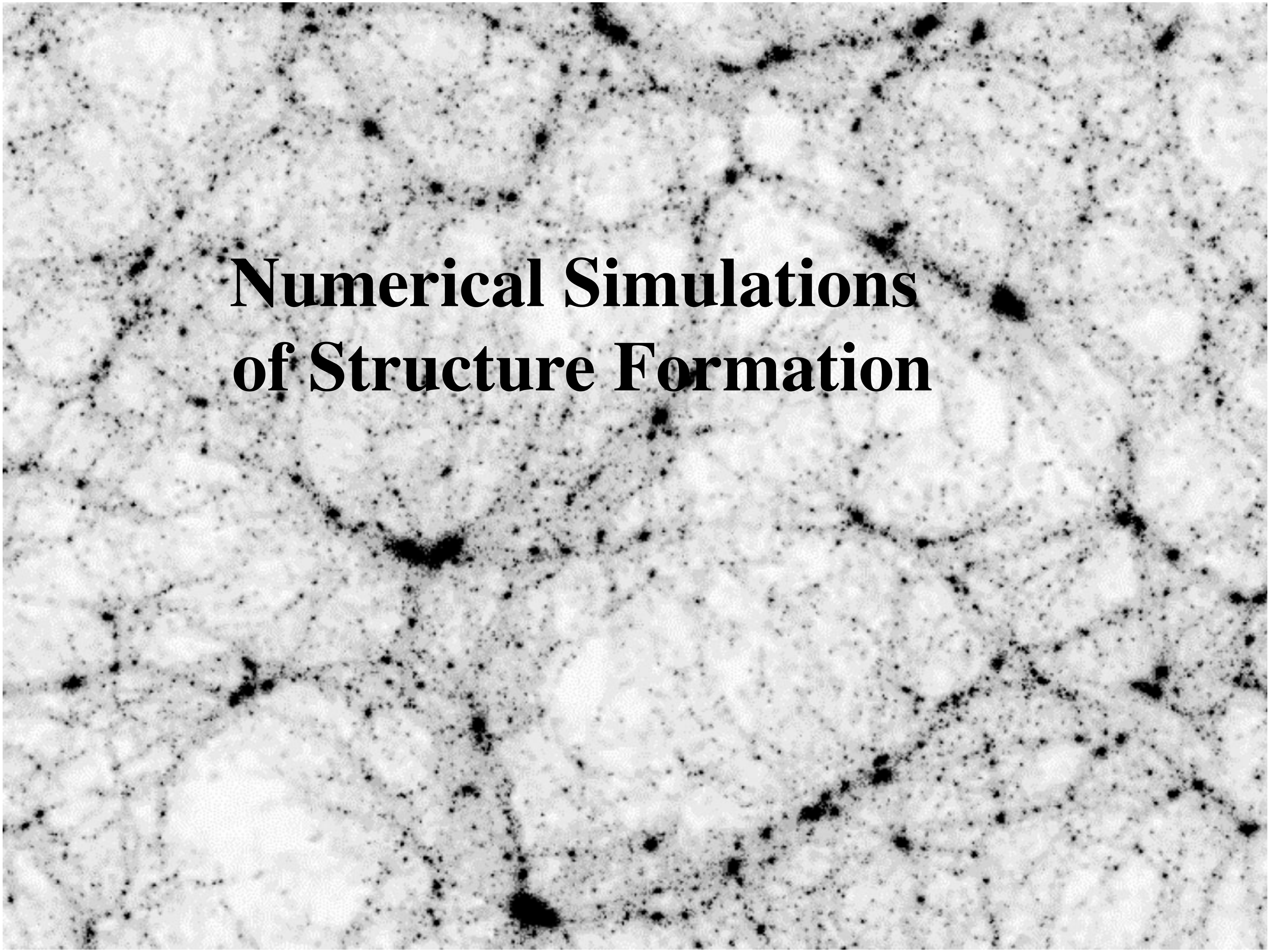
Numerical Simulations of Structure Formation



