Fundamentals of Electrical Engineering

The Frequency Domain

- Signal power in the frequency domain
- Building signals in the frequency domain
- Filtering periodic signals

RICE

Parseval's Theorem

$$s(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi kt}{T}} \qquad c_k = \frac{1}{T}\int_0^T s(t)e^{-j\frac{2\pi kt}{T}}\,dt$$

Power in time-domain equals power in the frequency domain

$$\text{power}[s] \equiv \frac{1}{T}\int_0^T s^2(t)\,dt$$

$$\boxed{\frac{1}{T}\int_0^T s^2(t)\,dt = \sum_{k=-\infty}^{\infty} |c_k|^2}$$

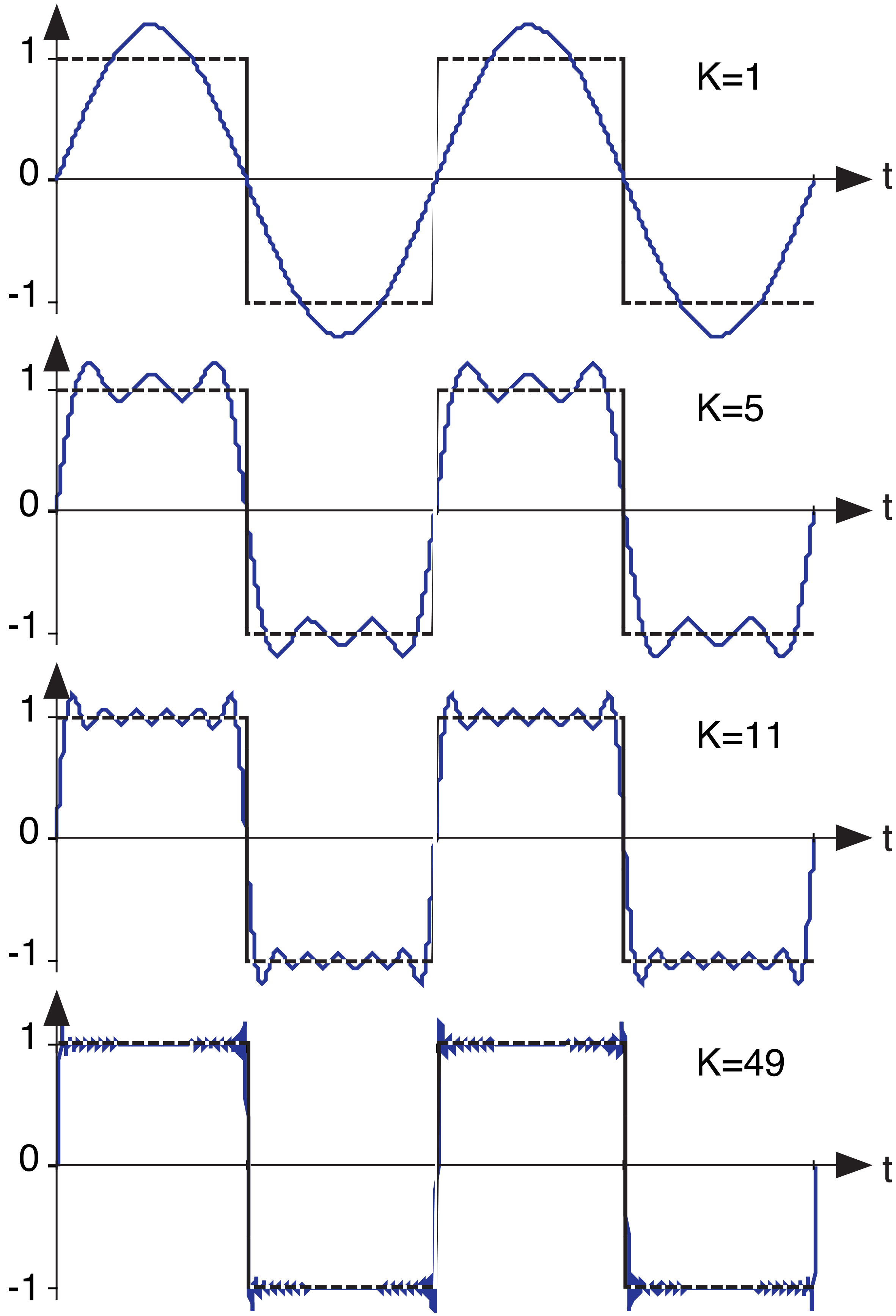
Approximation of Signals

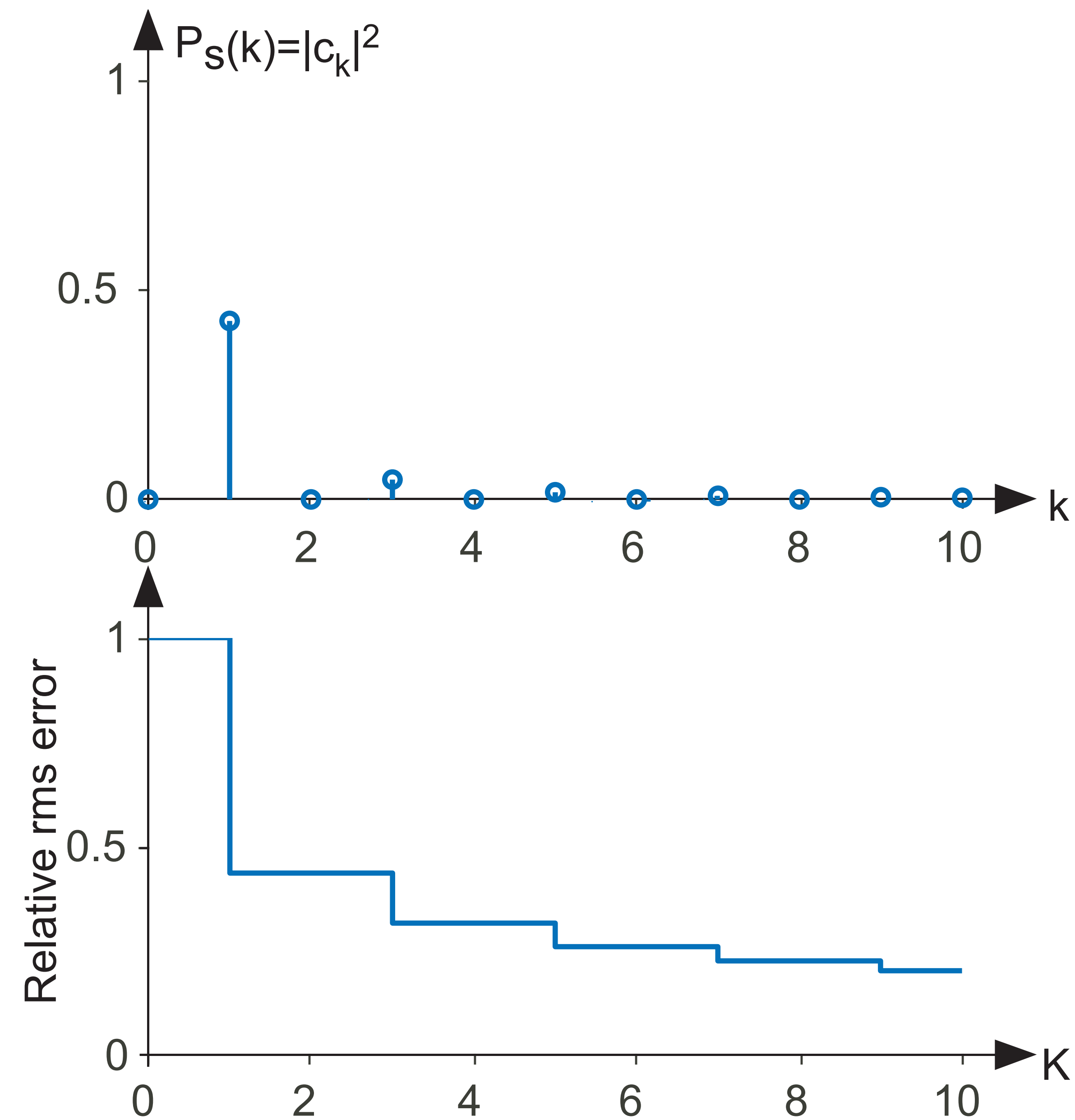
$$s(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi kt}{T}} \qquad s_K(t) = \sum_{k=-K}^{K} c_k e^{j\frac{2\pi kt}{T}}$$

