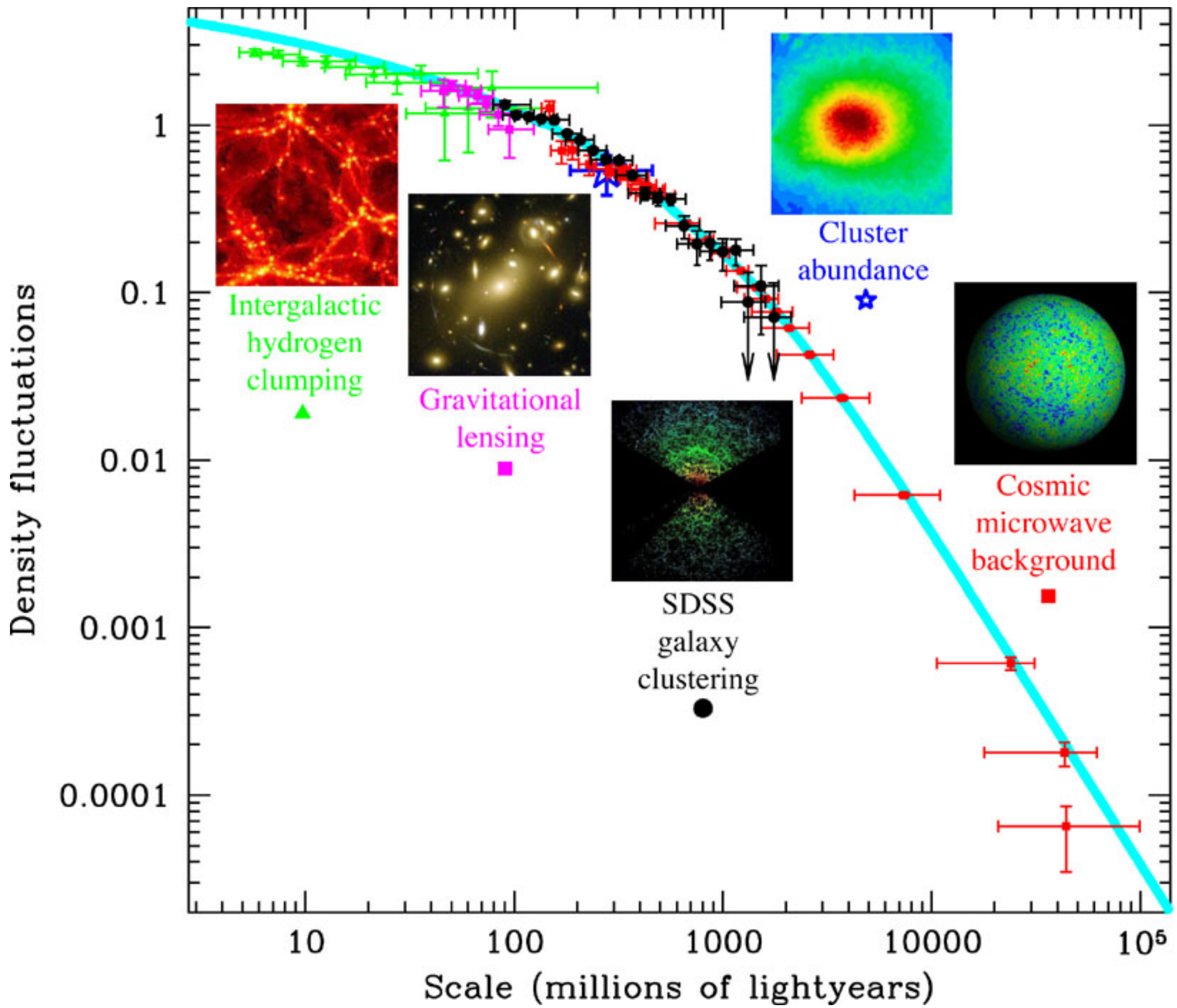
Large Scale Structure: Power Spectrum



Correlation Function and Power Spectrum

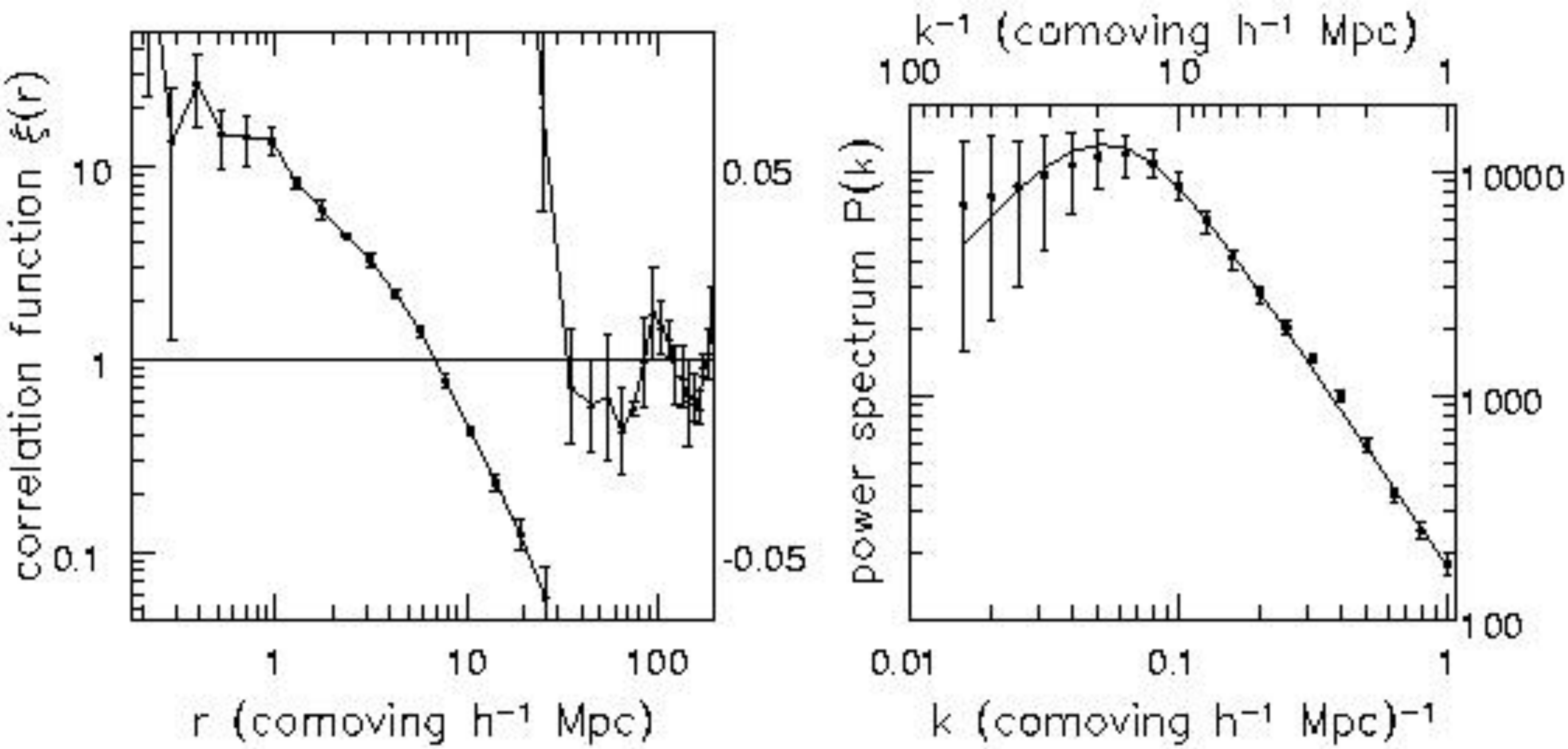
- Given the overdensity field $\delta(\mathbf{x}) = \frac{n(\mathbf{x})}{\langle n \rangle} - 1$
- Its Fourier transform is $\delta(\mathbf{x}) = \frac{1}{(2\pi)^3} \int d^3\mathbf{k}\, e^{i\mathbf{kx}} \delta(\mathbf{k})$
- Its inverse transform is $\delta(\mathbf{k}) = \int d^3\mathbf{x}\, e^{-i\mathbf{kx}} \delta(\mathbf{x})$