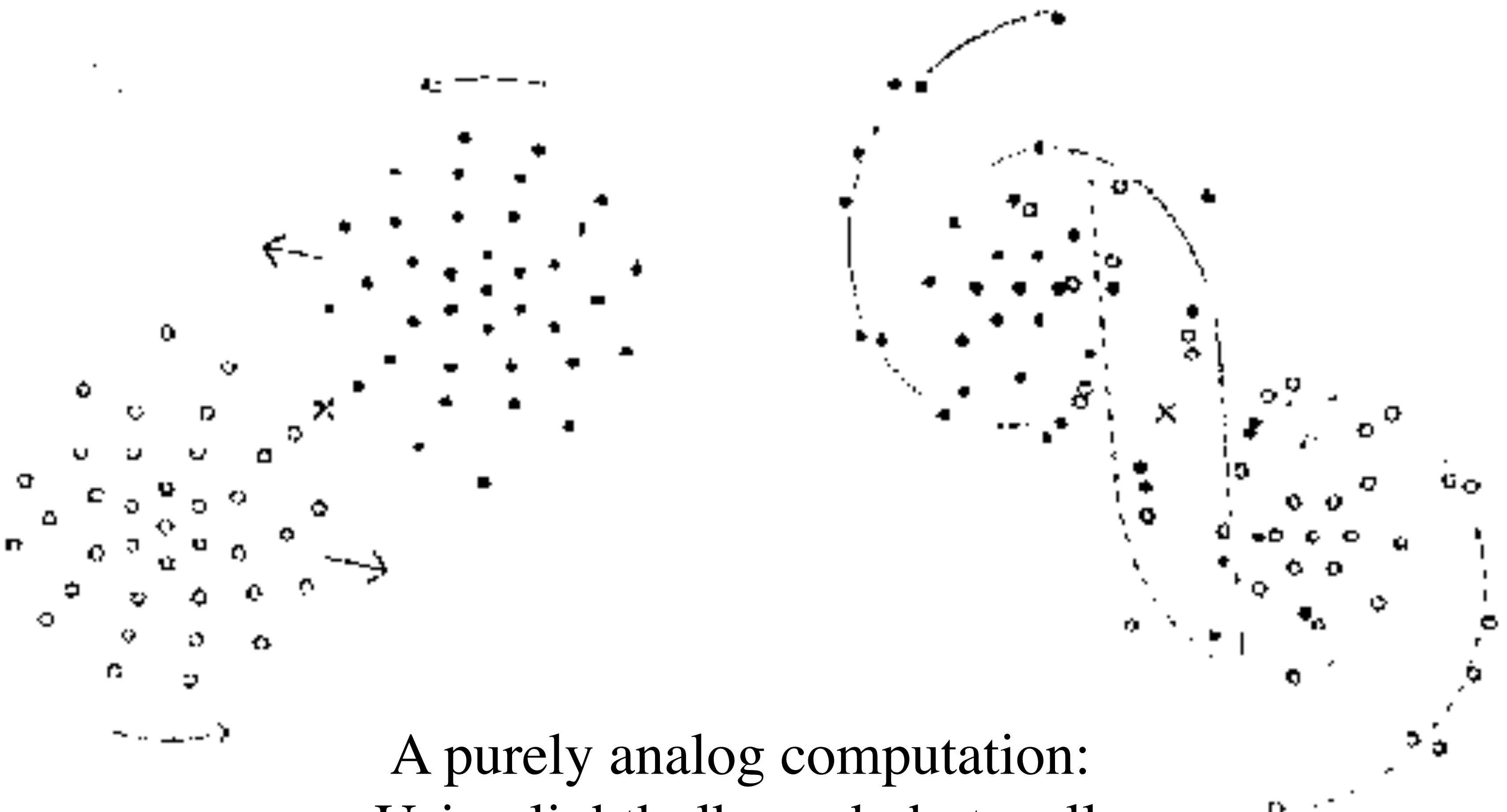
Numerical Simulations in Cosmology

- Many problems in astrophysics lack the symmetries that generally allow analytic solution. A full 3D numerical treatment is required
- For the Newtonian gravity, there are no analytical solutions beyond the 2-body (Kepler) problem
- Numerical simulations thus play a key role in cosmology
- In cosmology know both the governing equations (Newtonian gravity, cosmological expansion) *and* can assume the initial conditions (primordial density field)
- Pure gravity simulations are dissipationless; adding gas and radiation makes the problem much harder, due to dissipation, hydrodynamical effects, radiation transfer…
- The output of simulations can be compared to observations

The first N-body Simulations

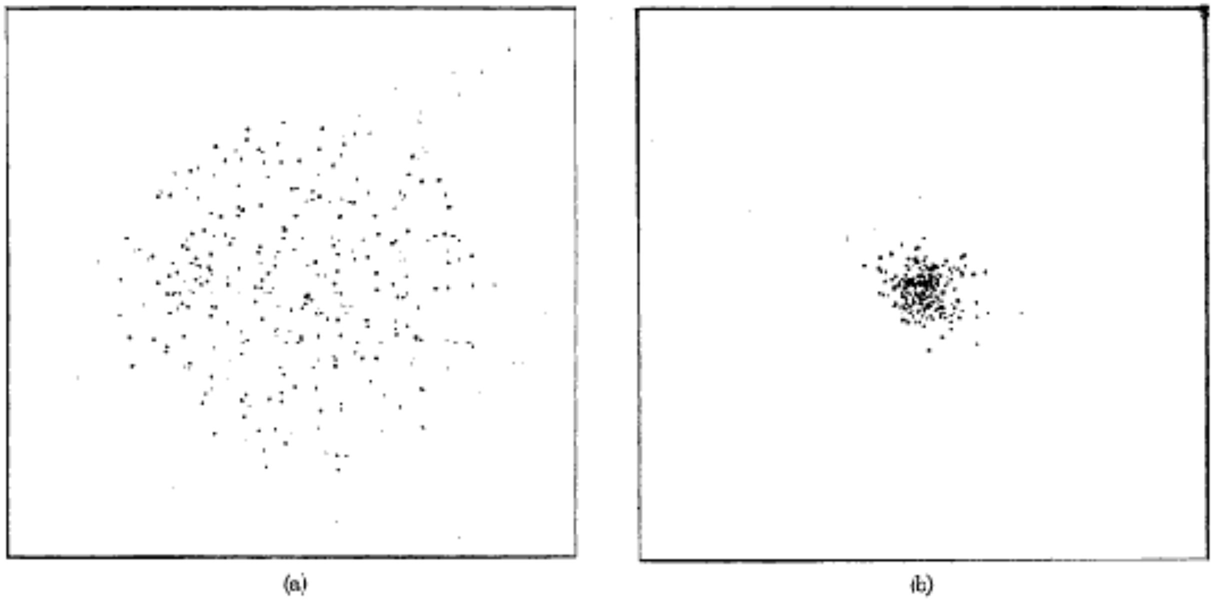
Erik Holmberg (1941)