$$\epsilon_K(t) = s(t) - s_K(t) = \sum_{k=-\infty}^{-K-1} c_k e^{j\frac{2\pi kt}{T}} + \sum_{k=K+1}^{\infty} c_k e^{j\frac{2\pi kt}{T}}$$

$$\mathrm{rms}(\epsilon_K) = \sqrt{2 \sum_{k=K+1}^{\infty} |c_k|^2}$$

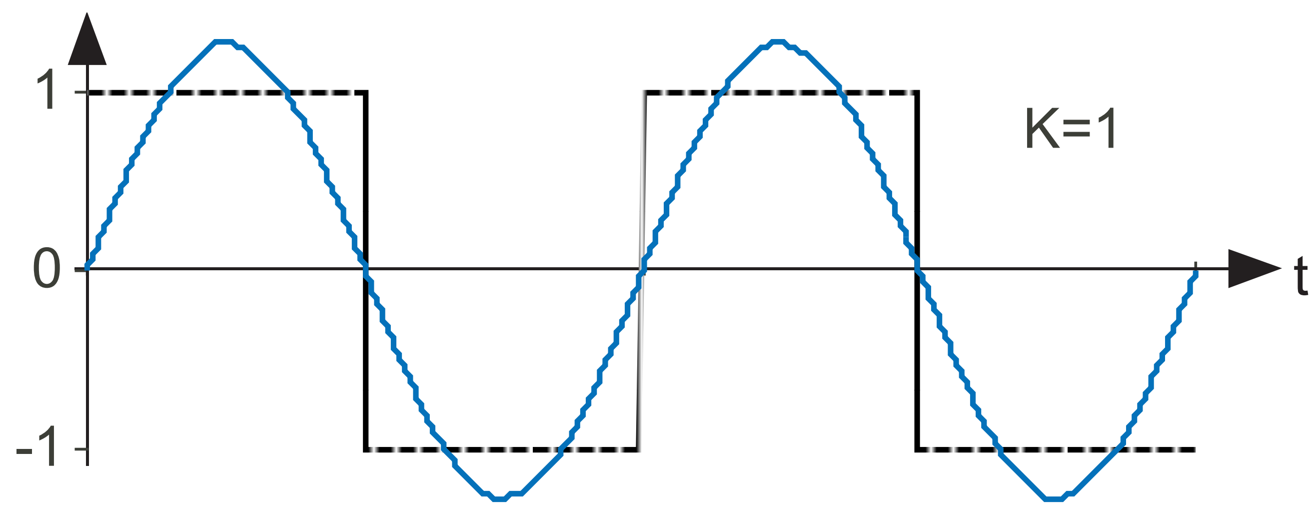
Approximation of Square Wave





Harmonic Distortion

How close to a sinusoid is a periodic waveform?



