Fundamentals of Electrical Engineering

Implementing Digital Filters

- IIR vs. FIR
- Computational complexity: time-domain vs. frequency-domain
- Frequency-domain filtering

RICE

IIR Filters

$$y(n) = a_1 y(n-1) + \cdots + a_p y(n-p) + b_0 x(n) + b_1 x(n-1) + \cdots + b_q x(n-q)$$

- For each output value,
  p multiplys and p-1 additions for "IIR" part
  q+1 multiplys and q additions for "FIR" part
- Complexity per output value: $O(p+q)$
- If input has duration N, output is *infinitely long*, meaning number of computations is theoretically infinite

RICE

FIR Filters

$$y(n) = b_0 x(n) + b_1 x(n-1) + \cdots + b_q x(n-q)$$

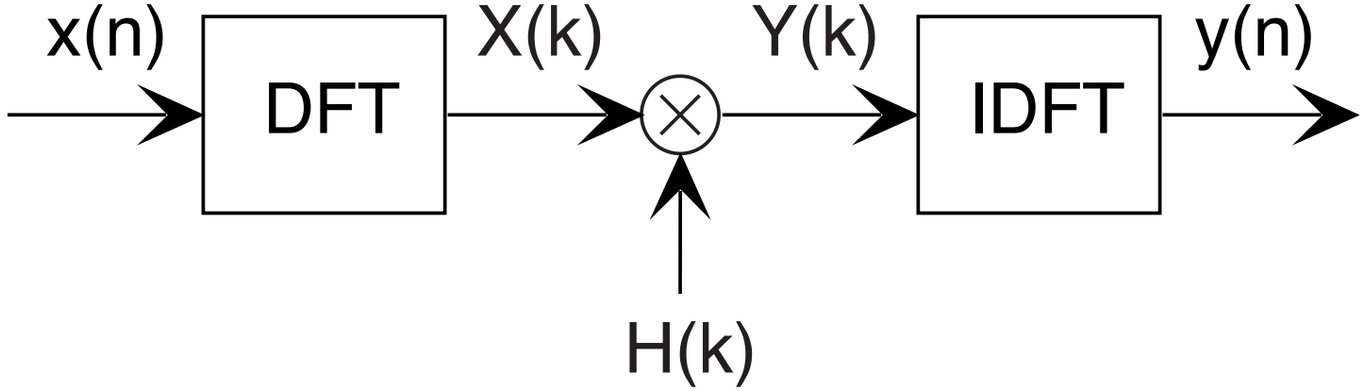
- For each output value, q+1 multiplys and q additions
- Complexity per output value: $O(q)$
- If input has duration N, output has duration N+q
- Complexity: $O(q \cdot (N+q))$

RICE

Frequency Domain Implementation?

We know $Y(e^{j2\pi f}) = H(e^{j2\pi f}) \cdot X(e^{j2\pi f})$

Can we compute this as $Y(k) = H(k) \cdot X(k)$?



IIR?

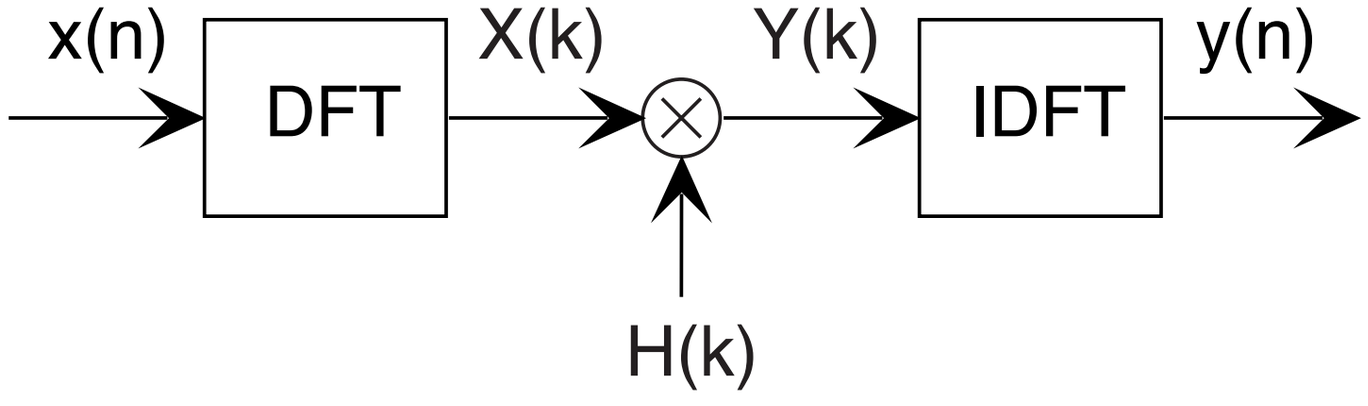
Unit-sample response has infinite duration ☹