  where $k = \frac{2\pi}{\lambda}$ is the wave number
- The power spectrum is $P(\mathbf{k}) = \left|\delta(\mathbf{k})\right|^2$
- Then

$$\xi(r) = \frac{1}{4\pi^2} \int d\ln k\, j_0(kr) \left[k^3 P(k)\right]$$

Correlation function and power spectrum are a Fourier pair

An Example from Las Campanas Redshift Survey



Correlation function is easier to evaluate, but power spectra is what we need to compare with the theory

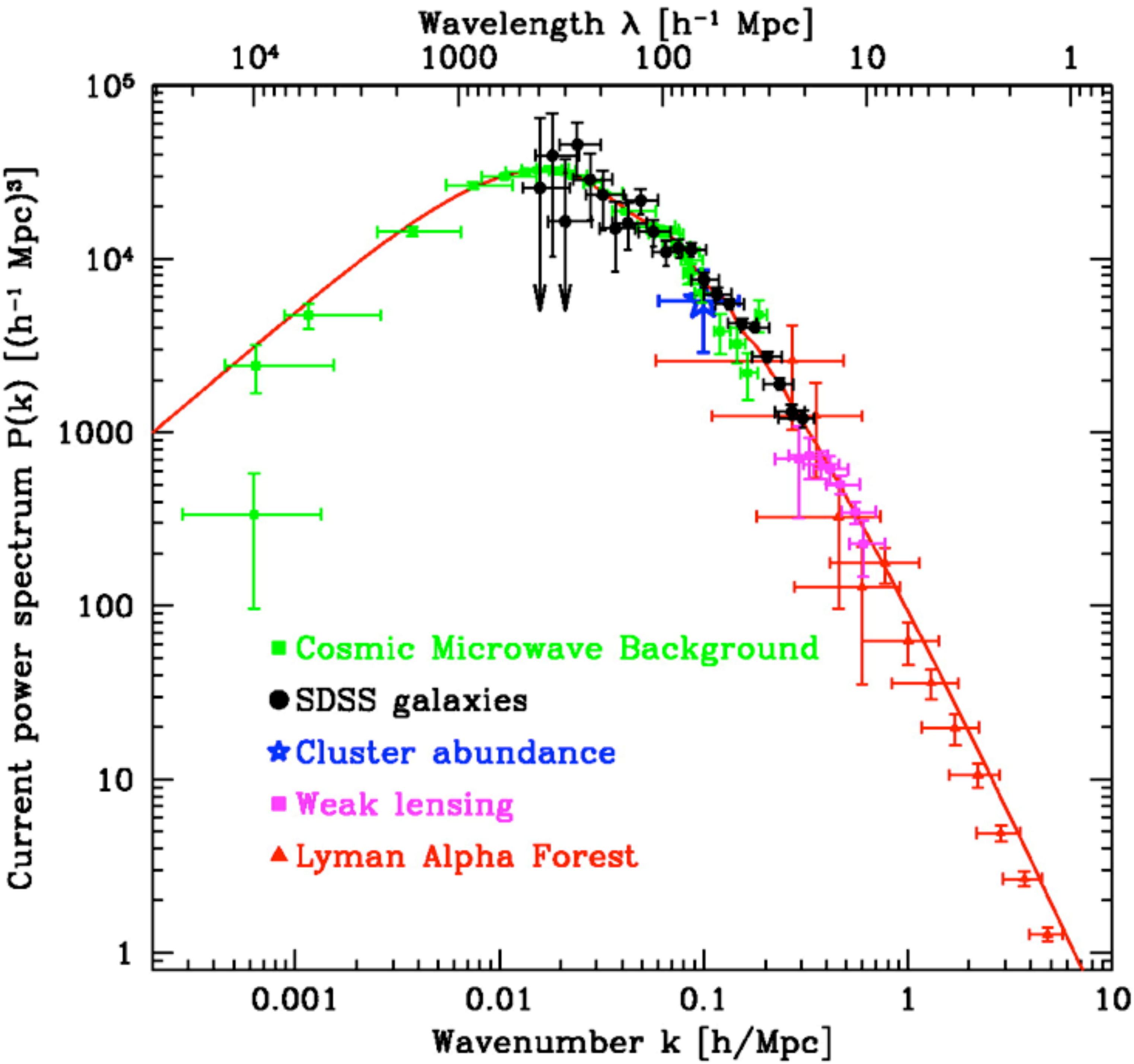
Normalizing the Power Spectrum

- Define σ_R as the r.m.s. of mass fluctuations on the scale R
- Typically a sphere with a radius $R = 8\ h^{-1}$ Mpc is used, as it gives $\sigma \approx 1$
- So, the amplitude of $P(k)$ is ~ 1 at $k = 2\pi / (8\ h^{-1}$ Mpc)
- This is often used to normalize the spectrum of the PDF
- Mathematically, where K_R is a convolving kernel, a spherical top-hat with a radius R:

$$\sigma_R^{\ 2} = \frac{1}{4\pi^2} \int d \ln k \left[k^3 P(k) \left| K_R(k) \right|^2 \right]$$

$$K_R(r) = \begin{cases} 1, & \text{if } r < R \\ 0, & \text{if } r \geq R \end{cases} \qquad K_R(k) = \left[\frac{j_1(kr)}{kr} \right]$$