$$c_1 = \frac{2}{j\pi}$$

$$\text{THD} = \frac{2\sum_{k=2}^{\infty}|c_k|^2}{\text{power}(s(t)-c_0)} = \frac{2\sum_{k=2}^{\infty}|c_k|^2}{2\sum_{k=1}^{\infty}|c_k|^2}$$

$$\frac{1-2\left(\frac{2}{\pi}\right)^2}{1}$$
$$= 0.1894$$

Note: $\quad c_0 = \frac{1}{T}\int_0^T s(t)\,dt = \text{average value of } s(t)$

A Glimpse of Digital Communication

Digital communication: “0” $\rightarrow 0$ $\quad 0 \leq t < T$

(one bit at a time) “1” $\rightarrow \sin 2\pi f_c t$

(two bits at a time)

“00” $\rightarrow 0 + 0$

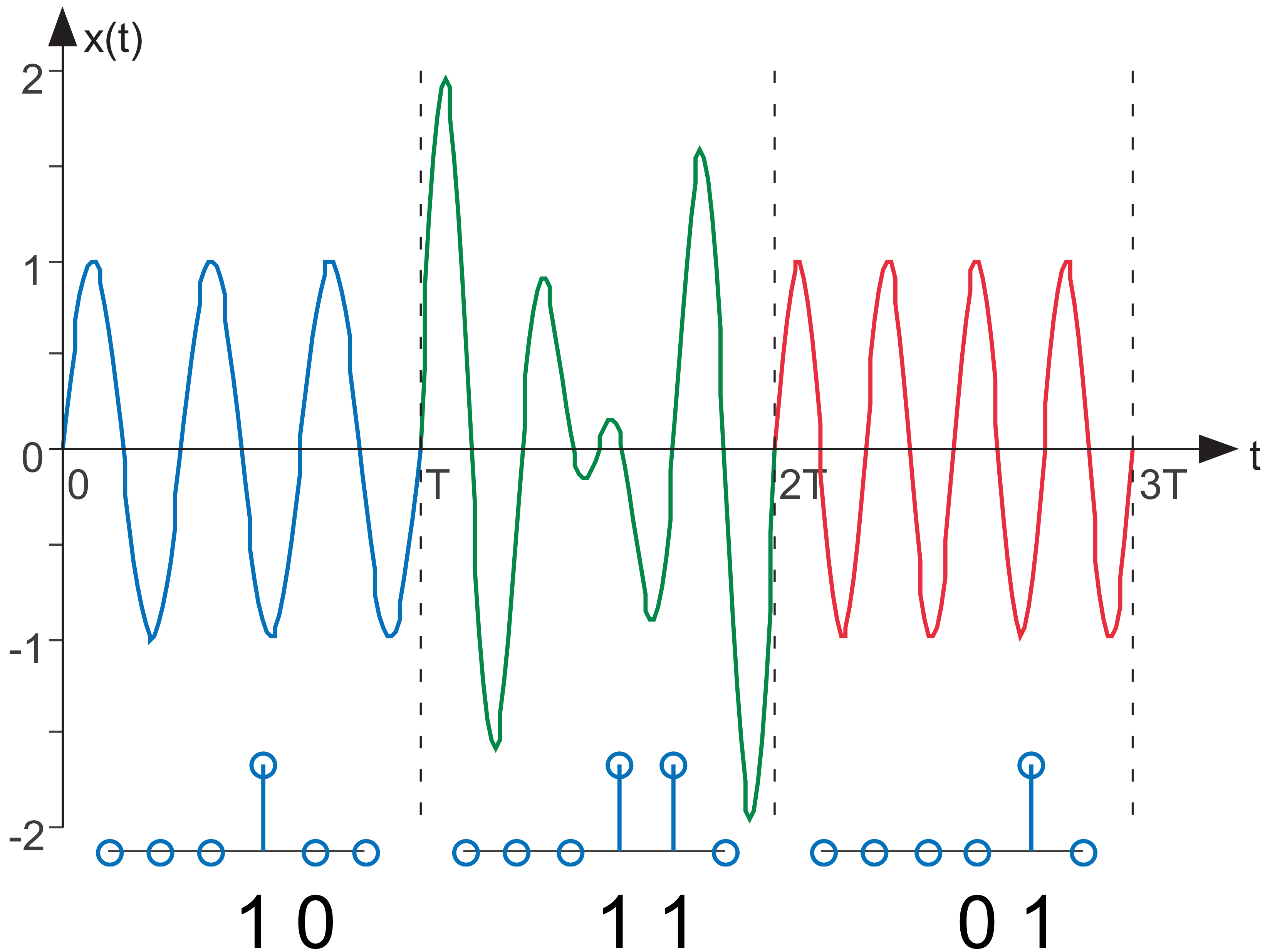
“01” $\rightarrow 0 + \sin 2\pi f_2 t$ $\quad 0 \leq t < T$

