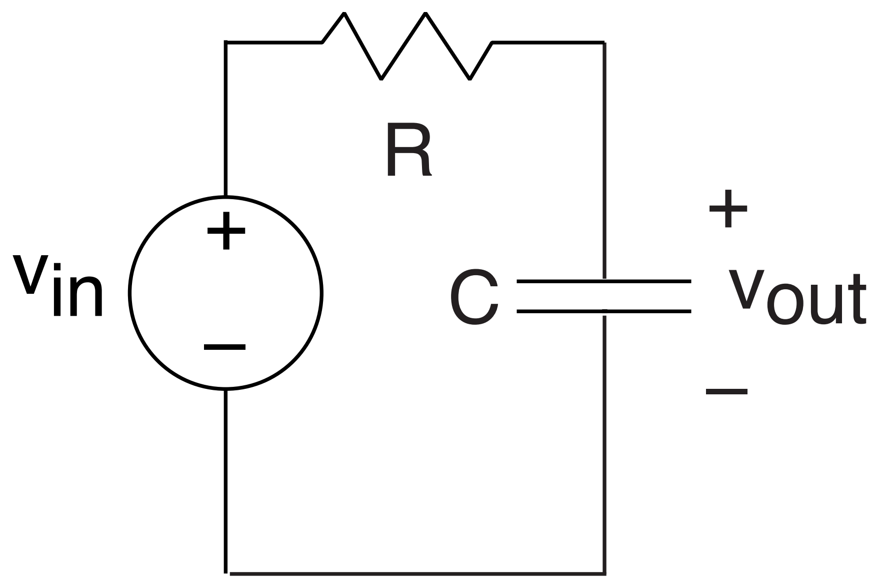
Fundamentals of Electrical Engineering

Circuits with Capacitors and Inductors

- Filters
- Thévenin and Mayer-Norton equivalents
- Complex power

What is the circuit doing?



$$v_{\text{in}}(t) = A\cos(2\pi f_0 t) = \text{Re}[Ae^{j2\pi f_0 t}]$$

$$V_{\text{out}} = \frac{1}{j2\pi f_0 RC + 1} V_{\text{in}}$$

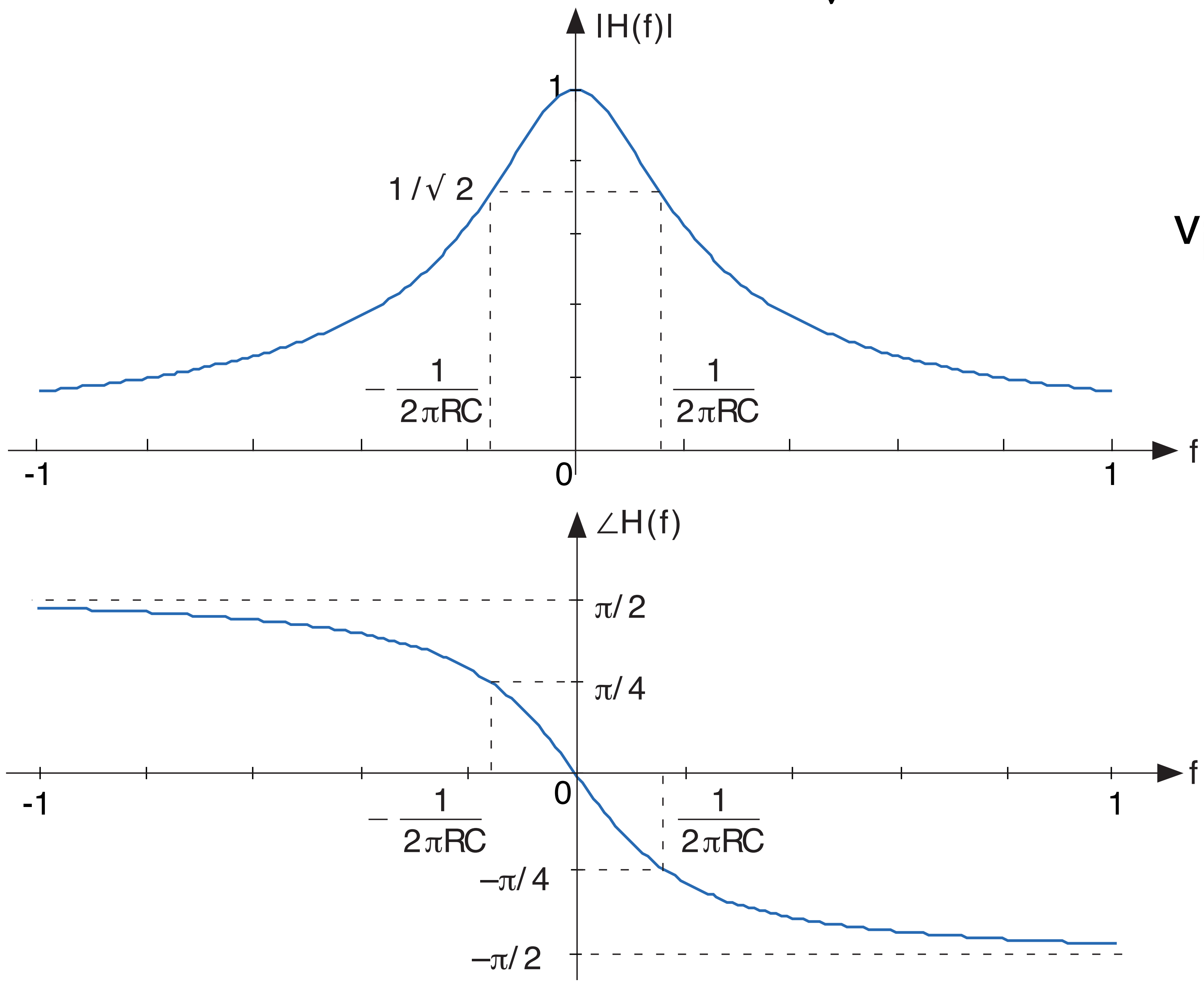
$$v_{\text{out}}(t) = \text{Re}\left[\frac{1}{j2\pi f_0 RC + 1} Ae^{j2\pi f_0 t}\right]$$

