Fundamentals of Electrical Engineering

The Fast Fourier Transform (FFT)

- Computing the DFT efficiently
- Computational Complexity

RICE

Computing the DFT

$$S(k) = \sum_{n=0}^{N-1} s(n) e^{-j\frac{2\pi nk}{N}} \quad k = 0, \ldots, N-1$$

How many computations are required to compute the spectrum for each frequency?

Multiplications (real): $2N$

Additions (real): $2(N-1)$

Total (real): $4N-2$

Since we have N frequencies, $N(4N-2)$ computations

Complexity: $O(N^2)$

RICE

The FFT (Gauss)

$$S(k) = \sum_{n=0}^{N-1} s(n) e^{-j\frac{2\pi nk}{N}} \quad k = 0, \ldots, N-1$$

Assume $N = 2^L$

$$S(k) = s(0) + s(2)\, e^{-j\frac{2\pi 2k}{N}} + \cdots + s(N-2)\, e^{-j\frac{2\pi (N-2)k}{N}}$$
$$+ s(1)\, e^{-j\frac{2\pi k}{N}} + s(3)\, e^{-j\frac{2\pi (2+1)k}{N}} + \cdots + s(N-1)\, e^{-j\frac{2\pi (N-2+1)k}{N}}$$

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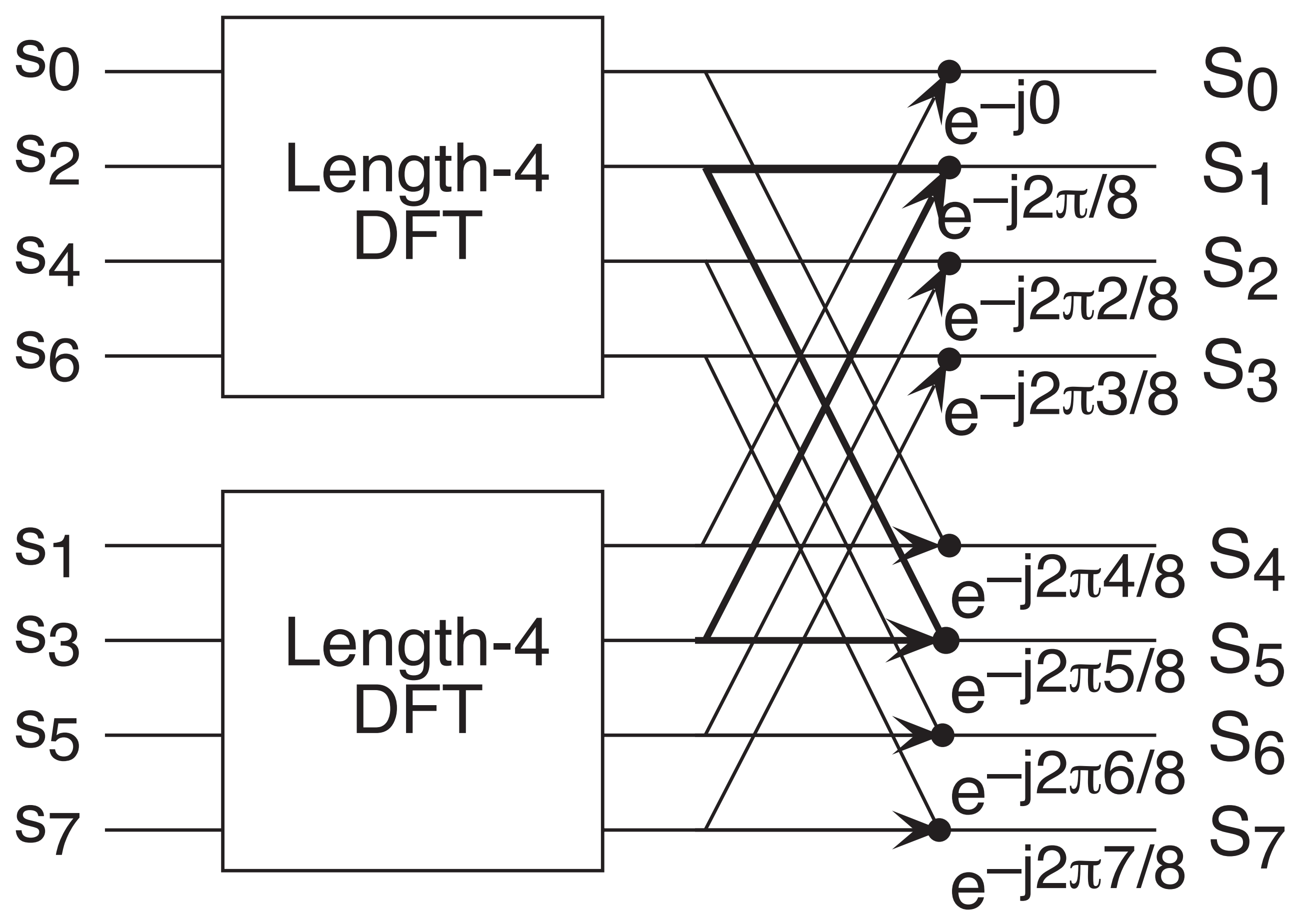
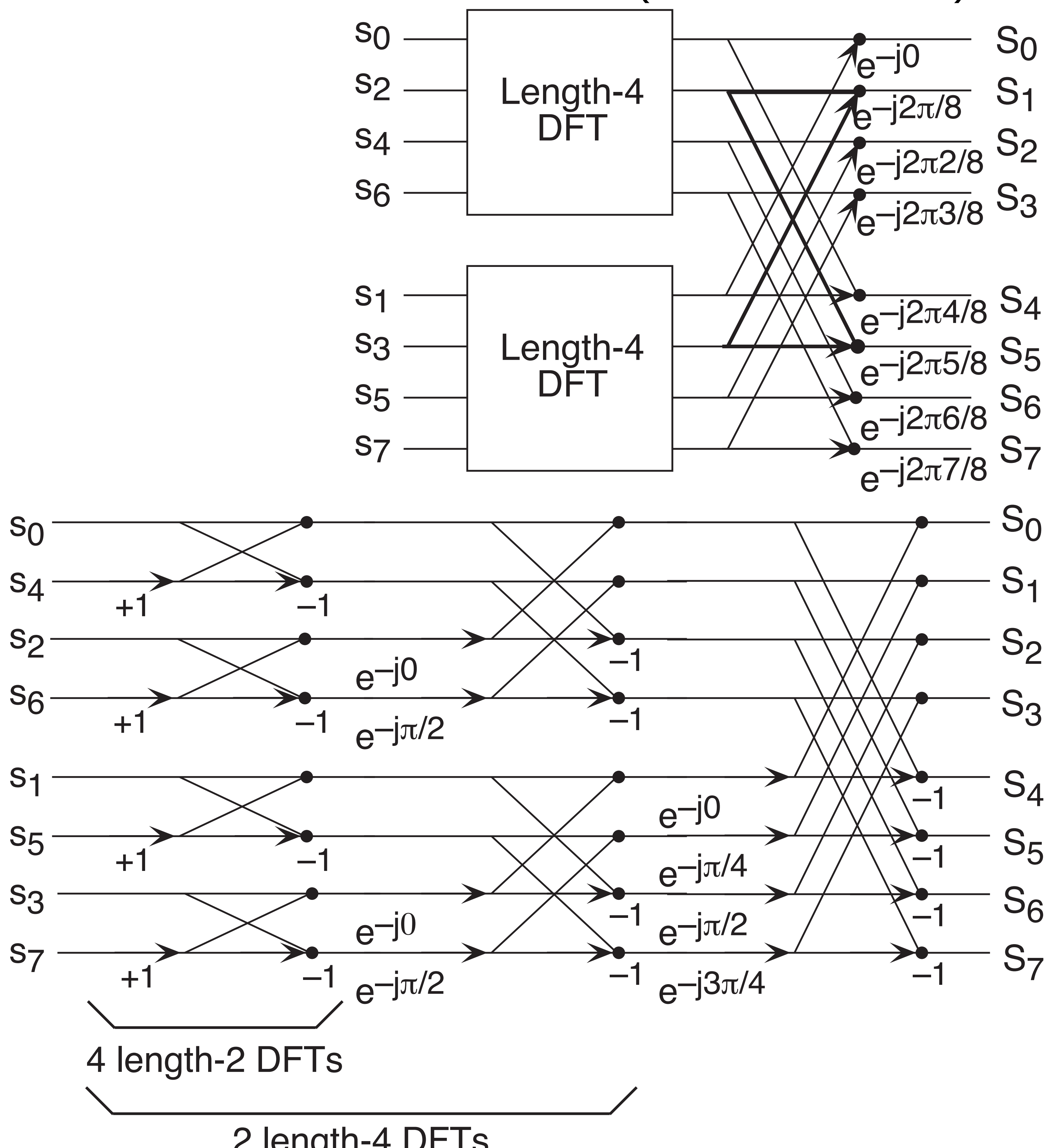
The FFT (Gauss)

$$S(k) = \left[s(0) + s(2)\, e^{-j\frac{2\pi k}{\frac{N}{2}}} + \cdots + s(N-2)\, e^{-j\frac{2\pi\left(\frac{N}{2}-1\right)k}{\frac{N}{2}}} \right]$$