“10” $\rightarrow \sin 2\pi f_1 t + 0$

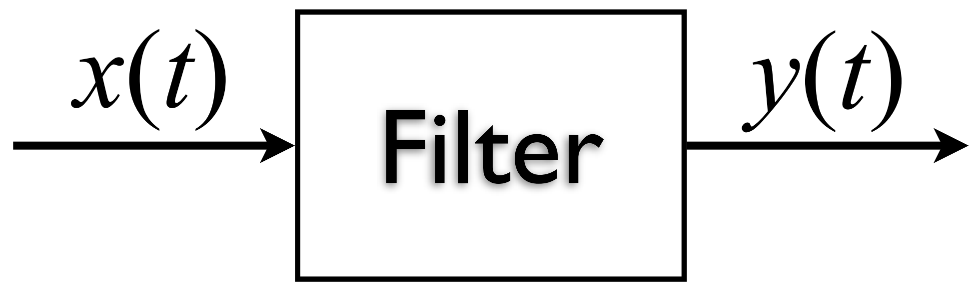
“11” $\rightarrow \sin 2\pi f_1 t + \sin 2\pi f_2 t$

RICE

Encoding Information in the Frequency Domain



Linear, Time-Invariant Filters

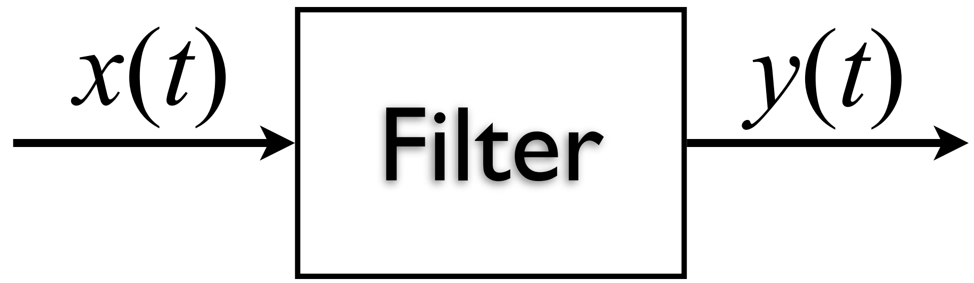


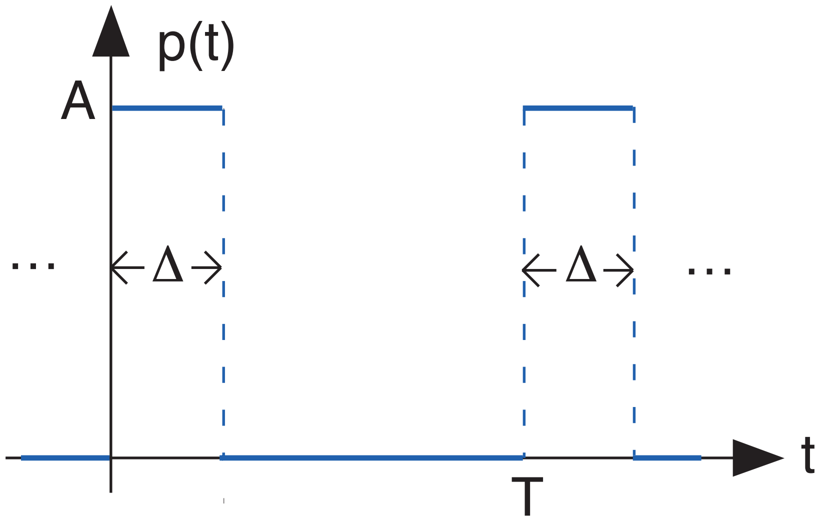
- If $x(t) = e^{j2\pi f t}, \; y(t) = H(f) e^{j2\pi f t}$
- If $x(t) = e^{j\frac{2\pi k t}{T}}, \; y(t) = H\left(\frac{k}{T}\right) e^{j\frac{2\pi k t}{T}}$