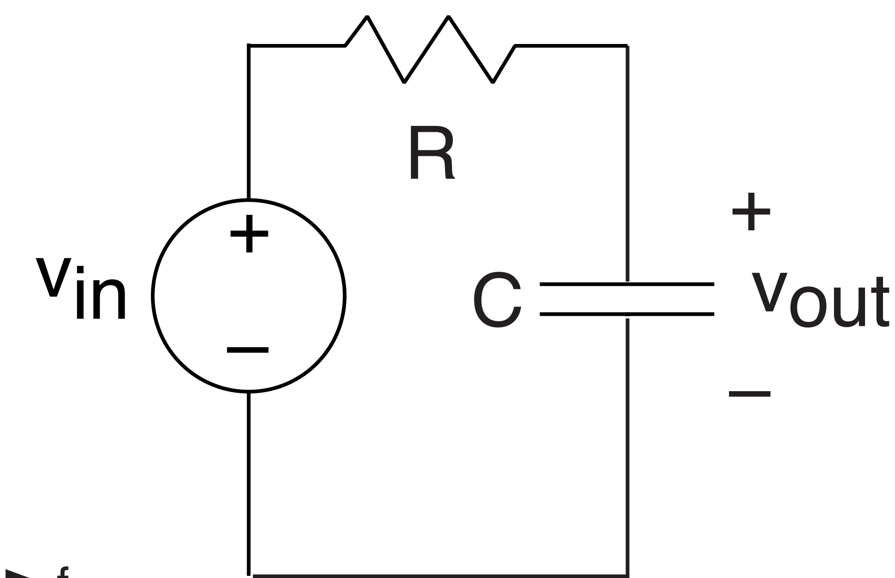
$$v_{\text{out}}(t) = \frac{A}{\sqrt{4\pi^2 f_0^2 R^2 C^2 + 1}} \cos(2\pi f_0 t - \tan^{-1} 2\pi f_0 RC)$$

$$H(f) = |H(f)|e^{j\angle H(f)} = \frac{1}{\sqrt{4\pi^2 f^2 R^2 C^2 + 1}} e^{-j\tan^{-1} 2\pi f RC}$$

What is the circuit doing?

$$H(f) = |H(f)|e^{j\angle H(f)} = \frac{1}{\sqrt{4\pi^2 f^2 R^2 C^2 + 1}} e^{-j\tan^{-1} 2\pi f RC}$$