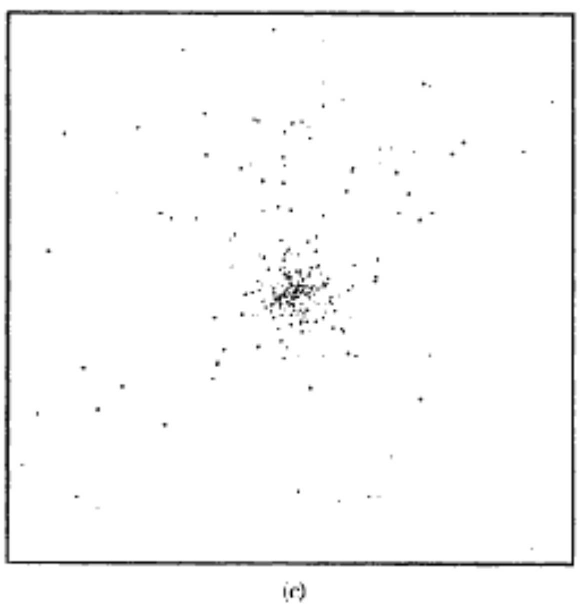
2D, N=74



A purely analog computation:
Using lightbulbs and photocells,
substituting light for gravity (both follow the inverse square law)

And Then With Computers



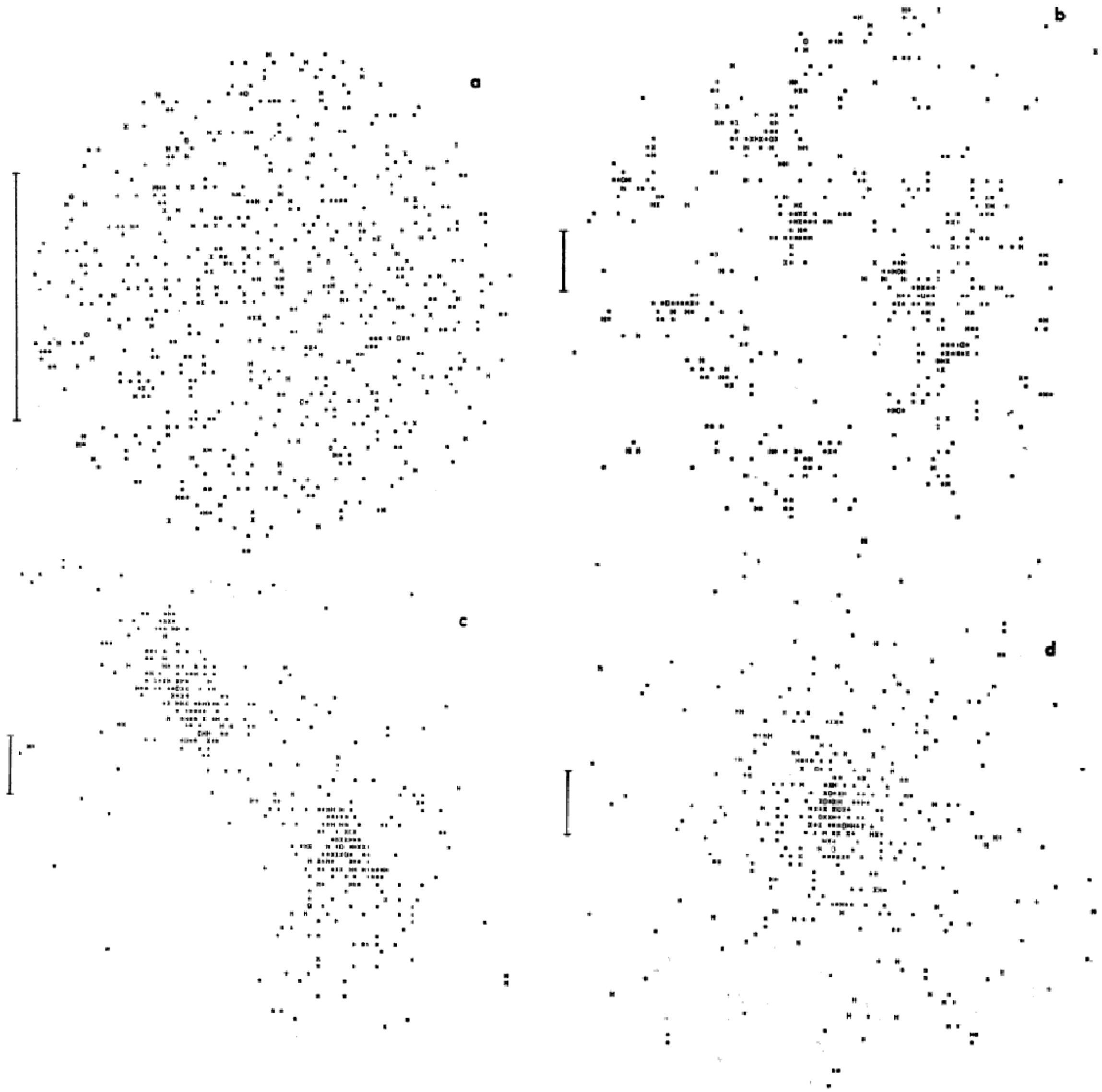


Jim Peebles (1970)

FIG. 1. Model 1 positions: (a) initial positions, $t=2.8$ b.y.; (b) $t=5.6$ b.y.; and (c) $t=8.4$ b.y.

N=300 (3D)

… And With Realistic Initial Conditions:



