Fundamentals of Electrical Engineering

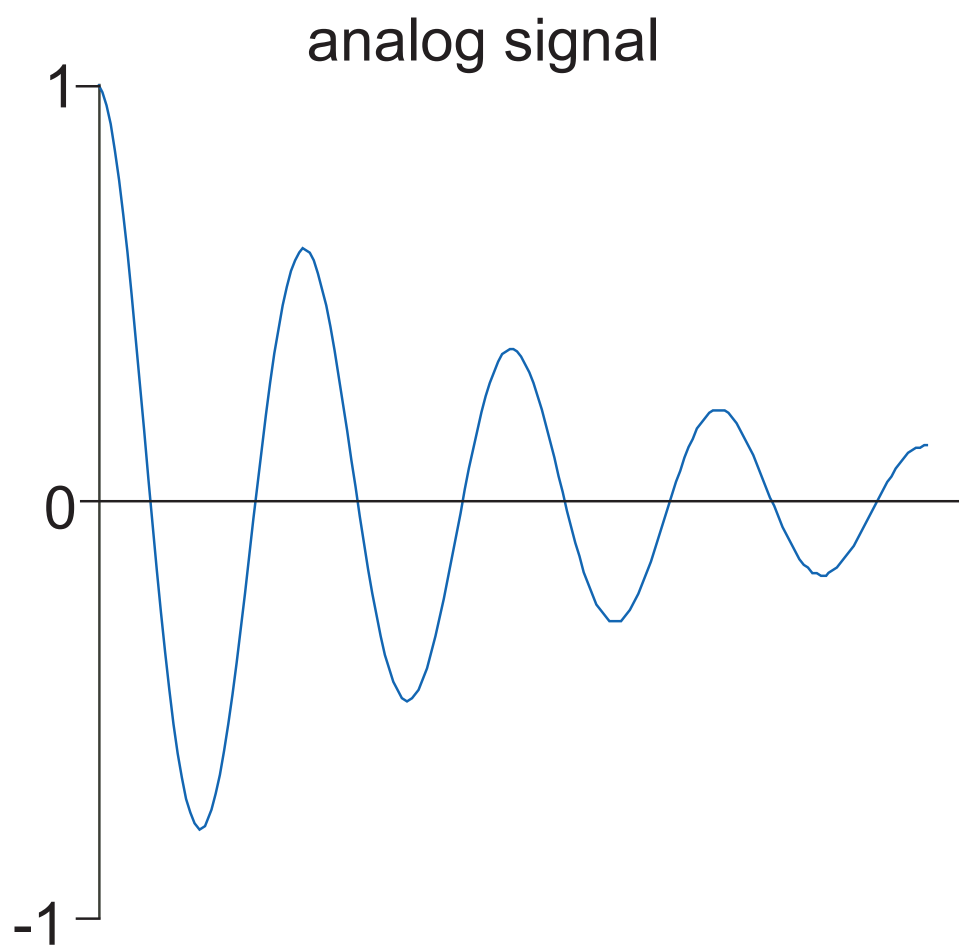
Analog-to-Digital Conversion

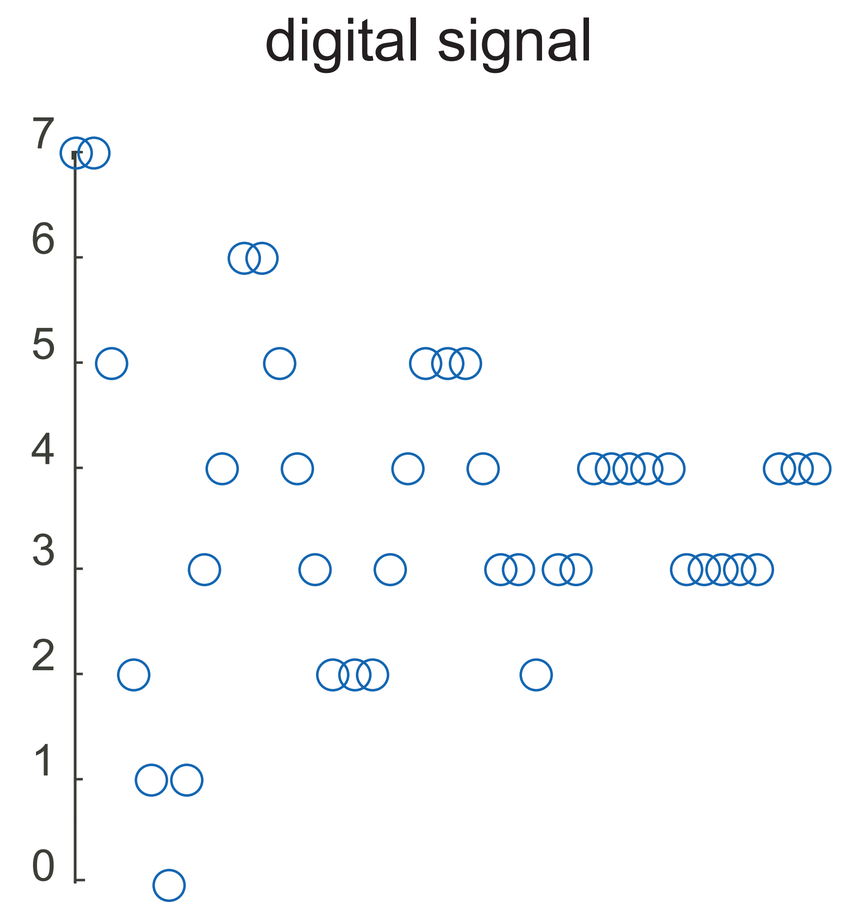
- Sampling Theorem
- Amplitude quantization

