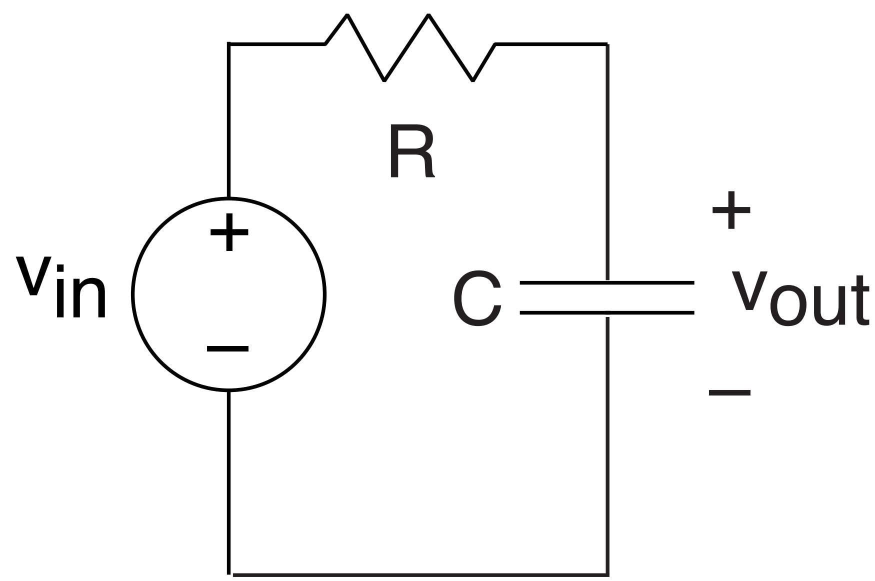
Fundamentals of Electrical Engineering

Circuits with Capacitors and Inductors

- Differential equations
- Impedance

A Simple RC Circuit



$$i = C\frac{d\,v}{dt}$$

$$RC\frac{d\,v_{\text{out}}}{dt} + v_{\text{out}} = v_{\text{in}}$$

A Different Approach

Suppose the voltage source is a complex exponential

$$v_{\text{in}} = V_{\text{in}} e^{j2\pi f t}$$

Assume that all voltages and currents in the circuit, once solved, are also complex exponentials

$$v = V e^{j2\pi f t} \qquad i = I e^{j2\pi f t} \qquad Z \equiv \frac{V}{I}$$

Resistor: $v = Ri \rightsquigarrow V e^{j2\pi f t} = R\, I e^{j2\pi f t} \implies Z_R = R$

Capacitor:

$$i = C\frac{dv}{dt} \rightsquigarrow I e^{j2\pi f t} = C\, j2\pi f\, V e^{j2\pi f t} \implies Z_C = \frac{1}{j2\pi f C}$$

Inductor:

$$v = L\frac{di}{dt} \rightsquigarrow V e^{j2\pi f t} = L\, j2\pi f\, I e^{j2\pi f t} \implies Z_L = j2\pi f L$$

A Different Approach

Assuming that all voltages and currents in the circuit are complex exponentials,

$$v = Ve^{j2\pi ft} \qquad i = Ie^{j2\pi ft}$$

complex amplitudes satisfy KVL and KCL

$$\sum_k v_k(t) = 0 \rightsquigarrow \sum_k V_k e^{j2\pi ft} = 0 \implies \sum_k V_k = 0$$
$$\sum_k i_k(t) = 0 \rightsquigarrow \sum_k I_k e^{j2\pi ft} = 0 \implies \sum_k I_k = 0$$

A Different Approach

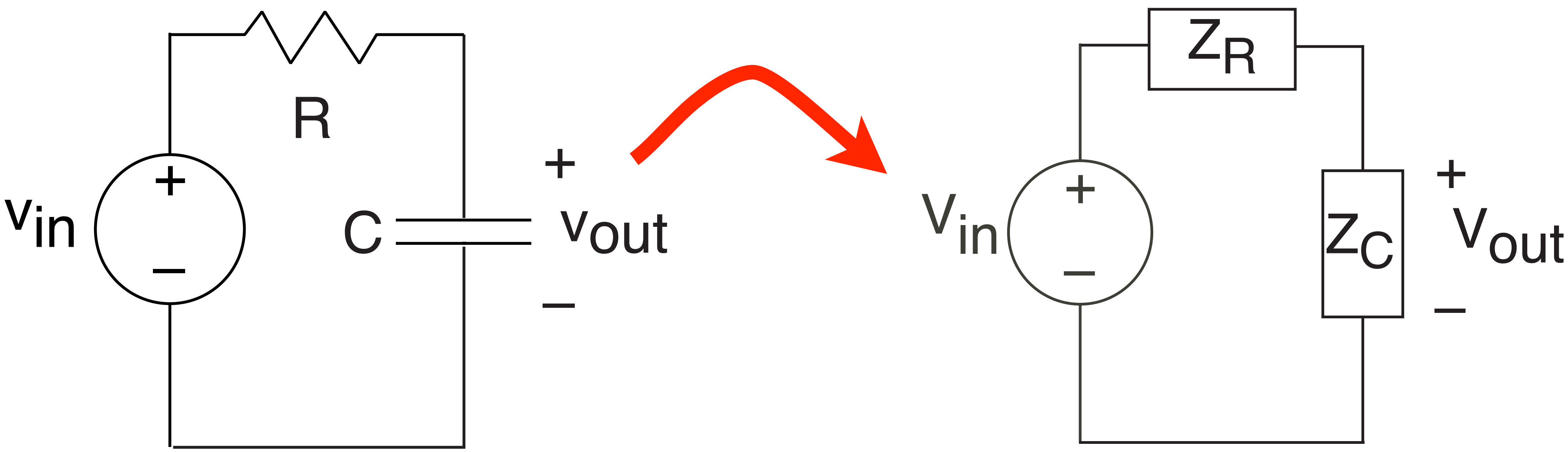
When the sources are complex exponentials,…