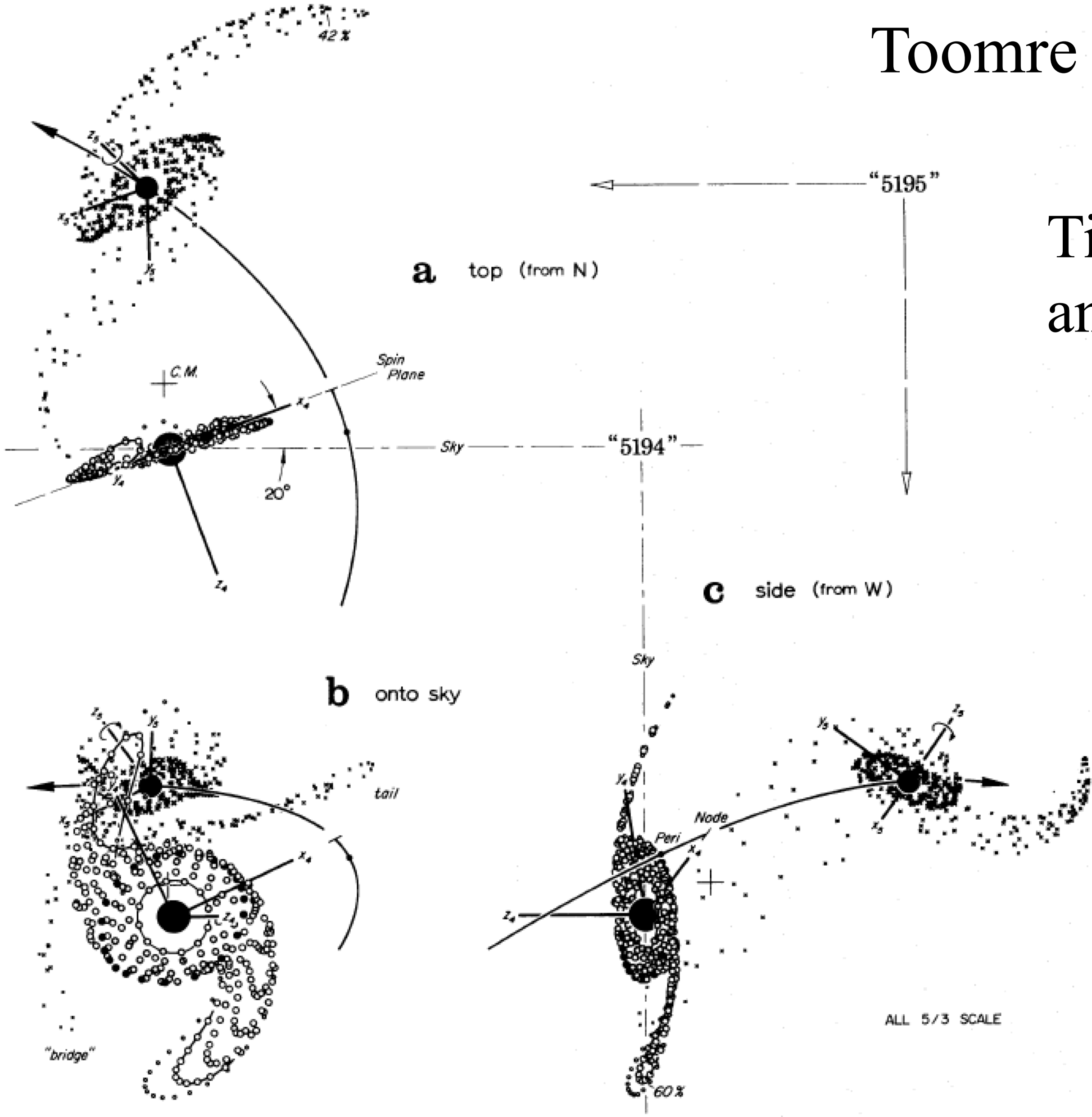
Simon White (1976)

N=700 (3D)

Galaxy Encounters and Mergers

Toomre & Toomre (1972)

Tidal tails, bridges, and other distortions



We are using simulations to:

- Understand the evolution of galaxy clustering
  - Compare with redshift surveys
  - Mock galaxy catalogues, galaxy formation models
- Make predictions for weak lensing and SZ surveys
  - Model non-linear evolution of DM
  - Simulated observations for experiments
  - Basic theory
- Quantify merger rates of galaxies and black holes
  - Main gravity wave source for LIGO, LISA, …
- Elucidate the structure of our dark matter halos
  - Direct detection experiments
- Study small-scale structure (Ly-α forest)
  - Strong limits on DM properties (& dark energy?)

Types of Physics Included:

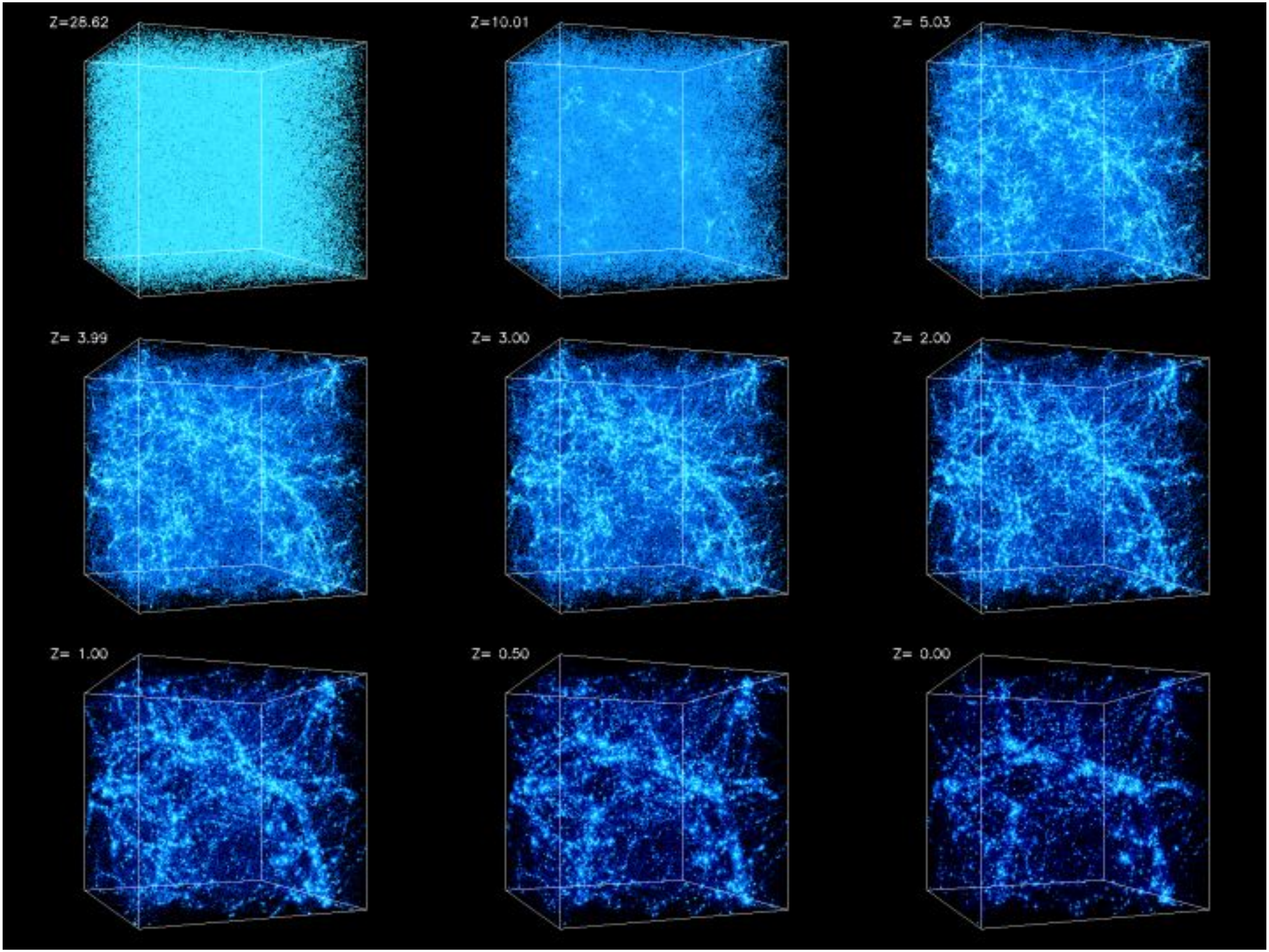
- Gravity
  - All cosmology codes have this! Dominates the forces above the scale of galaxies
- Hydrodynamics
  - Adiabatic physics
  - Cooling
  - Star "formation" and feedback
- Magnetic fields (MHD)
  - Not so common in cosmology, but may be important
- Radiative transfer
  - Still early days … resolution is a problem (photons are much smaller than galaxies)

Gravity: the N-body Method

- Original N-body work was for stellar systems (a true "N-body problem", not doable analytically)
- Taken over to cosmology – change in emphasis: modelling DM dominated or pure stellar systems as a **fluid**
- This is for dissipationless (i.e., gravity only) interaction: good for DM and stars only, but not for gas
- Various approximations used, and various methods:

Direct summation	$\mathcal{O}(N^2)$ [Special hardware]	Practical for $N<10^4$ [up to 10^7]
Particle mesh	$\mathcal{O}(N \log N)$	Uses FFTs to invert Poisson equation
Tree codes	$\mathcal{O}(N \log N)$	Multipole expansion