RICE

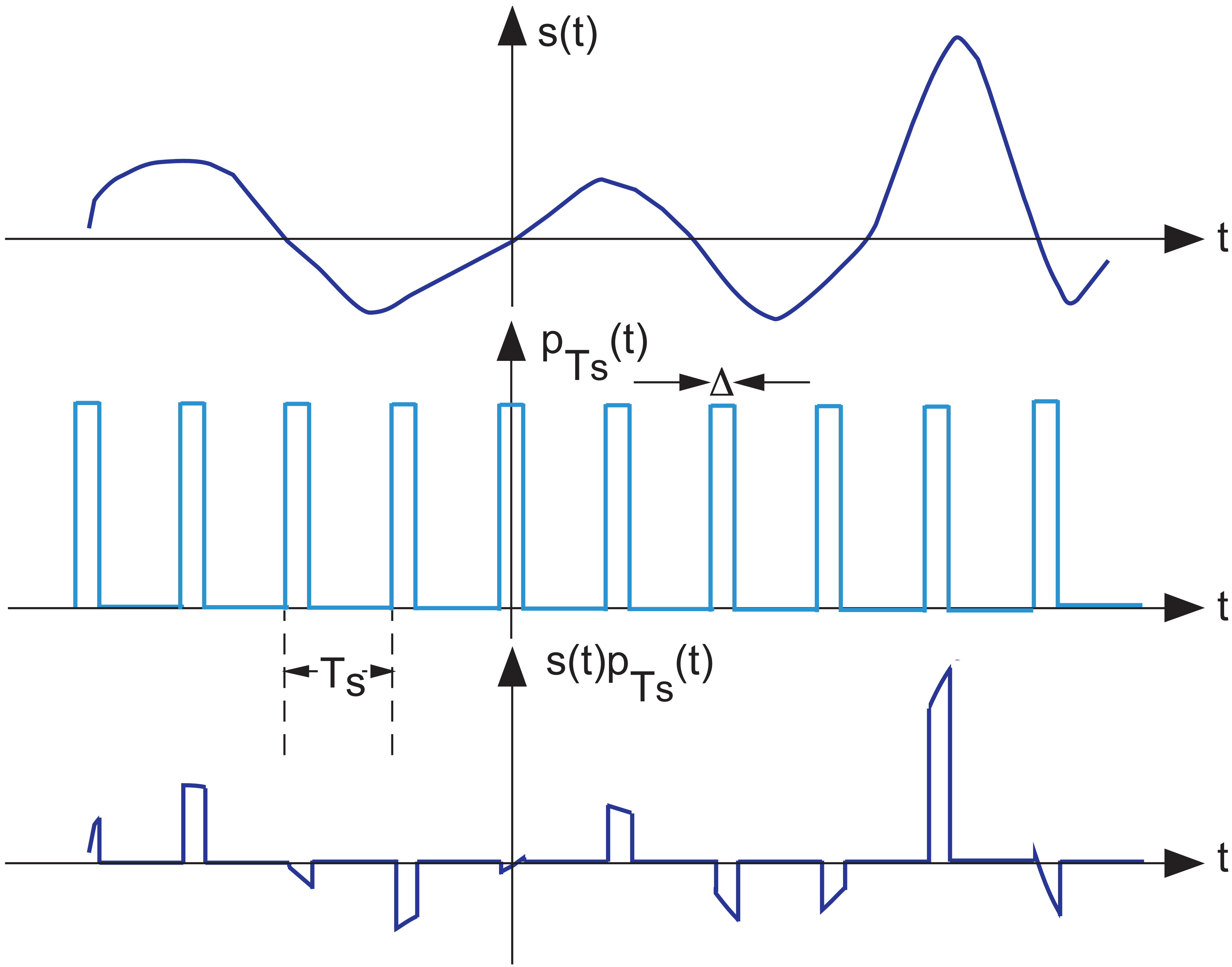
Analog and Digital Signals

- Analog signals: functions of a continuous variable
- Digital signals: discrete-valued functions of the integers