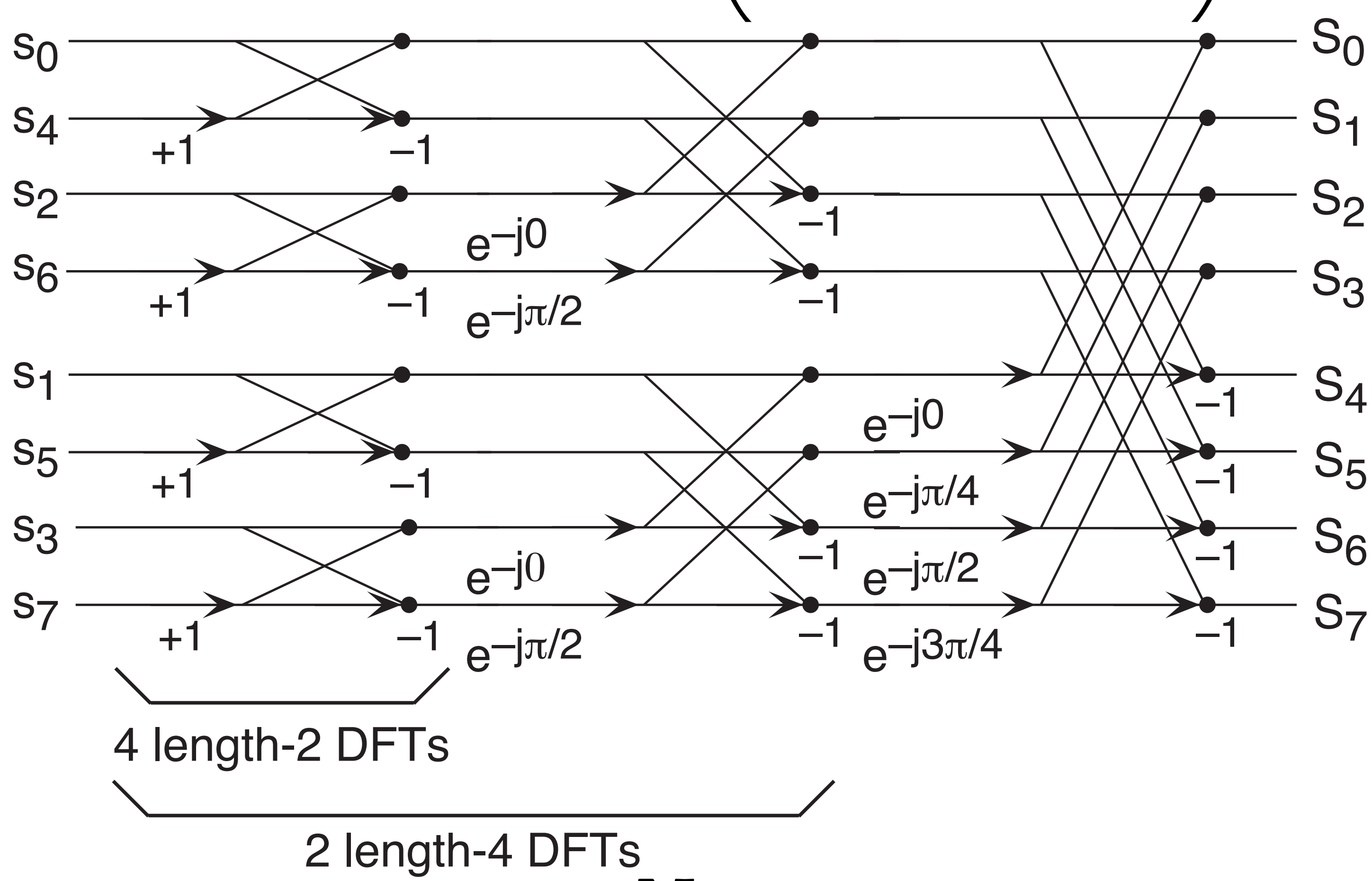
$$+ \left[s(1) + s(3)\, e^{-j\frac{2\pi k}{\frac{N}{2}}} + \cdots + s(N-1)\, e^{-j\frac{2\pi\left(\frac{N}{2}-1\right)k}{\frac{N}{2}}} \right] e^{-\frac{j2\pi k}{N}}$$