(Usually, some combination of these is used)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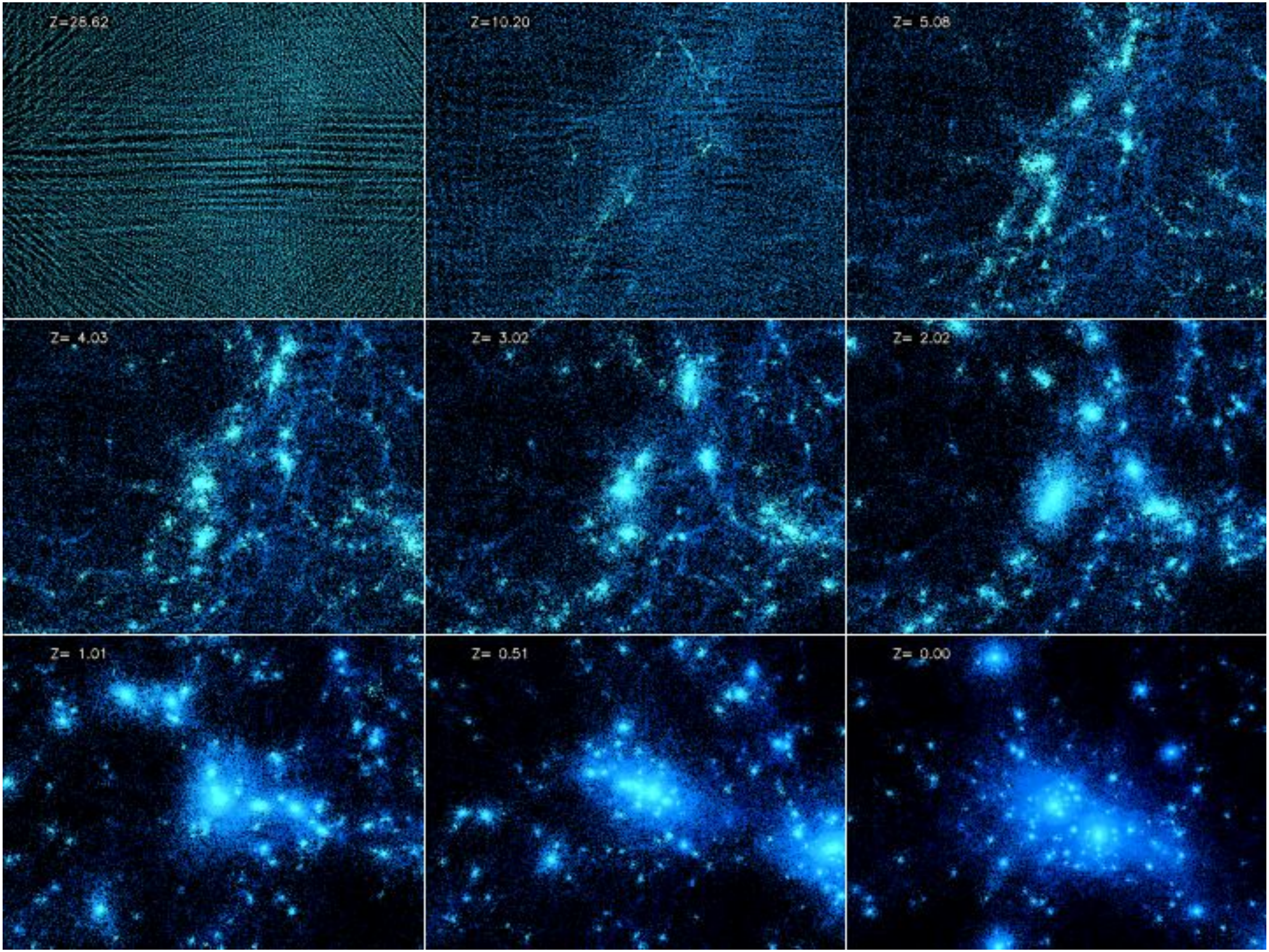
Z= 5.03
Z= 3.99
Z= 3.00
Z= 2.00
Z= 1.00
Z= 0.50