Sampling



What's the Spectrum?

$$s(t) \cdot p_{T_s}(t) \longleftrightarrow ?$$

$$p_{T_s}(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi k t}{T_s}}, \quad c_k = \frac{\sin\left(\frac{\pi k \Delta}{T_s}\right)}{\pi k}$$

$$s(t) \cdot p_{T_s}(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi k t}{T_s}} s(t)$$

$$e^{j\frac{2\pi k t}{T_s}} s(t) \longleftrightarrow S\left(f - \frac{k}{T_s}\right)$$

$$\boxed{s(t) \cdot p_{T_s}(t) \longleftrightarrow \sum_{k=-\infty}^{\infty} c_k S\left(f - \frac{k}{T_s}\right)}$$

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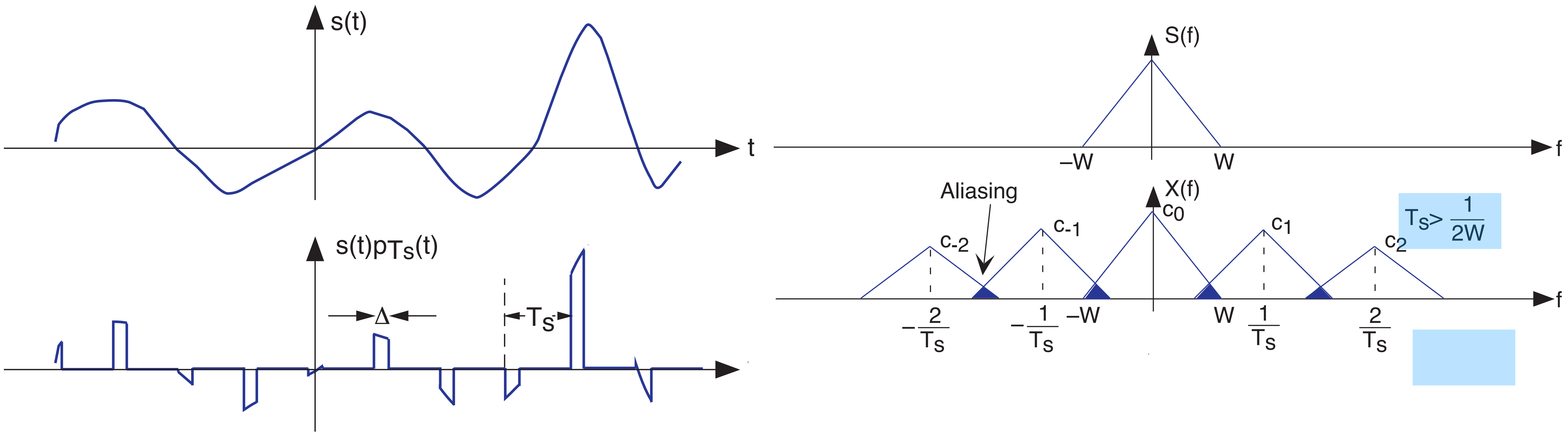
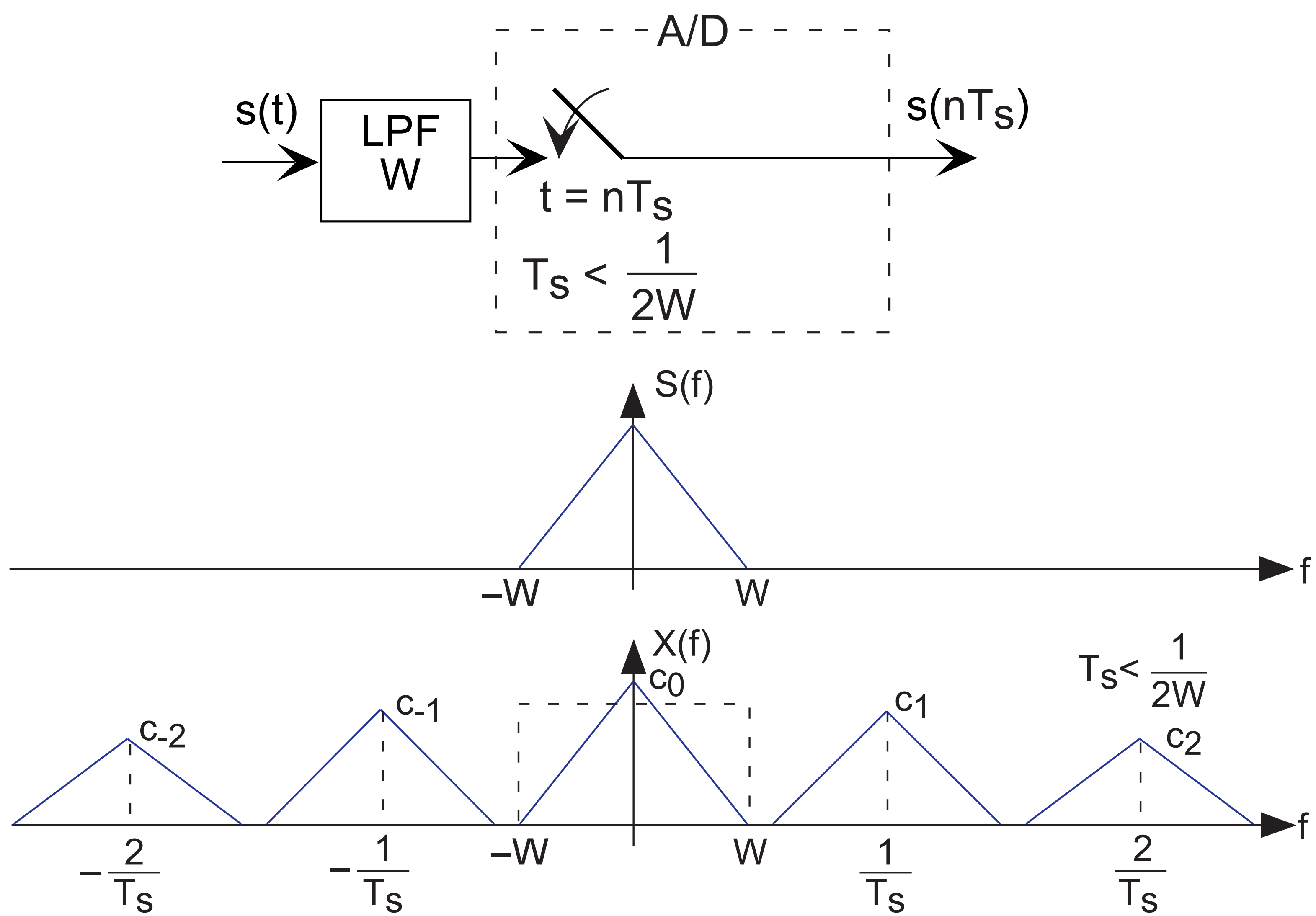
What's the Spectrum?