Lowpass filter

Filtering

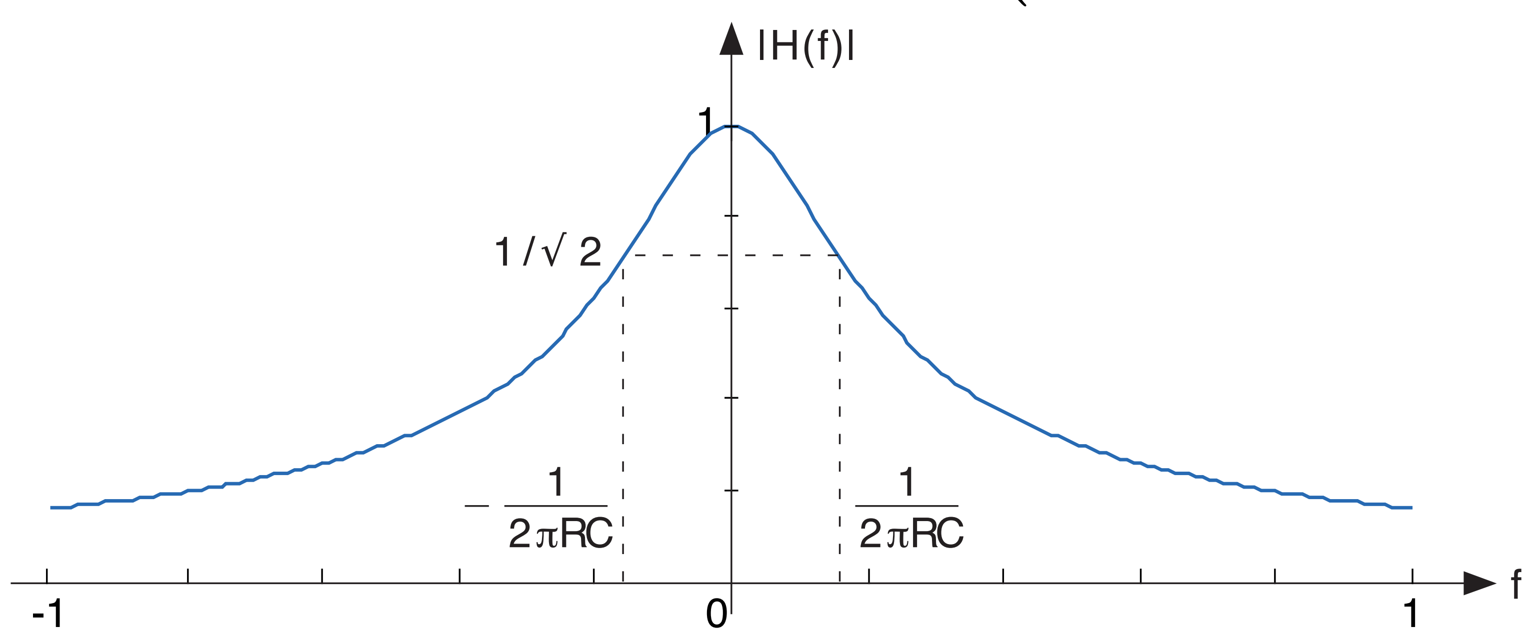
$$v_{\text{in}}(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t)$$

$$v_{\text{in}}(t) = A_1 \cos(2\pi f_1 t) \implies v_{\text{out}}(t) = |H(f_1)| A_1 \cos\big(2\pi f_1 t + \angle H(f_1)\big)$$

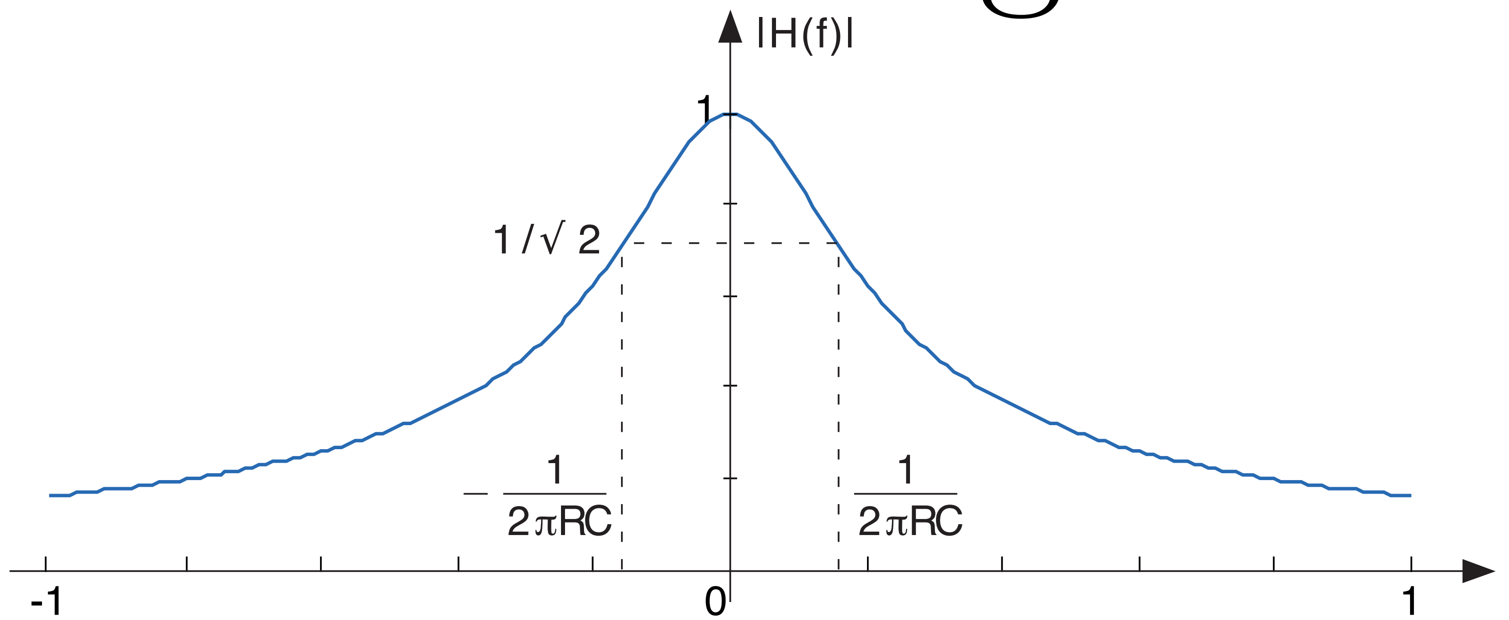
$$v_{\text{in}}(t) = A_2 \cos(2\pi f_2 t) \implies v_{\text{out}}(t) = |H(f_2)| A_2 \cos\big(2\pi f_2 t + \angle H(f_2)\big)$$