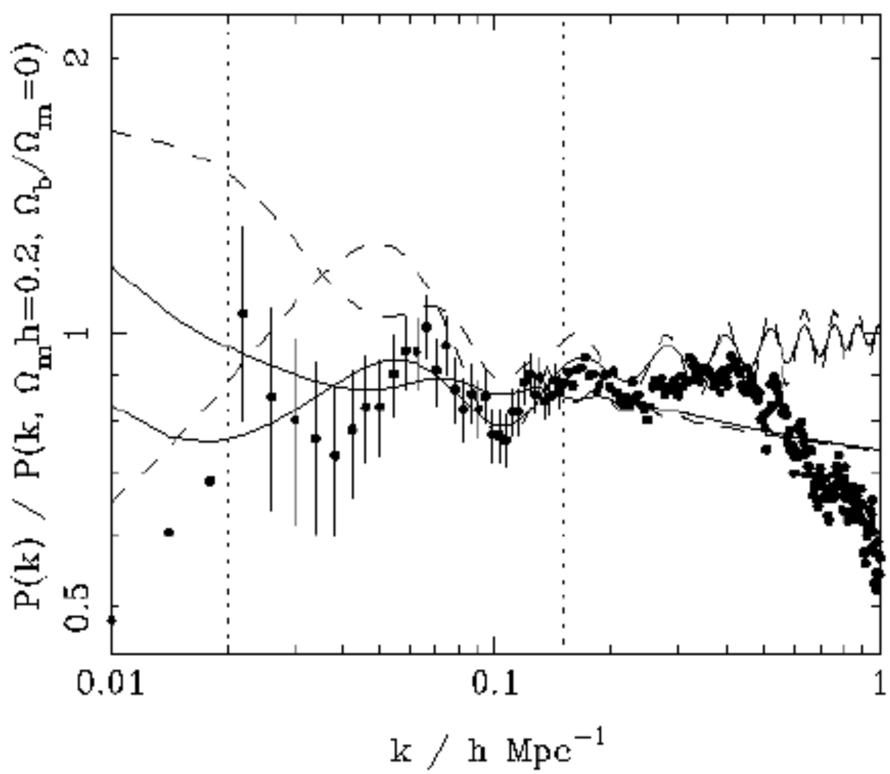
The Observed Power Spectrum



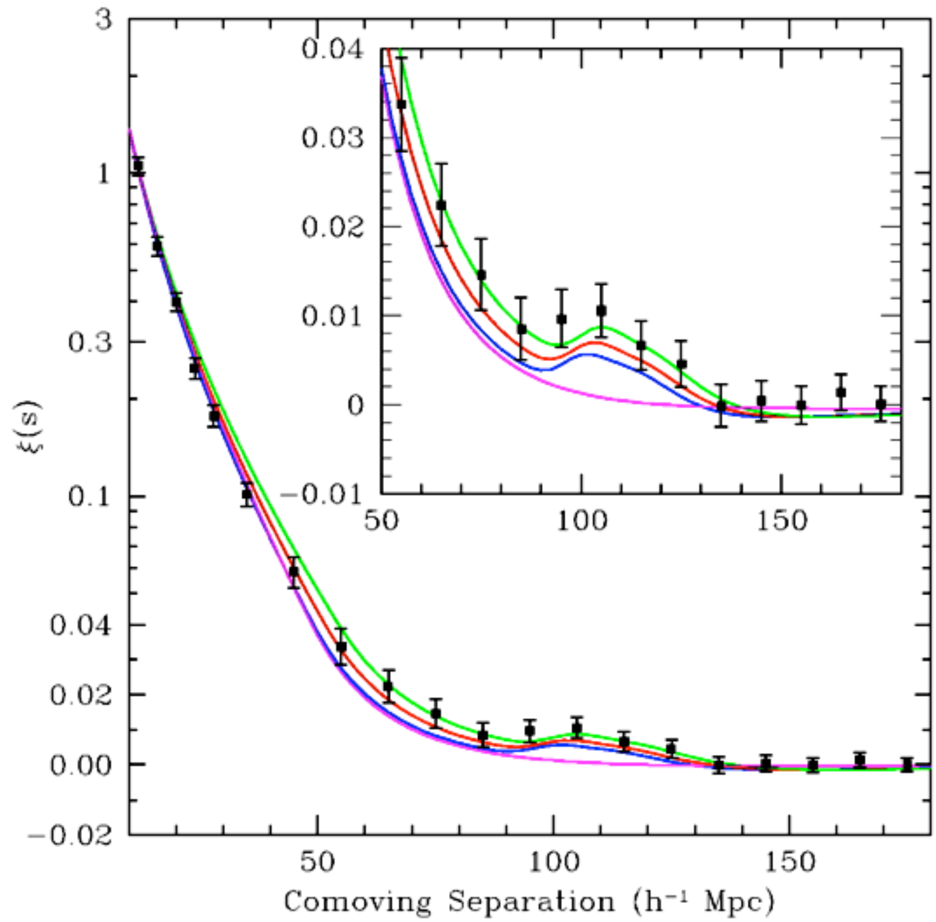
(Tegmark et al.)

Baryonic oscillations seen in the CMBR are detected in the LSS at lower redshifts

Thus, we can use the first peak as a stardard ruler at more than one redshift



2dF (Percival et al.)