$$s(t) \cdot p_{T_s}(t) \longleftrightarrow \sum_{k=-\infty}^{\infty} c_k S\left(f - \frac{k}{T_s}\right)$$

Sampling Theorem

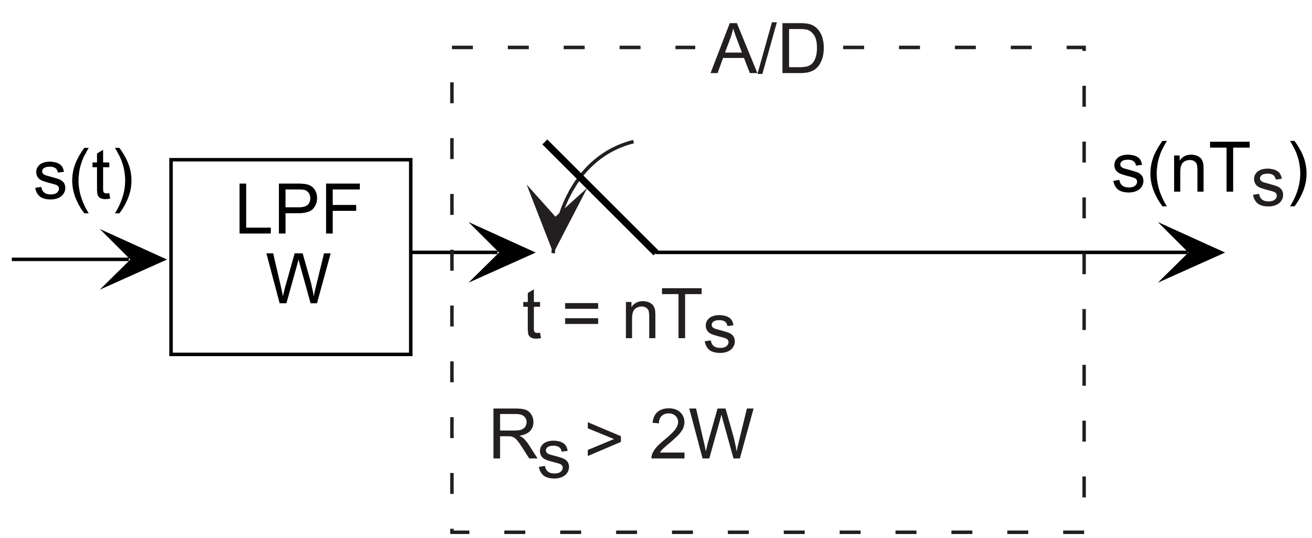
Theorem: If $s(t)$ is bandlimited to W Hz and is sampled by a pulse sequence so that $T_s < \frac{1}{2W}$, then $s(t)$ can be recovered from $s(t)p_{T_s}(t)$ by lowpass filtering