- If $x(t) = c_{k_1} e^{j\frac{2\pi k_1 t}{T}} + c_{k_2} e^{j\frac{2\pi k_2 t}{T}}$

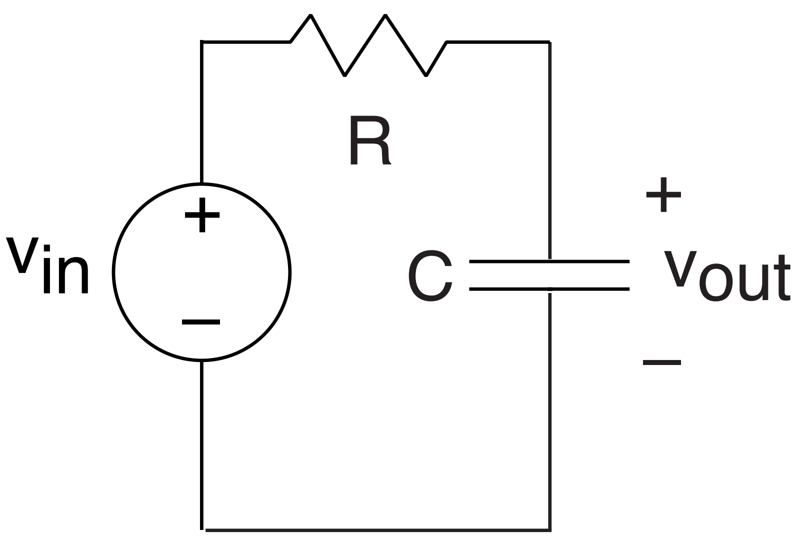
$$y(t) = H\left(\frac{k_1}{T}\right) c_{k_1} e^{j\frac{2\pi k_1 t}{T}} + H\left(\frac{k_2}{T}\right) c_{k_2} e^{j\frac{2\pi k_2 t}{T}}$$

- If $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi k t}{T}}, \; y(t) = \sum_{k=-\infty}^{\infty} H\left(\frac{k}{T}\right) c_k e^{j\frac{2\pi k t}{T}}$