The FFT (Gauss)



The FFT (Gauss)



Each stage of the FFT has $\frac{N}{2}$ length-2 DFTs

Every pair of length-2 DFTs combined after one is multiplied by a complex exponential, giving 10 computations each, totaling $5N/2$ for each stage

Number of stages: $\log_2^* N$ $O(N \log N)$

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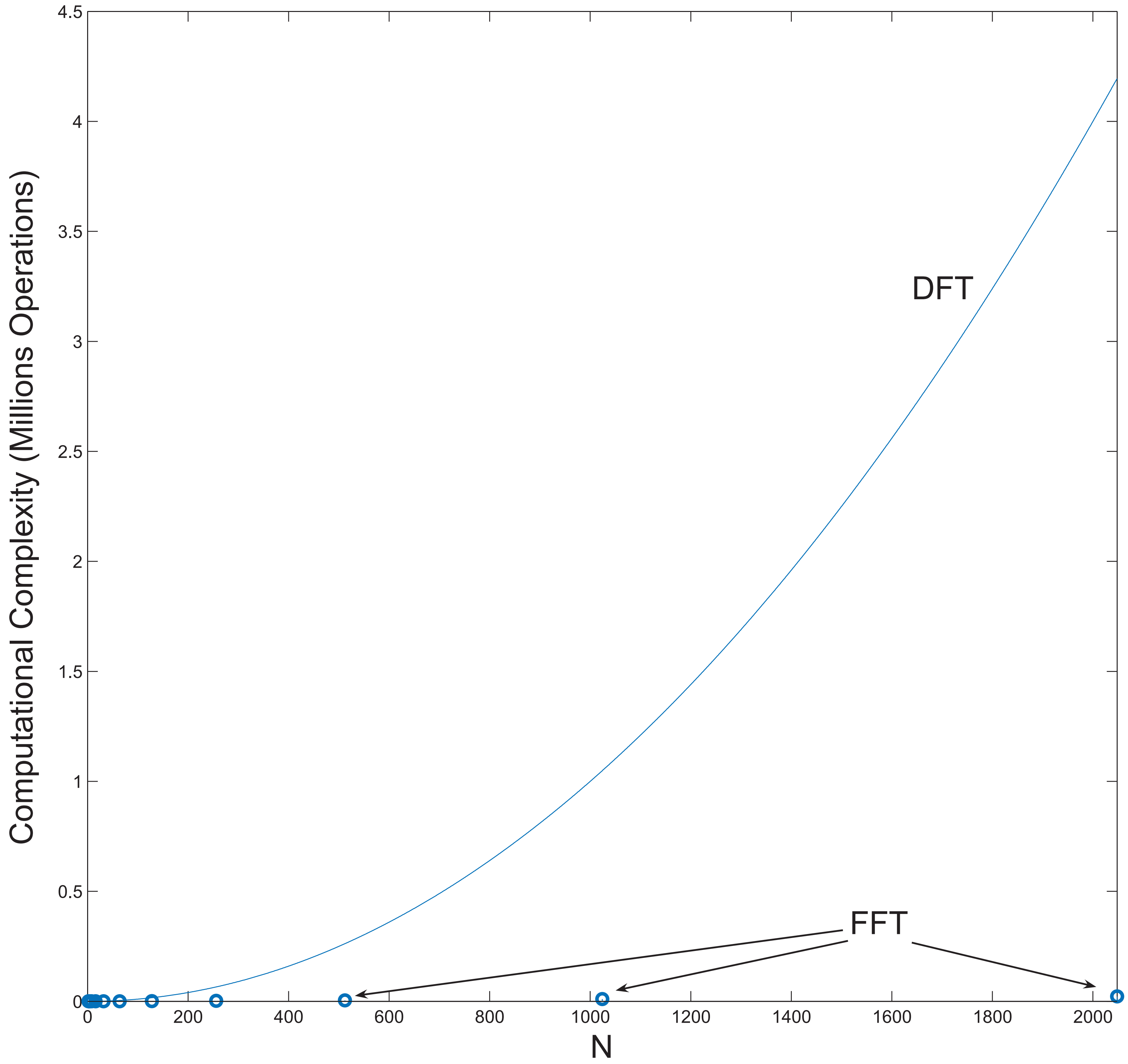
The FFT (Gauss)



The FFT (Gauss)

- Computes the DFT efficiently
- Not a new Fourier transform, but an *algorithm* for computing the DFT
- Opens the door to many signal processing ideas

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