Sampling

Usually, sampling *rate* is used rather than sampling interval

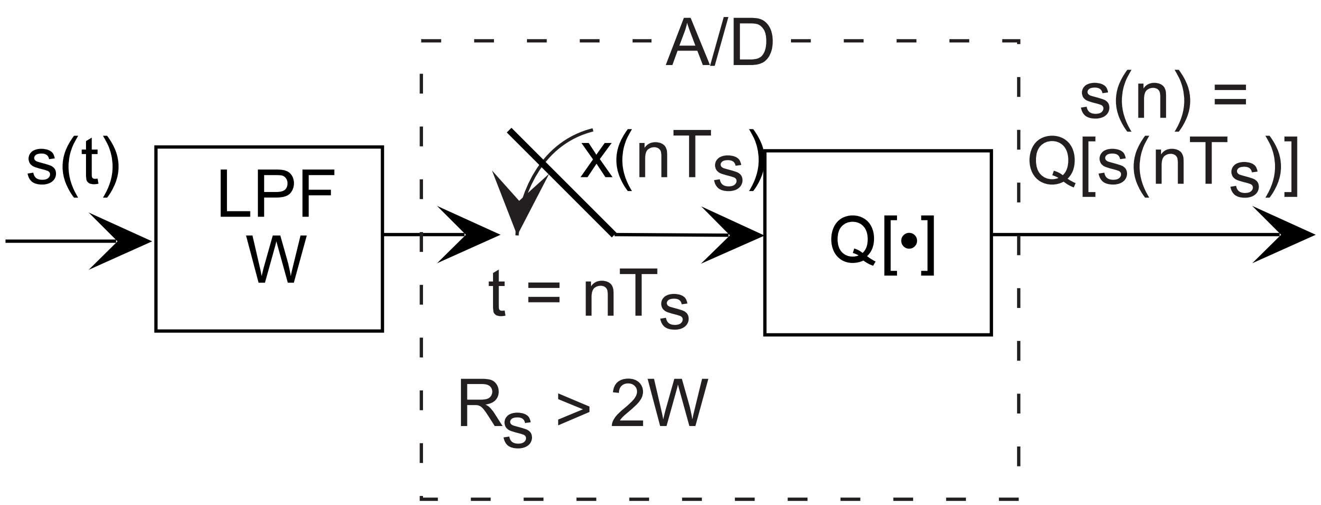
$$R_s = \frac{1}{T_s} > 2W$$