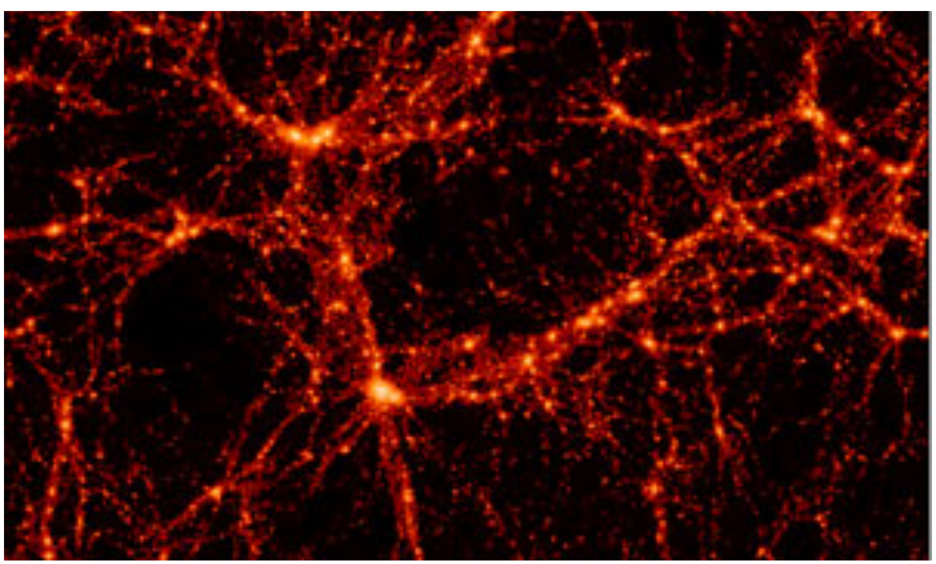


SDSS (Eisenstein et al.)

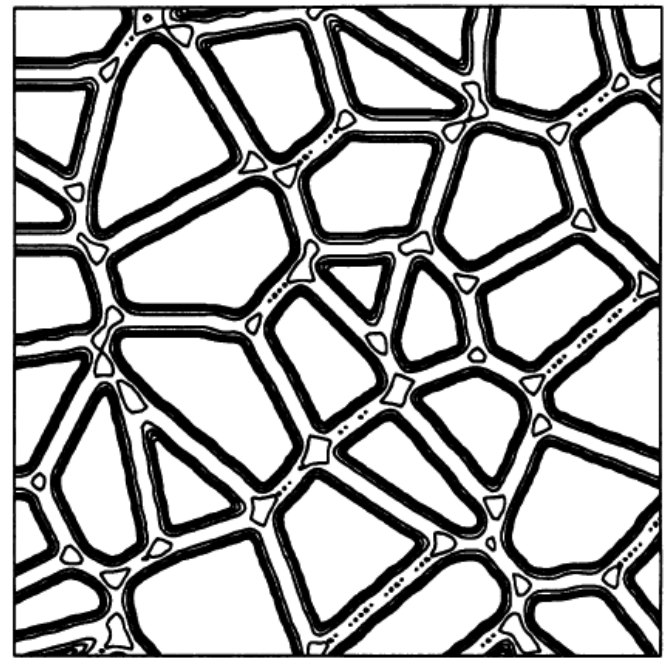
Is the Power Spectrum Enough?

These two images have ***identical power spectra*** (by construction)

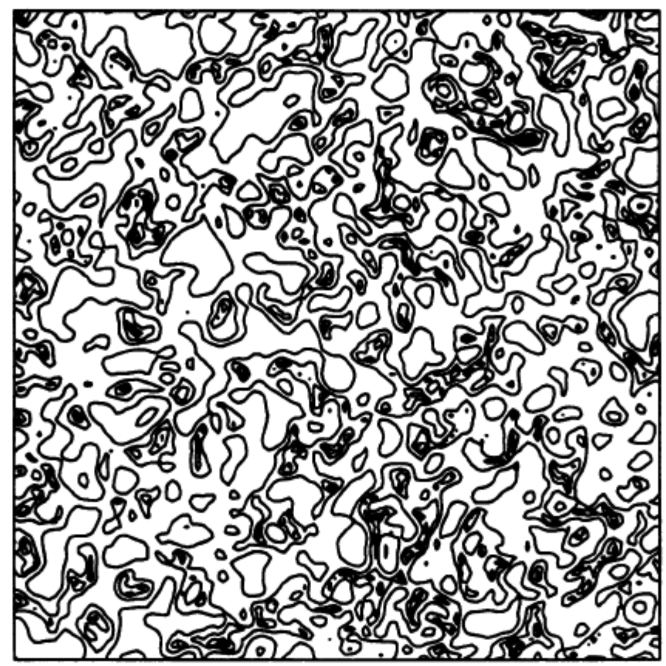
The power spectrum alone does not capture the phase information: the coherence of cosmic structures (voids, walls, filaments …)



