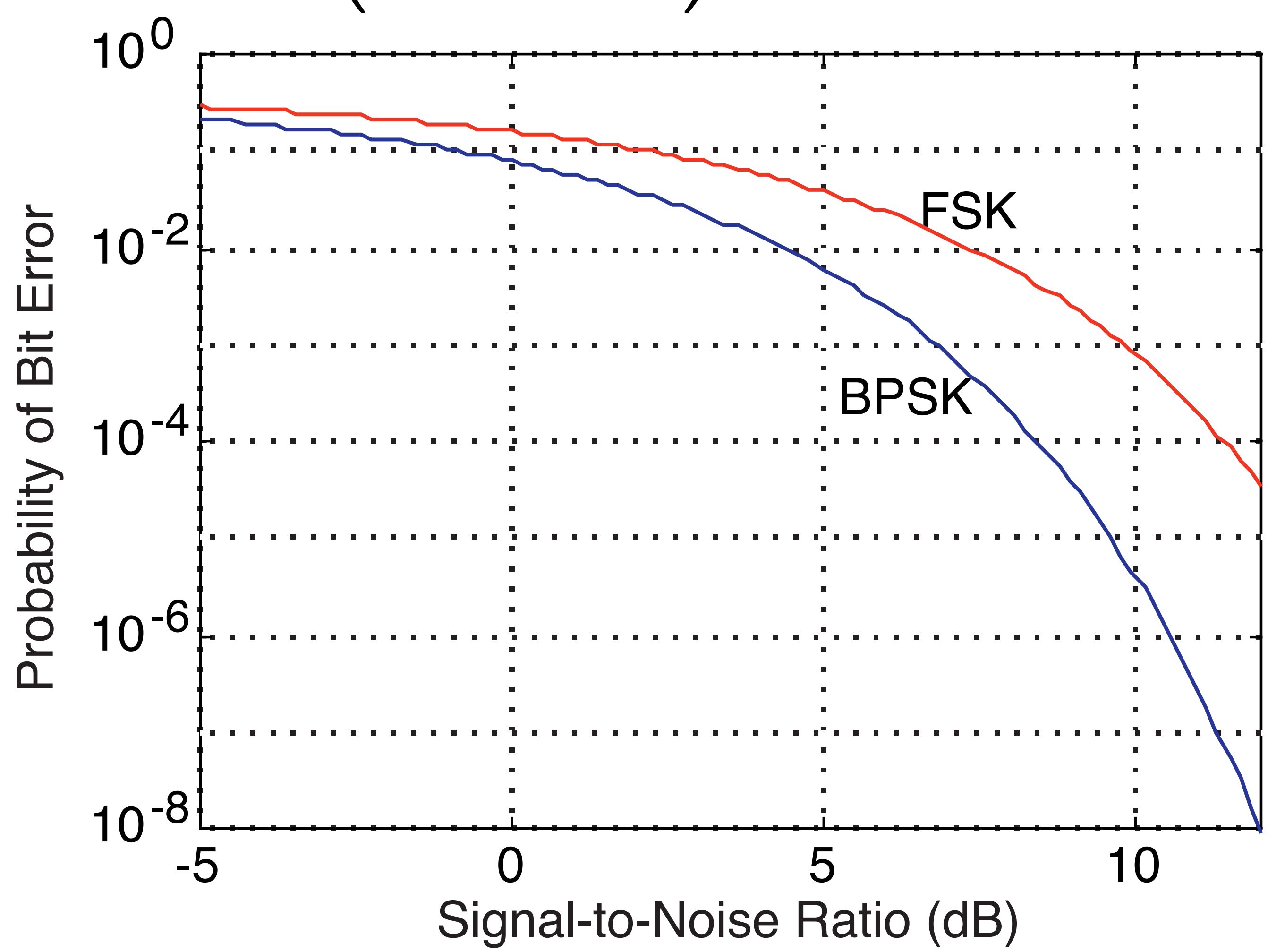
$$Q(x) \equiv \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\beta^2/2}\, d\beta$$



Correlation Receiver Performance

BPSK: $$P_e = Q\left(\sqrt{\frac{2\alpha^2 E_b}{N_0}}\right) \qquad E_b = A^2T$$

FSK: $$P_e = Q\left(\sqrt{\frac{\alpha^2 E_b}{N_0}}\right) \qquad E_b = A^2T$$



Digital Communication

- Digital communication systems are not perfect: a noisy channel introduces bit errors with some probability
- Signal set choice can have significant impact on performance
- As the received signal becomes increasingly noisy, probability of error approaches 1/2
- How small should P_e be?

  Assuming a datarate of R bits/s and $P_e = \frac{1}{R}$ means, on the average, one error/second!