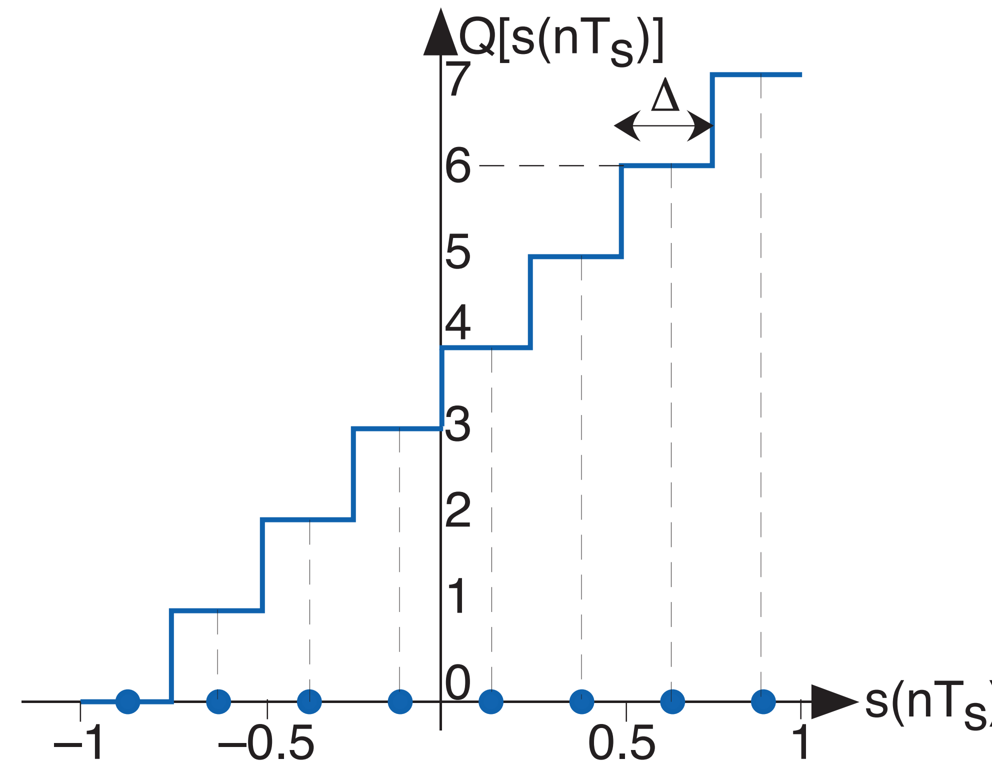


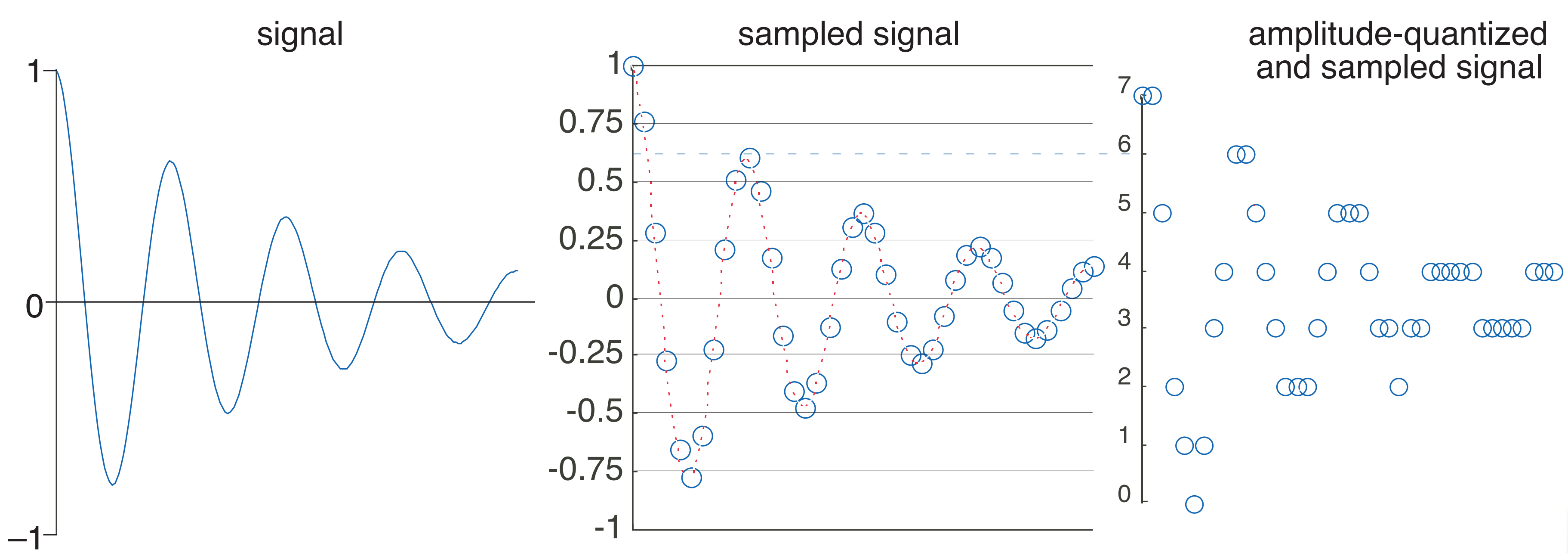
We have a discrete-time signal, not a digital one (yet)