Z= 0.00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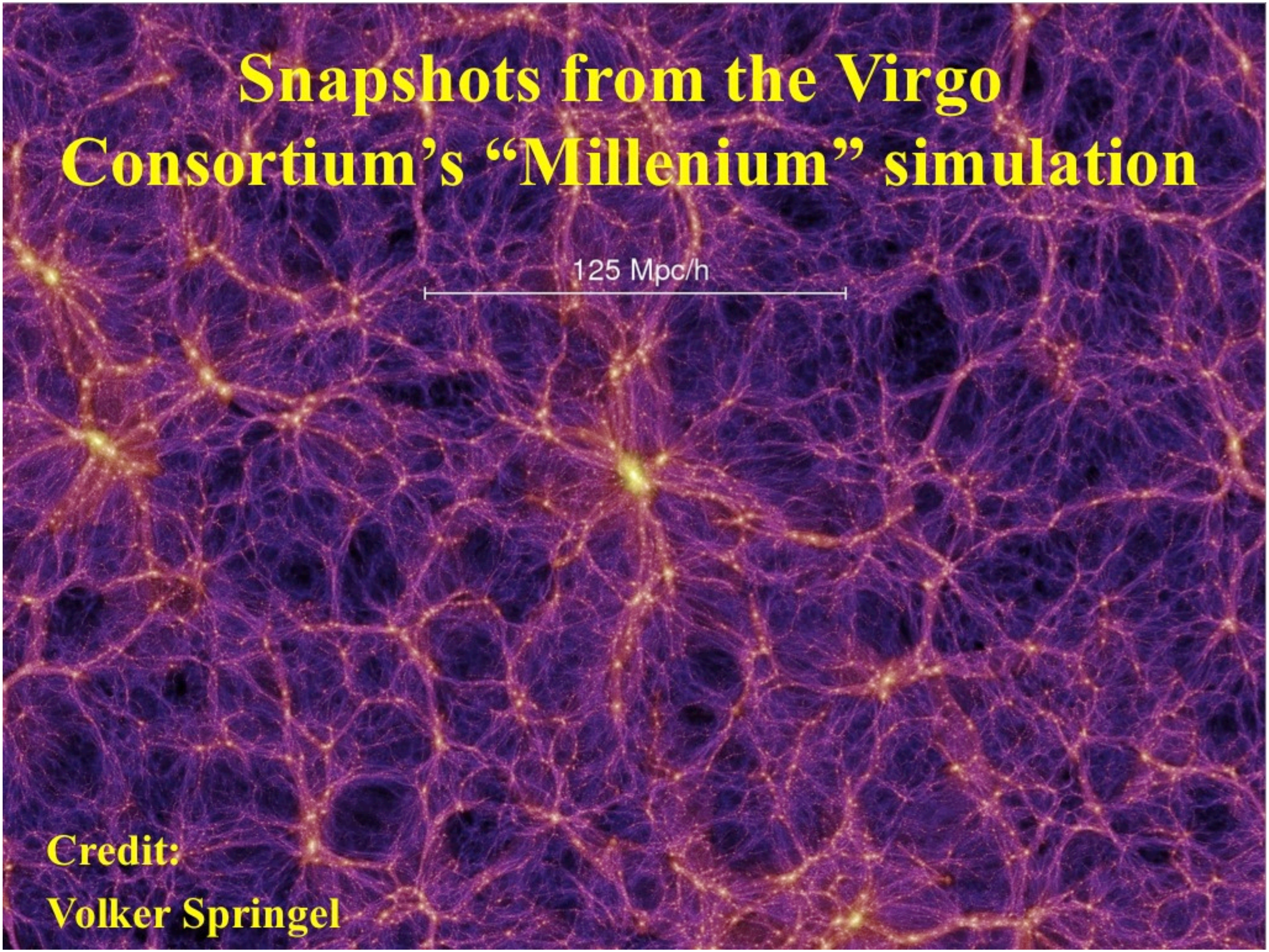
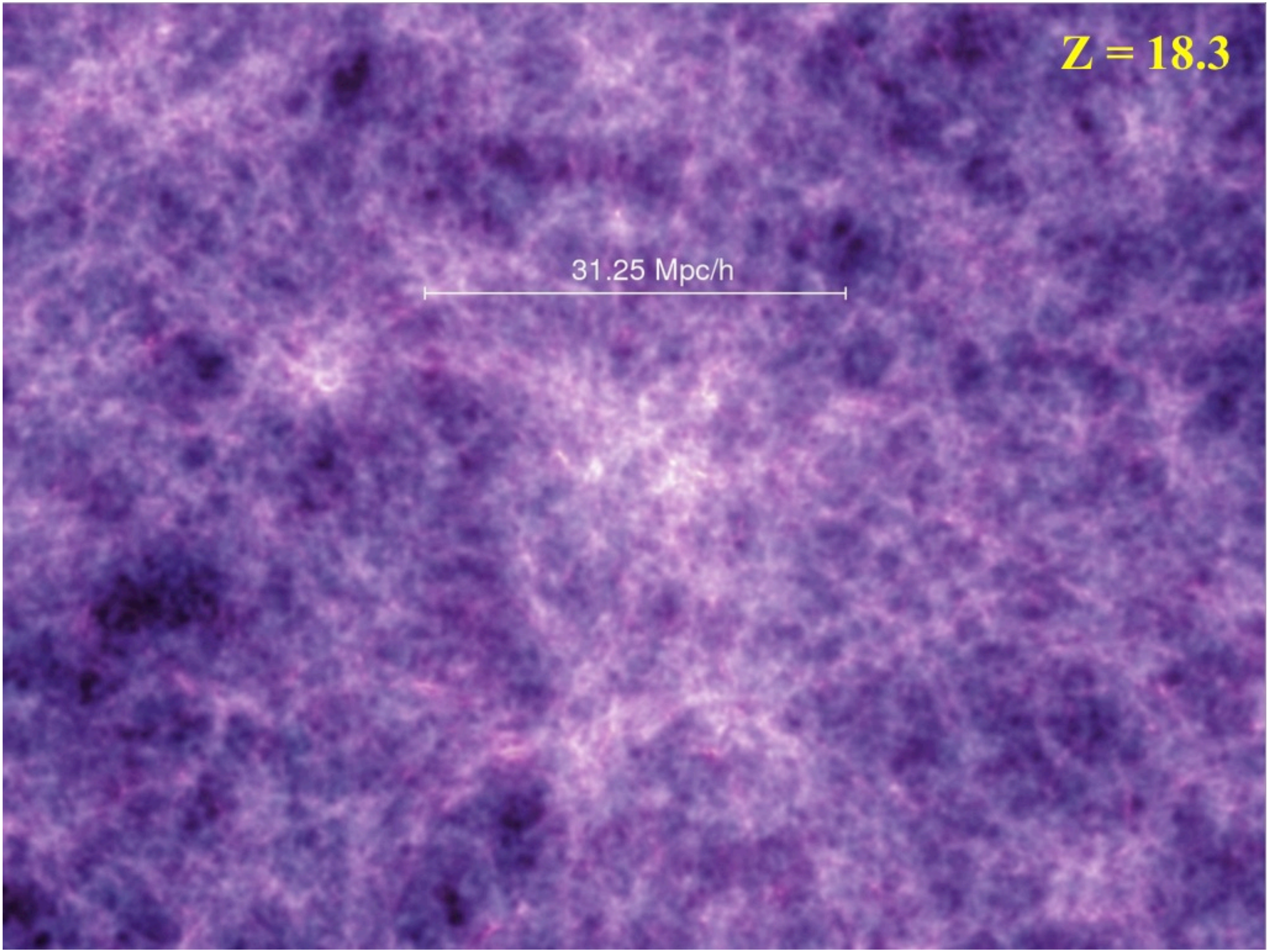
Z=10.20
Z= 5.08
Z= 4.03
Z= 3.02
Z= 2.02
Z= 1.01
Z= 0.51
Z= 0.00