Because the circuit is *linear*, superposition applies

$$\begin{aligned} v_{\text{out}}(t) = {} & |H(f_1)| A_1 \cos\big(2\pi f_1 t + \angle H(f_1)\big) \\ & + |H(f_2)| A_2 \cos\big(2\pi f_2 t + \angle H(f_2)\big) \end{aligned}$$



Filtering



Cutoff frequency: frequency f_c at which

$$|H(f_c)| = \frac{1}{\sqrt{2}} \max_f |H(f)|$$

For the RC lowpass,

$$|H(f)| = \frac{1}{\sqrt{4\pi^2 f^2 R^2 C^2 + 1}}$$

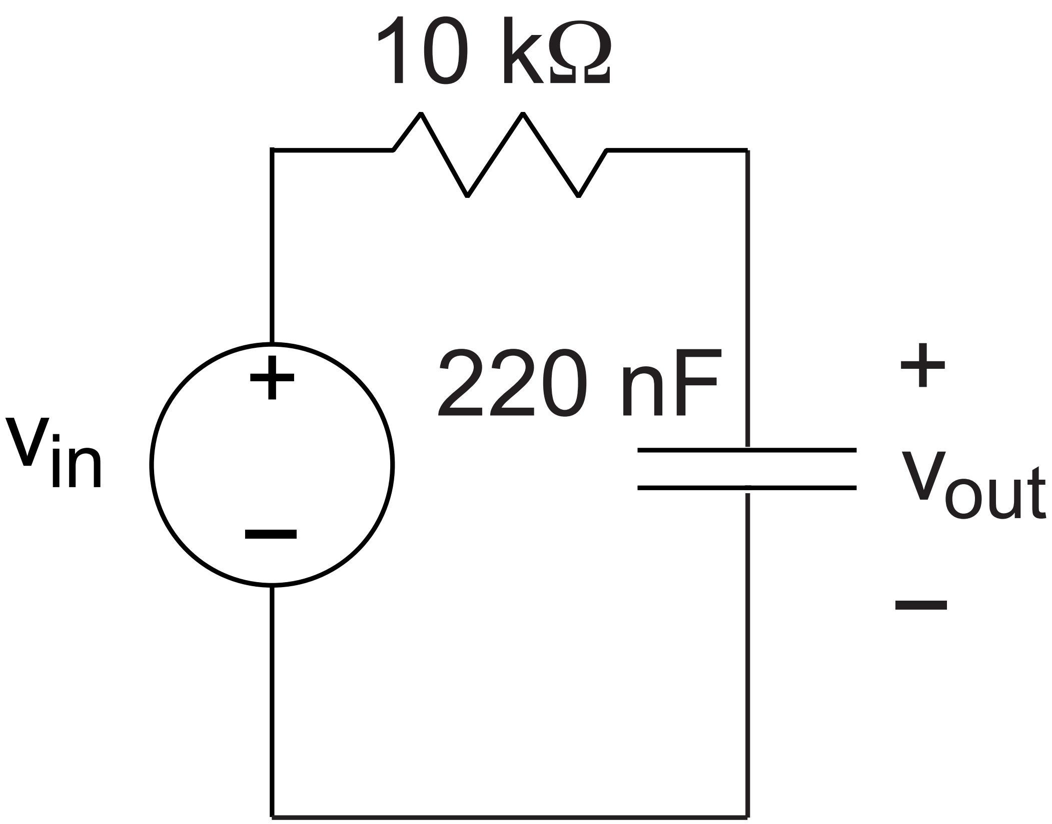
$$f_c = \frac{1}{2\pi RC}$$

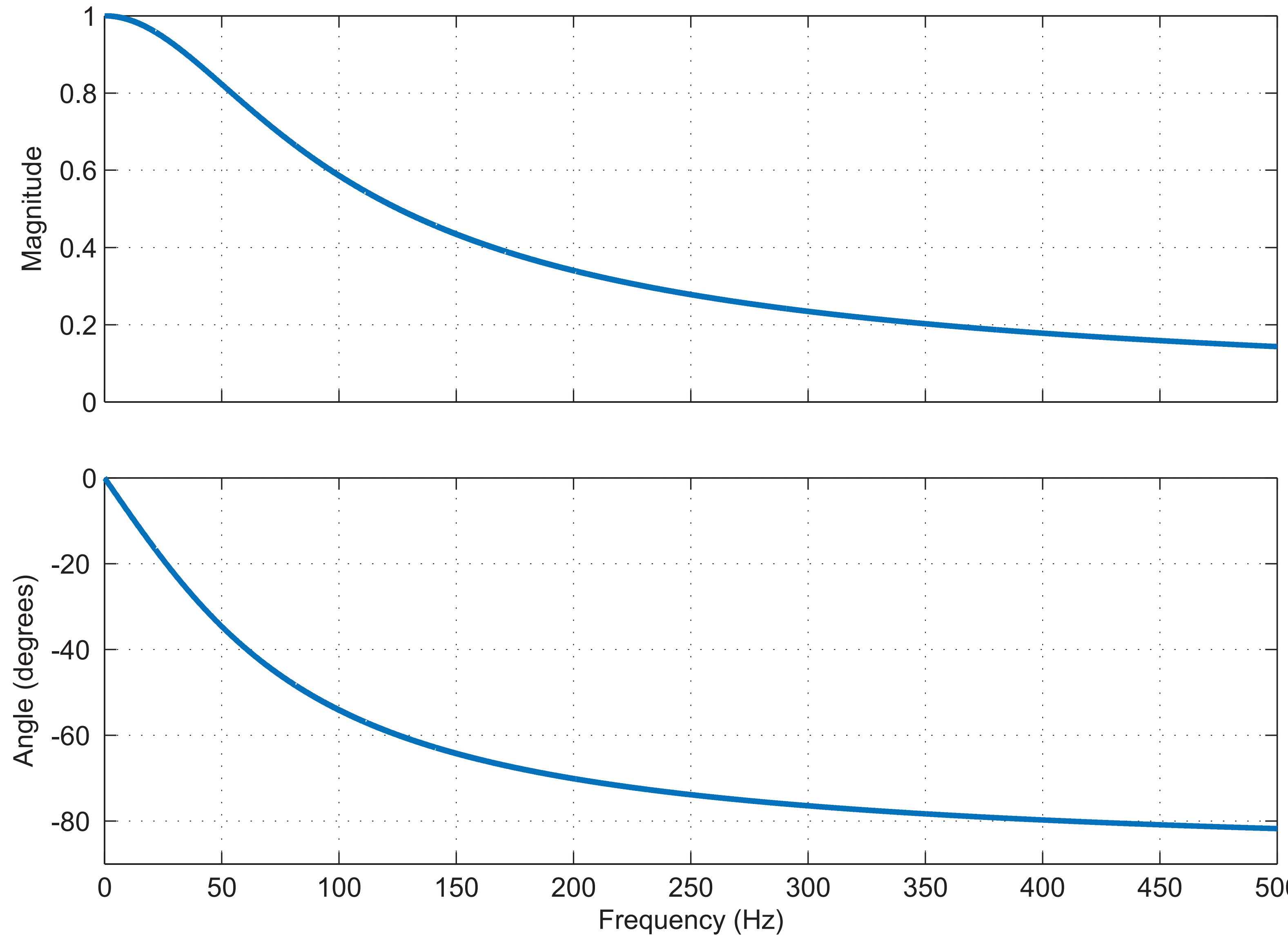
Filtering

Several filter types are common.