Amplitude Quantization

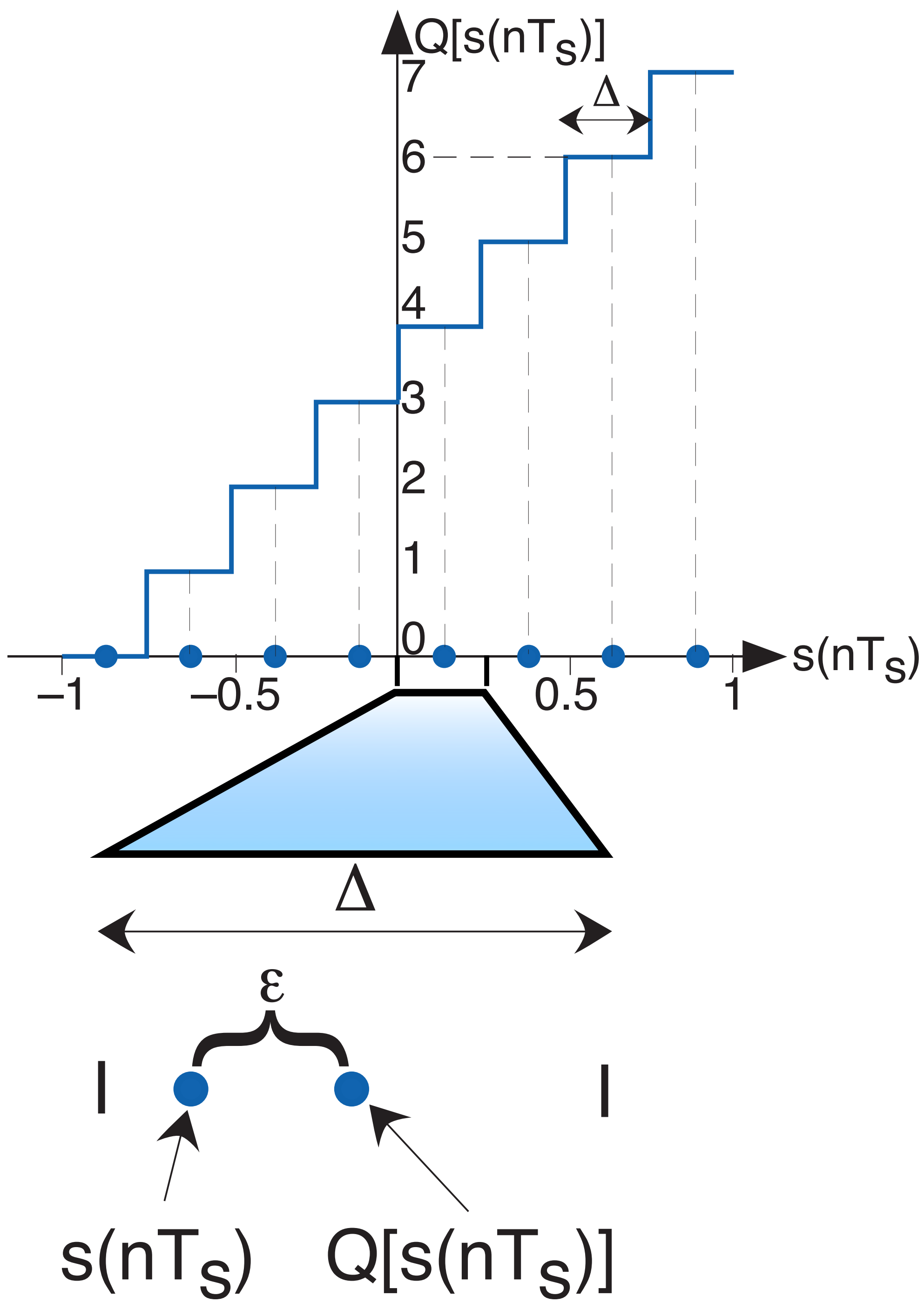
Need to convert continuous-valued amplitudes into discrete values







Amplitude Quantization Introduces Error



$$\mathrm{rms}(\epsilon) = \sqrt{\frac{1}{\Delta}\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \epsilon^2 \, d\epsilon}$$

$$= \sqrt{\frac{\Delta^2}{12}}$$

$$\Delta = \frac{2 \max |s(t)|}{2^B}$$

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Signal-to-Noise Ratio

Definition: Signal-to-noise ratio (SNR) is the ratio of signal power to "noise" power