Voronoi foam, R=1.6, smoothed original

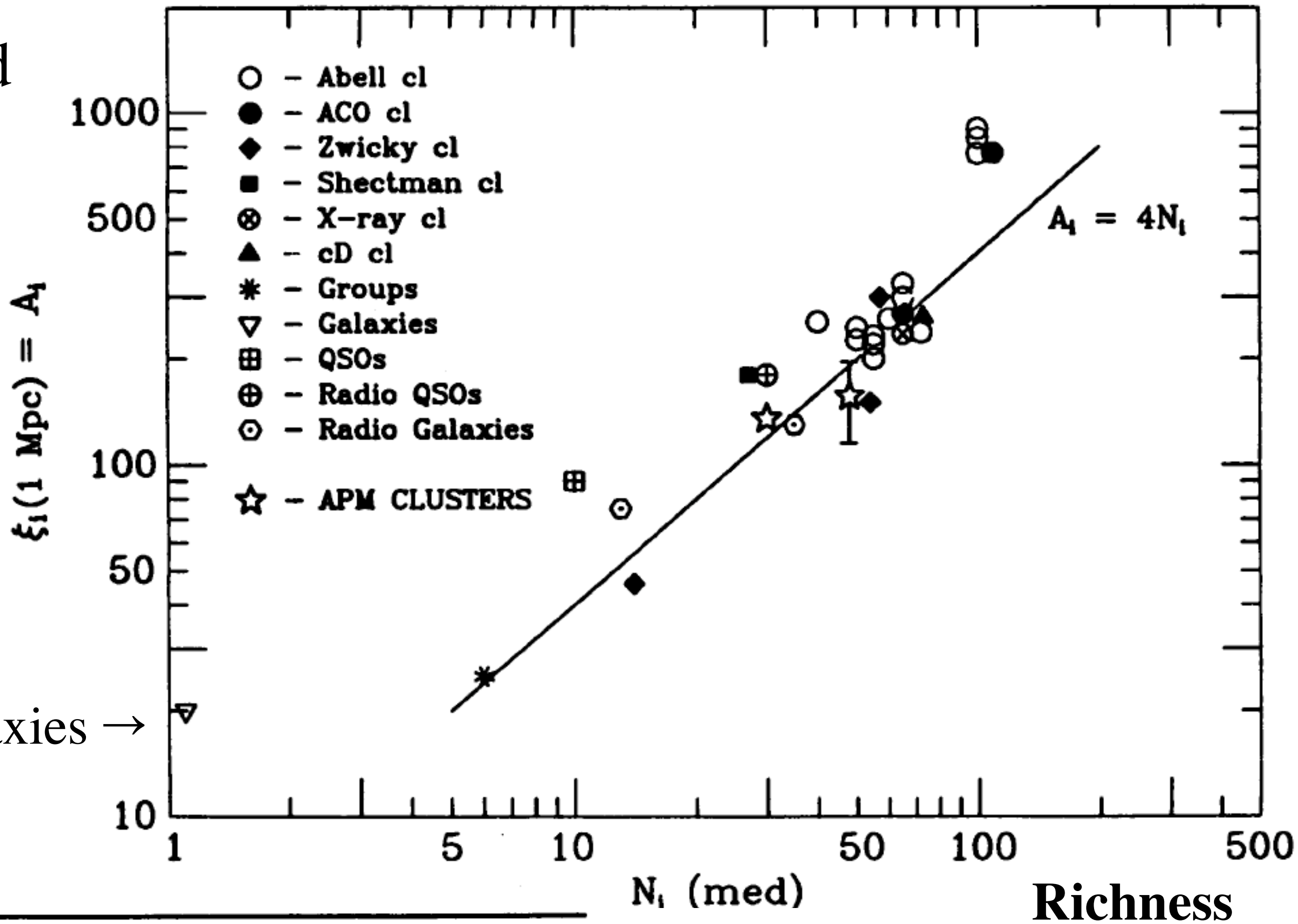


Voronoi foam, R=1.6, random phases



Cluster-Cluster Clustering

Clusters are clustered more strongly than individual galaxies, and rich ones more than the poor ones