Lowpass Filtering





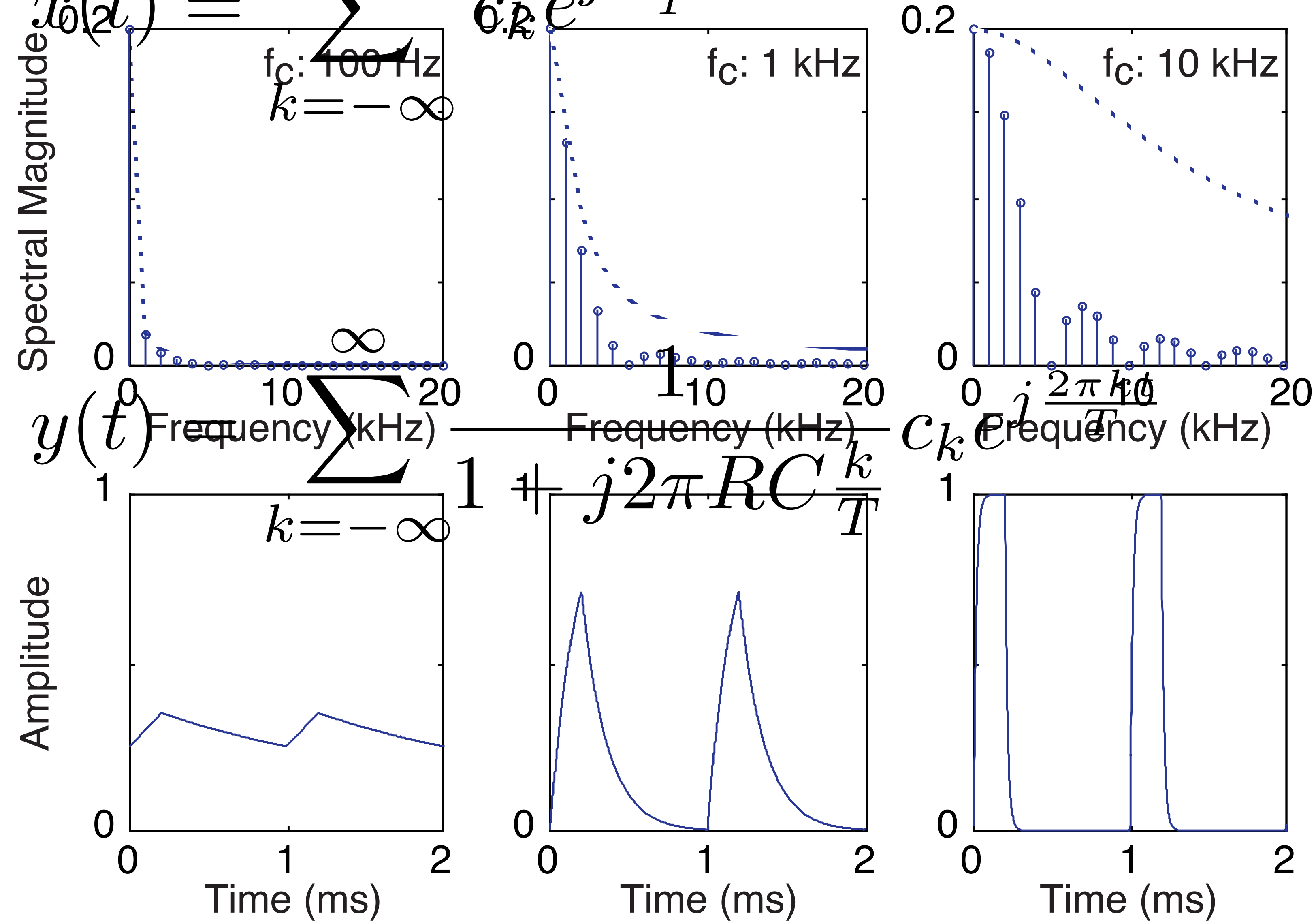


$$H(f) = \frac{1}{1 + j2\pi RCf}$$

$$T = 0.001 \text{ s } (1 \text{ ms}) \qquad \frac{\Delta}{T} = 0.2$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi k t}{T}}$$

$$y(t) = \sum_{k=-\infty}^{\infty} \frac{1}{1 + j2\pi RC\frac{k}{T}} c_k e^{j\frac{2\pi k t}{T}}$$



Signals in Time and Frequency

$$s(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi k t}{T}} \qquad c_k = \frac{1}{T}\int_0^T s(t) e^{-j\frac{2\pi k t}{T}}\, dt$$

$$s(t) \longleftrightarrow c_k$$

- Signals can be defined in either the time domain or the frequency domain
- Can study a signal's structure in either domain
- For linear, time-invariant systems, we can determine their outputs for periodic inputs