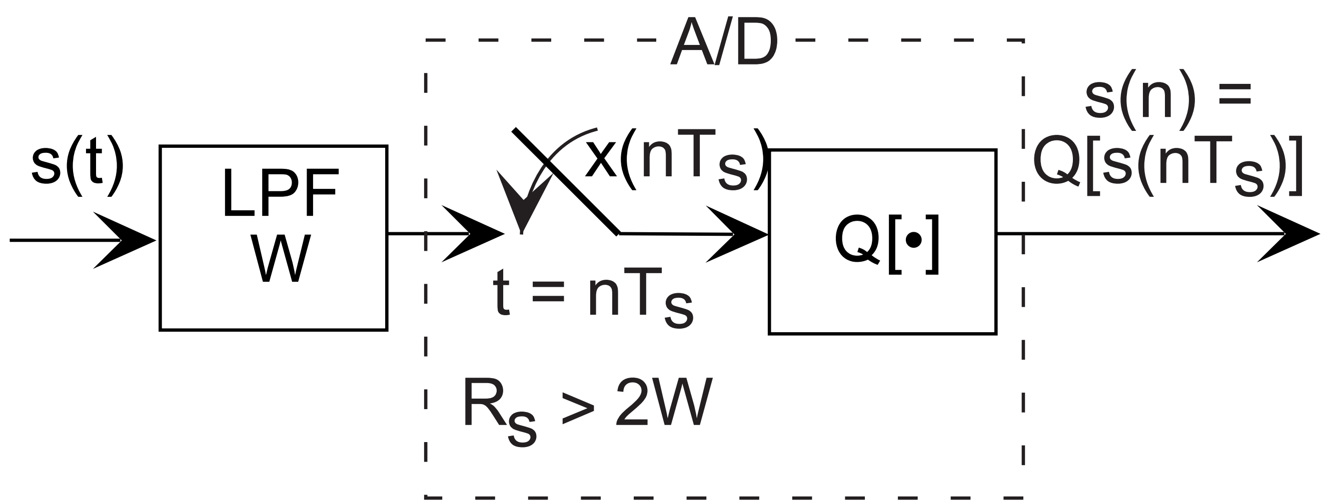
$$\text{SNR} = \frac{\text{power}(s)}{\text{power}(\epsilon)}$$

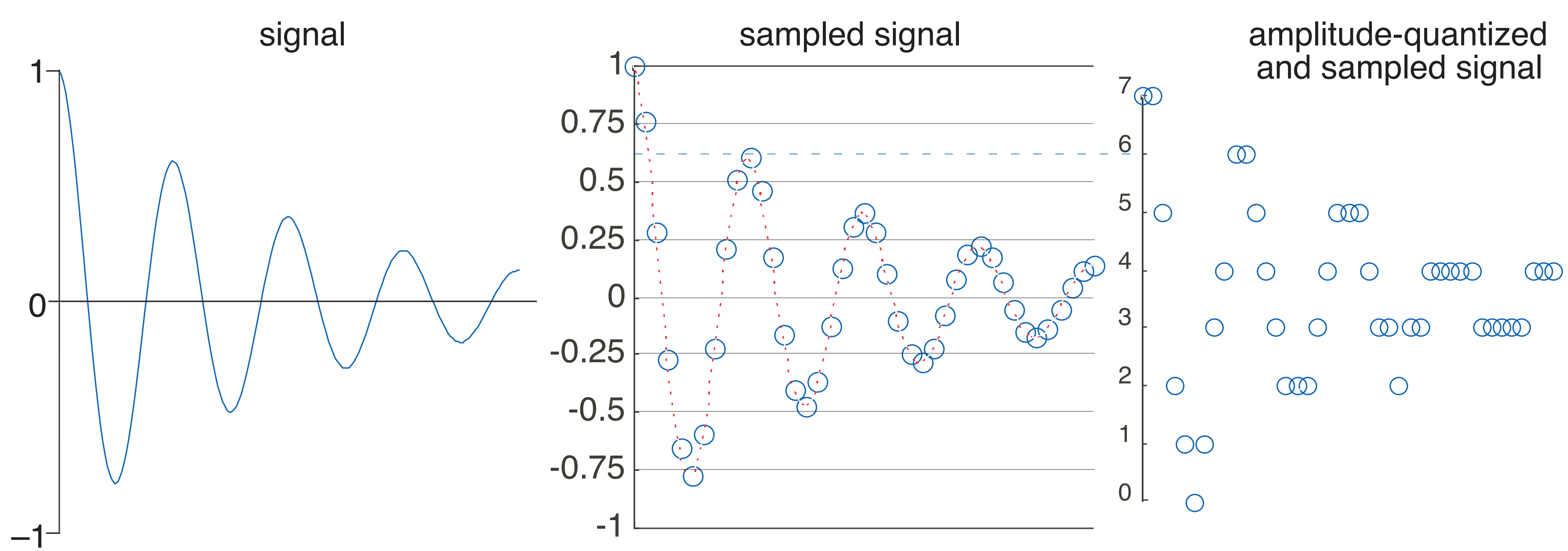
For amplitude quantization,

$$\text{power}(s) = \frac{A^2}{2} \qquad \text{power}(\epsilon) = \frac{\Delta^2}{12} \qquad \Delta = \frac{2A}{2^B}$$

$$\text{SNR} = \frac{\frac{A^2}{2}}{\frac{A^2}{3 \cdot 2^{2B}}} = \frac{3}{2} 2^{2B} \quad = 6B + 10 \log_{10} 1.5 \text{ dB}$$

Analog-to-Digital Conversion





$$\mathrm{SNR} = \frac{3}{2}2^{2B}$$