- Lowpass
- Highpass
- Bandpass

RC Lowpass in Action



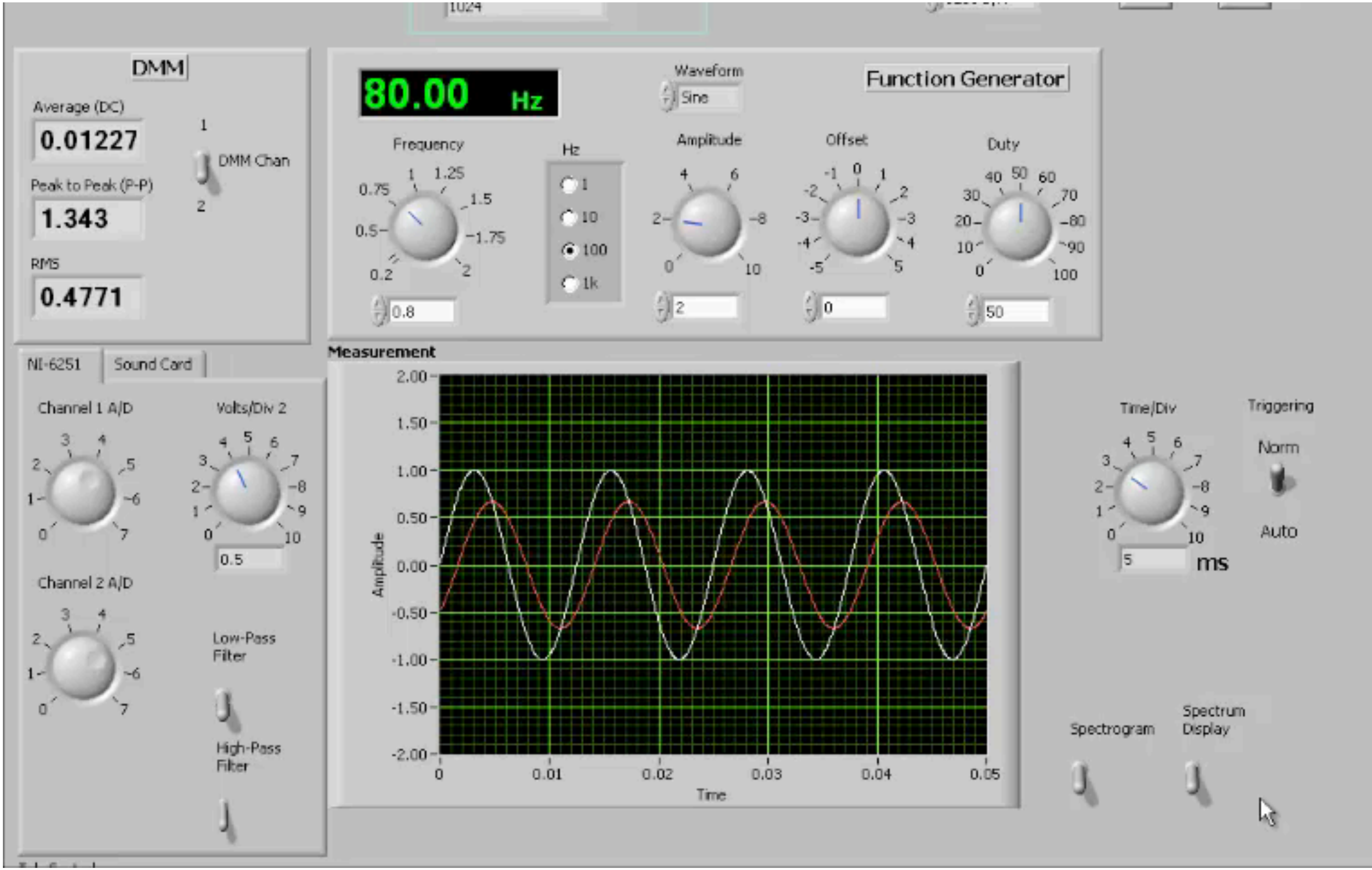


$$v_{\text{in}}(t) = \sin(2\pi f t) \qquad v_{\text{out}}(t) = |H(f)| \sin\left(2\pi f t + \angle H(f)\right)$$

$$H(f) = |H(f)| e^{j\angle H(f)} = \frac{1}{\sqrt{4\pi^2 f^2 R^2 C^2 + 1}} e^{-j\tan^{-1} 2\pi f RC}$$

