Fundamentals of Electrical Engineering

Discrete-Time Spectral Analysis

- Discrete-time Fourier transform (DTFT)
- Properties

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DTFT

$$S(e^{j2\pi f}) = \sum_{n=-\infty}^{\infty} s(n)e^{-j2\pi fn}$$

$S(e^{j2\pi f})$ periodic with period $= 1$

Example: $s(n) = a^n \mathrm{u}(n)$

$$S(e^{j2\pi f}) = \sum_{n=-\infty}^{\infty} a^n \mathrm{u}(n)e^{-j2\pi fn}$$

$$= \sum_{n=0}^{\infty} a^n e^{-j2\pi fn}$$

$$= \sum_{n=0}^{\infty} \left(ae^{-j2\pi f}\right)^n$$

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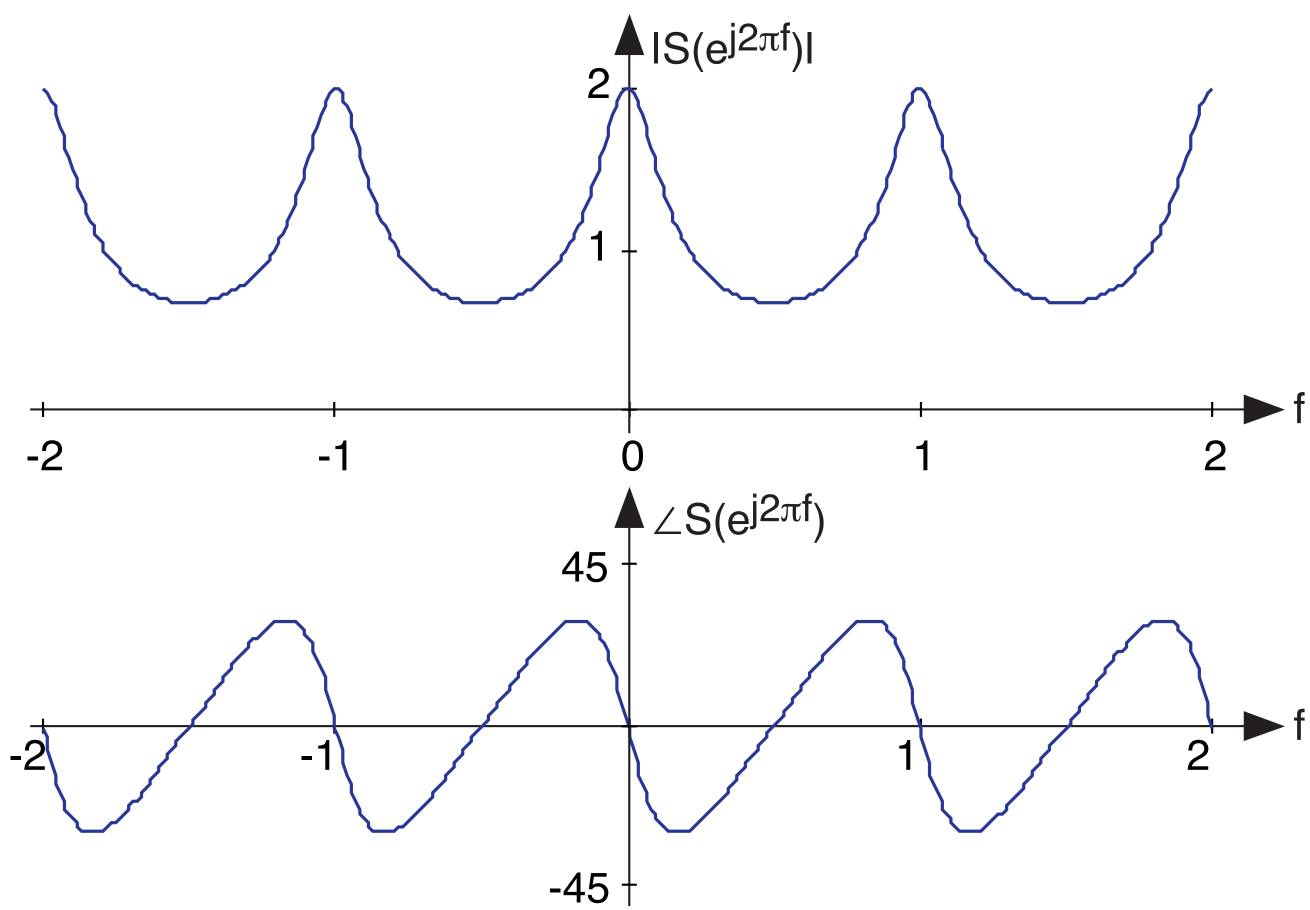
DTFT

$$S(e^{j2\pi f}) = \sum_{n=0}^{\infty} a^n e^{-j2\pi f n} = \sum_{n=0}^{\infty} \left(ae^{-j2\pi f}\right)^n$$

Geometric Series: $$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}, \quad |\alpha| < 1$$

$$S\left(e^{j2\pi f}\right) = \frac{1}{1 - ae^{-j2\pi f}}, \quad |a| < 1$$



$a = 0.5$

Plot Video

Inverse DTFT

Because $\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j2\pi fn} e^{j2\pi fm}\, df = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} = \delta(n-m)$

IDTFT: $$s(n) = \int_{-\frac{1}{2}}^{\frac{1}{2}} S\left(e^{j2\pi f}\right) e^{j2\pi fn}\, df$$

DTFT: $$S(e^{j2\pi f}) = \sum_{n=-\infty}^{\infty} s(n) e^{-j2\pi fn}$$

$$\boxed{s(n) \longleftrightarrow S\left(e^{j2\pi f}\right)}$$

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Properties of DTFT

Linearity:

$$a_1 s_1(n) + a_2 s_2(n) \longleftrightarrow a_1 S_1\left(e^{j2\pi f}\right) + a_2 S_2\left(e^{j2\pi f}\right)$$

Conjugate symmetry:

$$S\left(e^{-j2\pi f}\right) = S\left(e^{j2\pi(1-f)}\right) = S^*\left(e^{j2\pi f}\right)$$

"Time" delay: $s(n-n_0) \longleftrightarrow e^{-j2\pi f n_0} S\left(e^{j2\pi f}\right)$

Complex modulation: $e^{j2\pi f_0 n} s(n) \longleftrightarrow S\left(e^{j2\pi(f-f_0)}\right)$

Parseval's Theorem: $\sum_{n=-\infty}^{\infty} |s(n)|^2 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left|S\left(e^{j2\pi f}\right)\right|^2 df$

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Spectra for Discrete-Time Signals

- Discrete-time Fourier transform (DTFT) shares many properties of the Fourier transform for analog signals
- Important to note that *all* spectra are periodic