FIR? ✔

☺ Finite-duration input yields finite-duration output

RICE

FIR Filters in Frequency Domain

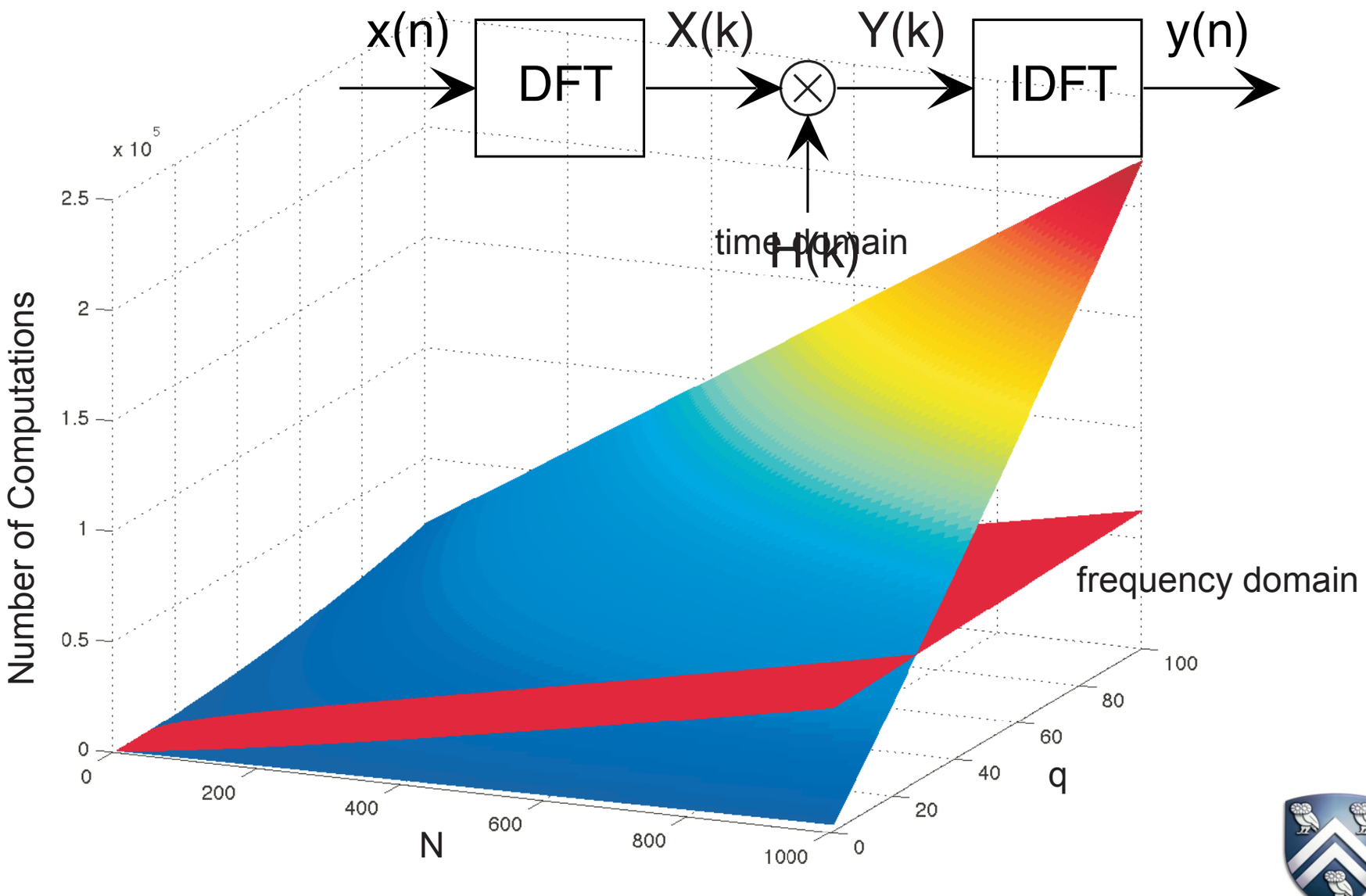


- *All* transform lengths must be the same
- Transform length governed by the longest signal
- duration(output) = duration(input)+duration(h)−1
- Transform length ≥ N+q

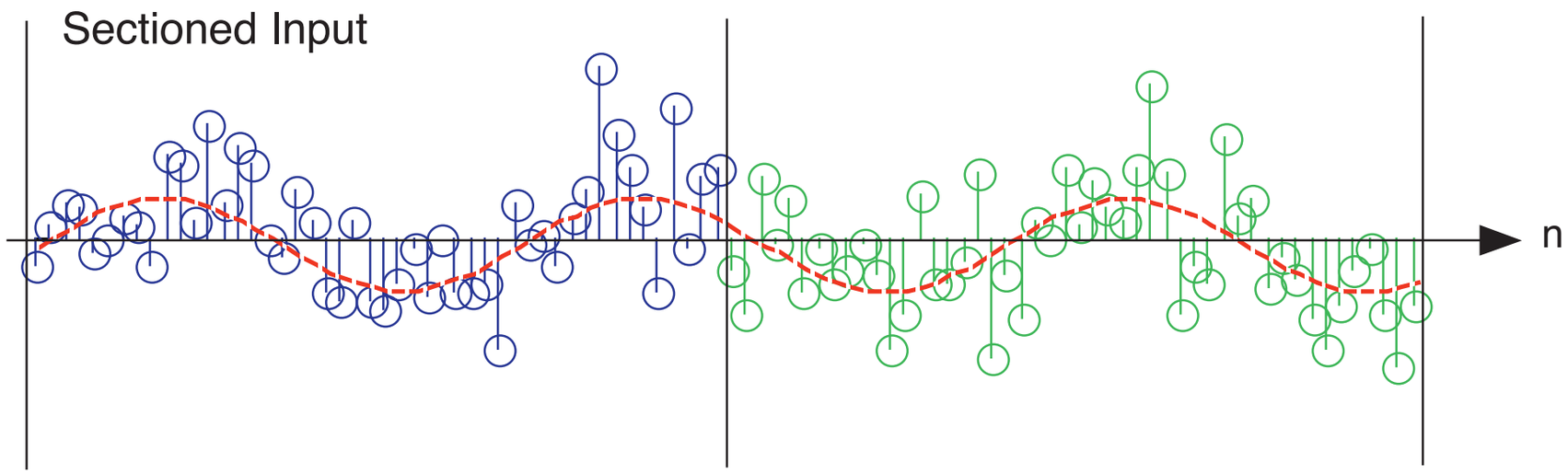
FIR Filters in Frequency Domain

time-domain vs. frequency domain ✔

$$(N+q)(2q+1) \overset{?}{\gtrless} 6(N+q) + 5(N+q)\log_2(N+q)$$



Long Inputs



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Implementing Filters

- IIR filters usually require fewer coefficient values than the FIR filter that has a similar transfer function
- If an IIR filter is chosen, usually implemented with its difference equation
- FIR filters are used because they can have linear phase characteristics *and* they can be implemented in the frequency domain using the FFT