- *All* currents and voltages are complex exponentials
- *All* sources and element voltages and currents can be considered to be complex amplitudes (the complex exponential at the source frequency is understood)
- *All* elements except the sources can be considered complex-valued resistors (impedances)

RICE

Circuits Built from Impedances

$$Z \equiv \frac{V}{I} \qquad v = Ve^{j2\pi ft}, \quad i = Ie^{j2\pi ft}$$

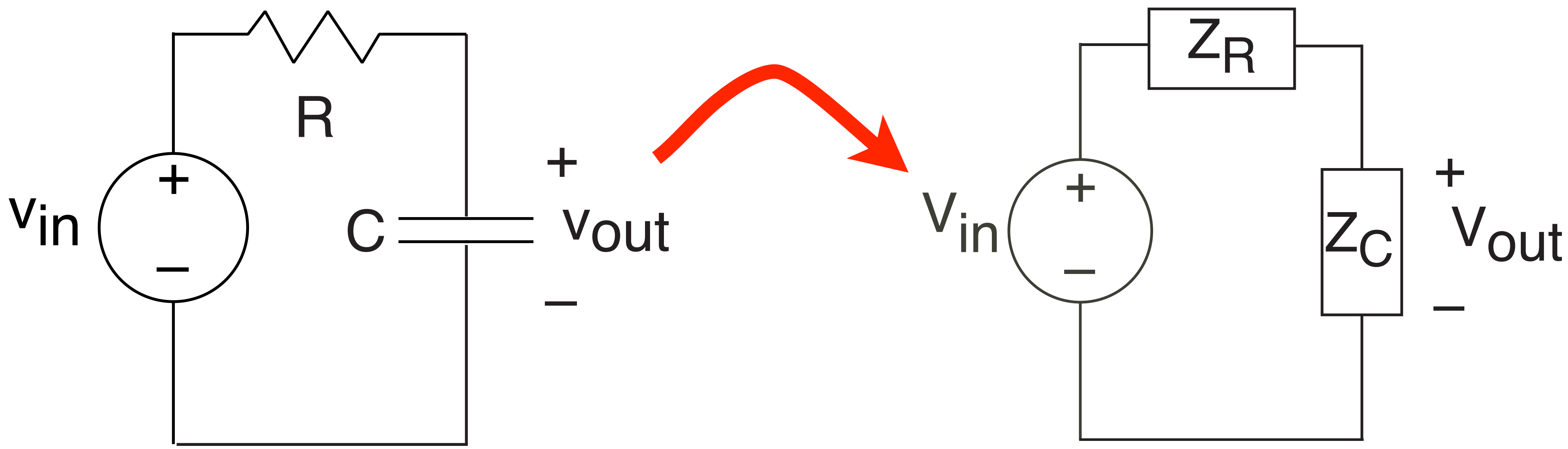


Can now use voltage divider!

$$\begin{aligned} V_{\text{out}} &= \frac{Z_C}{Z_R + Z_C} V_{\text{in}} \\ &= \frac{\frac{1}{j2\pi fC}}{R + \frac{1}{j2\pi fC}} V_{\text{in}} \\ &= \frac{1}{j2\pi fRC + 1} V_{\text{in}} \end{aligned}$$

RICE

Circuits Built from Impedances



$$V_{\text{out}} = \frac{1}{j2\pi f RC + 1} V_{\text{in}}$$

Using Impedances

- When the circuit consists of sources and any number of resistors, capacitors and inductors…
- Pretend the sources are complex exponentials having a frequency f
- Consider each element an impedance

element	impedance
R	R
C	$\dfrac{1}{j2\pi fC}$
L	$j2\pi fL$

- Use voltage divider, current divider, series/parallel rules to relate output variable's complex amplitude to the complex amplitude of the source

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