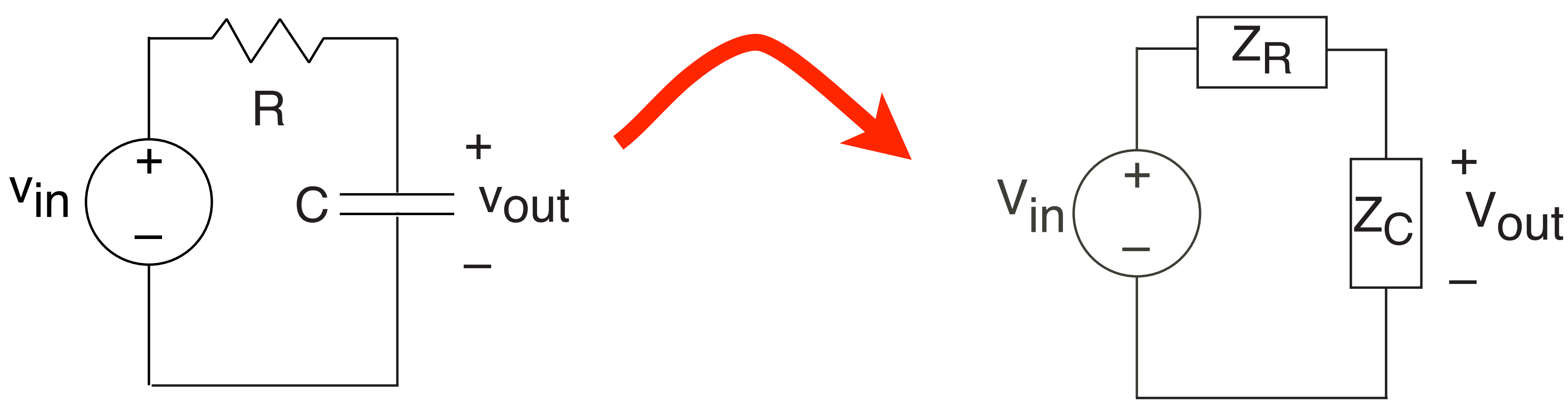
$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 10^4 \times 220 \times 10^{-9}} = 72.34 \text{ Hz}$$