Field galaxies →

Richness

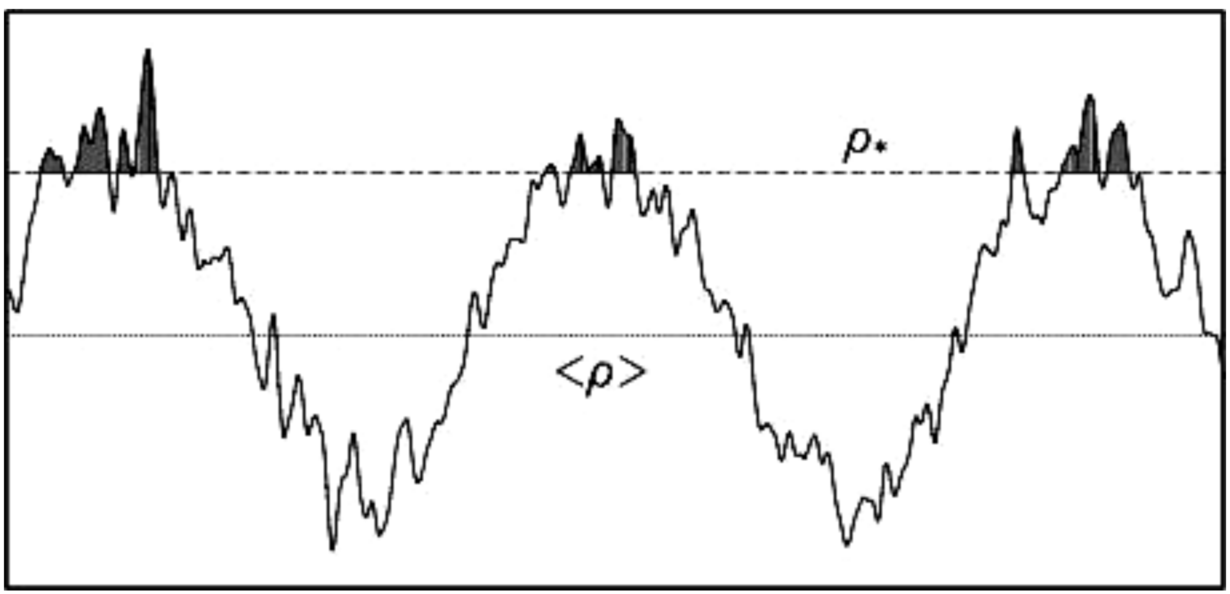
Table 1. Cluster correlations

Catalog	n_c, h^3 Mpc^{-3}	d, h^{-1} Mpc	r_o(obs)	$r_o = 0.4d$
Abell, $R \geq 2$	1.2×10^{-6}	94	42 ± 10	37.6
Abell, $R \geq 1$	6×10^{-6}	55	22 ± 3	22.0
EDCC	15×10^{-6}	40.5	16 ± 4	16.2
APM	24×10^{-6}	34.7	13 ± 2	13.9

(from N. Bahcall)

Clustering of Different Structures

- The more massive systems (e.g., elliptical vs. spiral galaxies; groups and clusters of increasing richness) cluster more strongly
- They correspond to increasingly higher peaks of the density field, and thus increasingly rare
- This can be naturally explained with the concept of ***biasing***



LSS Observations Summary

- A range of structures: galaxies (~10 kpc), groups (~ 0.3 - 1 Mpc), clusters (~ few Mpc), superclusters (~ 10 - 100 Mpc)
- Redshift surveys are used to map LSS; ~ 10^6 galaxies now
- LSS topology is prominent: voids, sheets, filaments…
- LSS quantified through 2-point (and higher) correlation function(s), well fit by a power-law:

  $$\xi(r) = (r / r_0)^{-\gamma}$$

  typical $\gamma \sim 1.8$, $r_0 \sim$ few Mpc
- Equivalent description: power spectrum *P(k)* - useful for comparisons with the theory
- CDM model fits the data over a very broad range of scales
- Objects of different types have different clustering strengths
- Generally more massive structure cluster more strongly

Next: Peculiar Velocities