RC Lowpass in Action

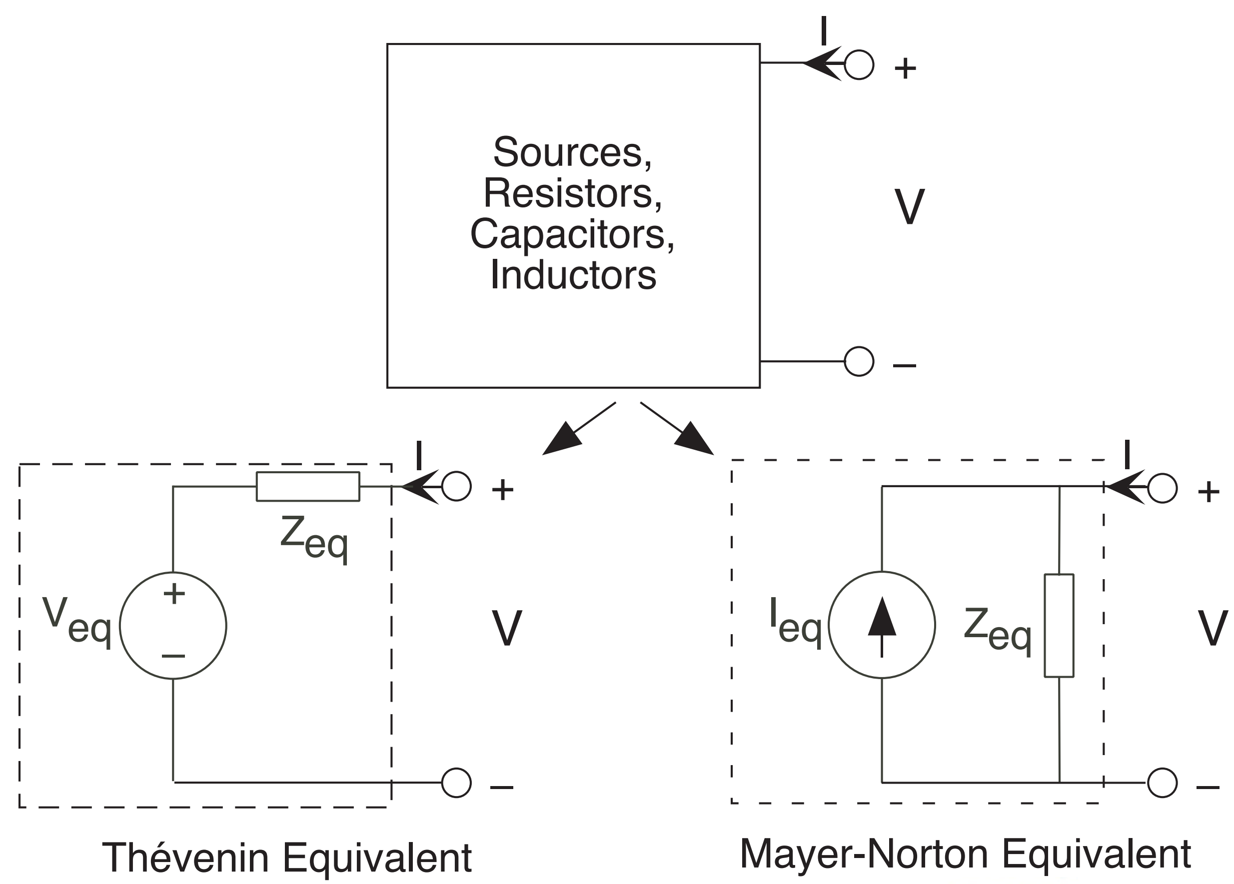


The Big Picture

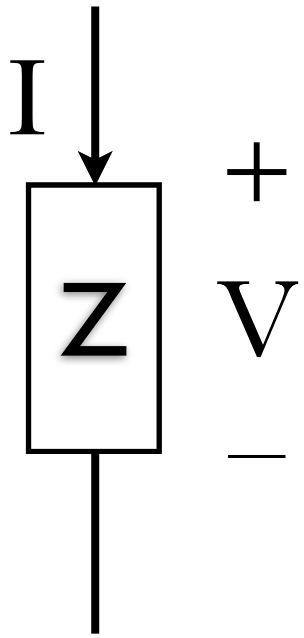
$$v = Ve^{j2\pi ft}, \quad i = Ie^{j2\pi ft} \qquad\qquad Z \equiv \frac{V}{I}$$



- Series/parallel "shortcuts" can be used
- Thévenin/Mayer-Norton equivalents have impedance "versions"



Complex Power



$$Z \equiv \frac{V}{I}$$

$$v = Ve^{j2\pi ft}, \quad i = Ie^{j2\pi ft}$$

$$V = |V|e^{j\phi},\ I = |I|e^{j\theta}$$

$$v(t) = |V|\cos(2\pi ft + \phi) \quad i(t) = |I|\cos(2\pi ft + \theta)$$

$$p(t) = v(t) \cdot i(t)$$

Complex Power

$$V = |V|e^{j\phi},\ I = |I|e^{j\theta}$$

$$v(t) = \frac{1}{2}\left(Ve^{j2\pi ft} + V^*e^{-j2\pi ft}\right)$$

$$i(t) = \frac{1}{2}\left(Ie^{j2\pi ft} + I^*e^{-j2\pi ft}\right)$$

$$p(t) = v(t) \cdot i(t)$$

$$\begin{aligned} p(t) &= \frac{1}{4}\left(VI^* + V^*I + VIe^{j4\pi ft} + V^*I^*e^{-j4\pi ft}\right) \\ &= \frac{1}{2}\mathrm{Re}\left[VI^*\right] + \frac{1}{2}\mathrm{Re}\left[VIe^{j4\pi ft}\right] \\ &= \frac{1}{2}\mathrm{Re}\left[VI^*\right] + \frac{1}{2}|V||I|\cos\left(4\pi ft + \phi + \theta\right) \end{aligned}$$

$$P_{\text{ave}} = \frac{1}{2}\mathrm{Re}[VI^*]$$

RICE

Complex Power

$$P = \frac{1}{2} V I^* \qquad P_{\text{ave}} = \frac{1}{2} \text{Re}[V I^*]$$

$$V = Z I$$

$$V I^* = Z |I|^2 = \frac{1}{Z^*} |V|^2$$

element	P	P_{ave}
R	$\frac{1}{2} R \vert I \vert^2$	$\frac{1}{2} R \vert I \vert^2$
C	$\frac{1}{2} \cdot \frac{1}{j 2 \pi f C} \vert I \vert^2$	0
L	$\frac{1}{2} \cdot j 2 \pi f L \vert I \vert^2$	0

Using Impedances

- By "thinking" of each element as a complex-valued resistor—as an impedance—a general picture emerges of what circuits do and how they behave
  - ✳ Series/parallel rules
  - ✳ Thévenin and Mayer-Norton equivalents
  - ✳ Transfer functions and filtering
  - ✳ Complex power
- *Only* applies when the source is a sinusoid

RICE