Snapshots from the Virgo Consortium's "Millenium" simulation



Credit:

Volker Springel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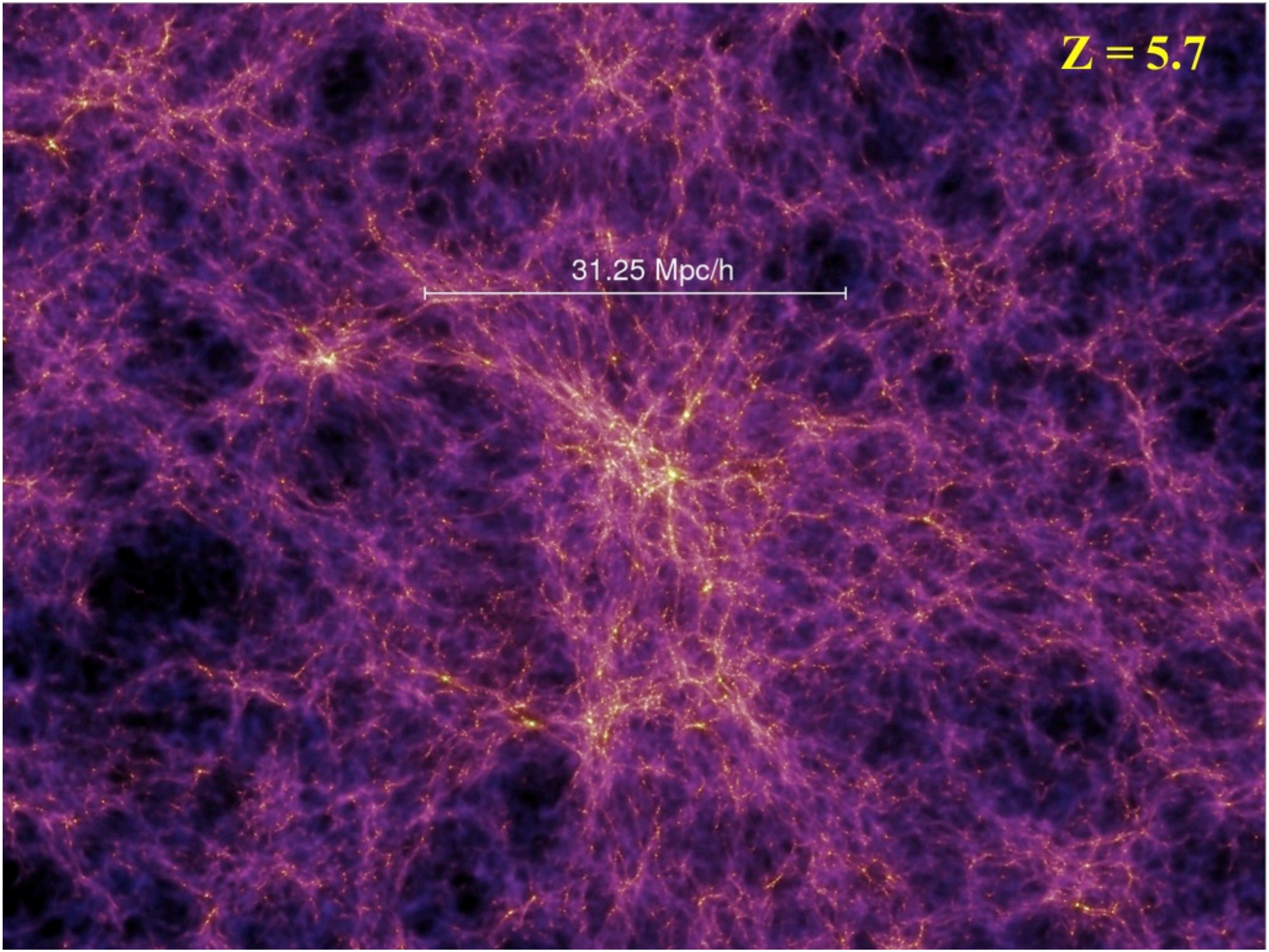
31.25 Mpc/h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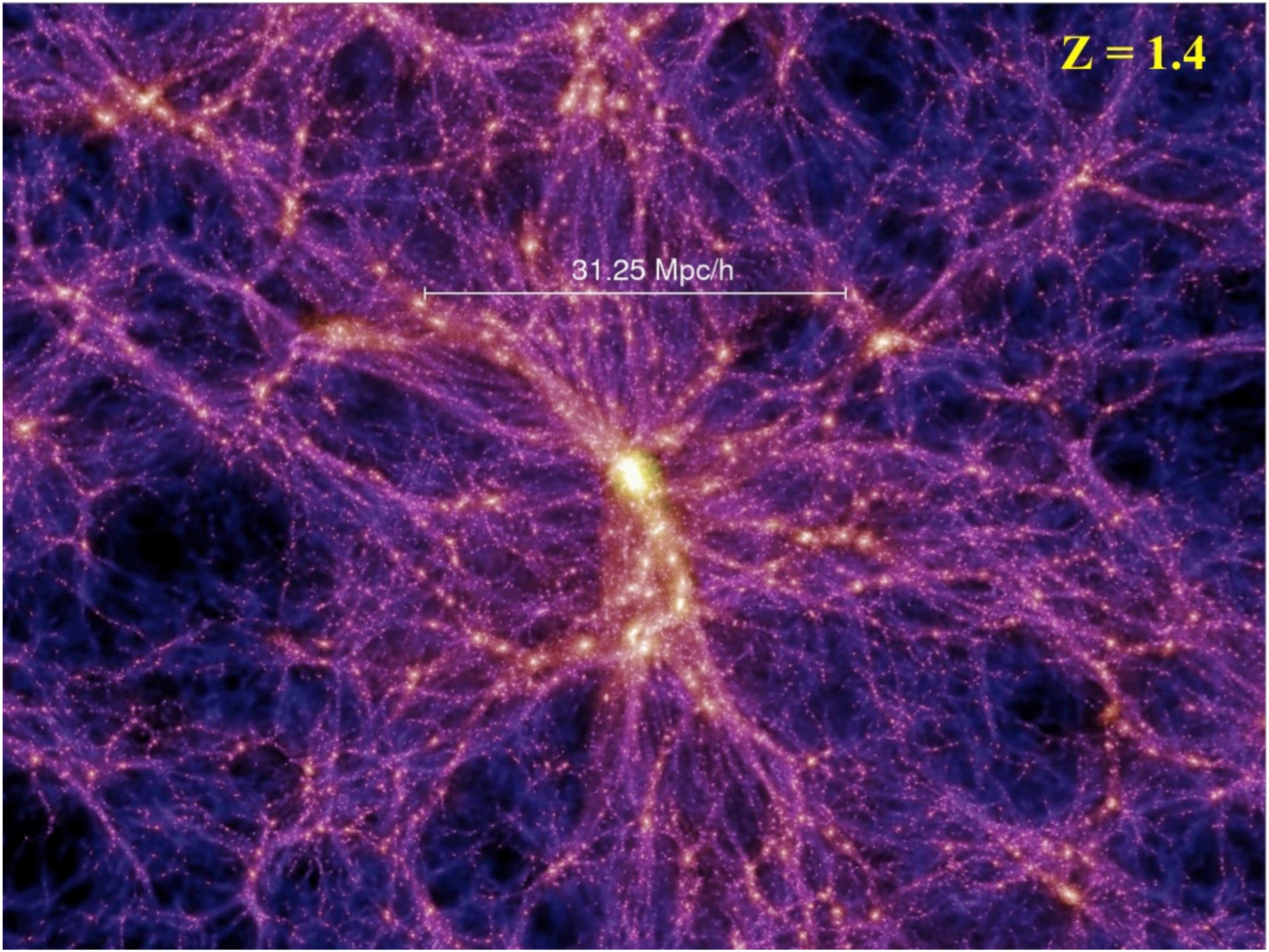
31.25 Mpc/h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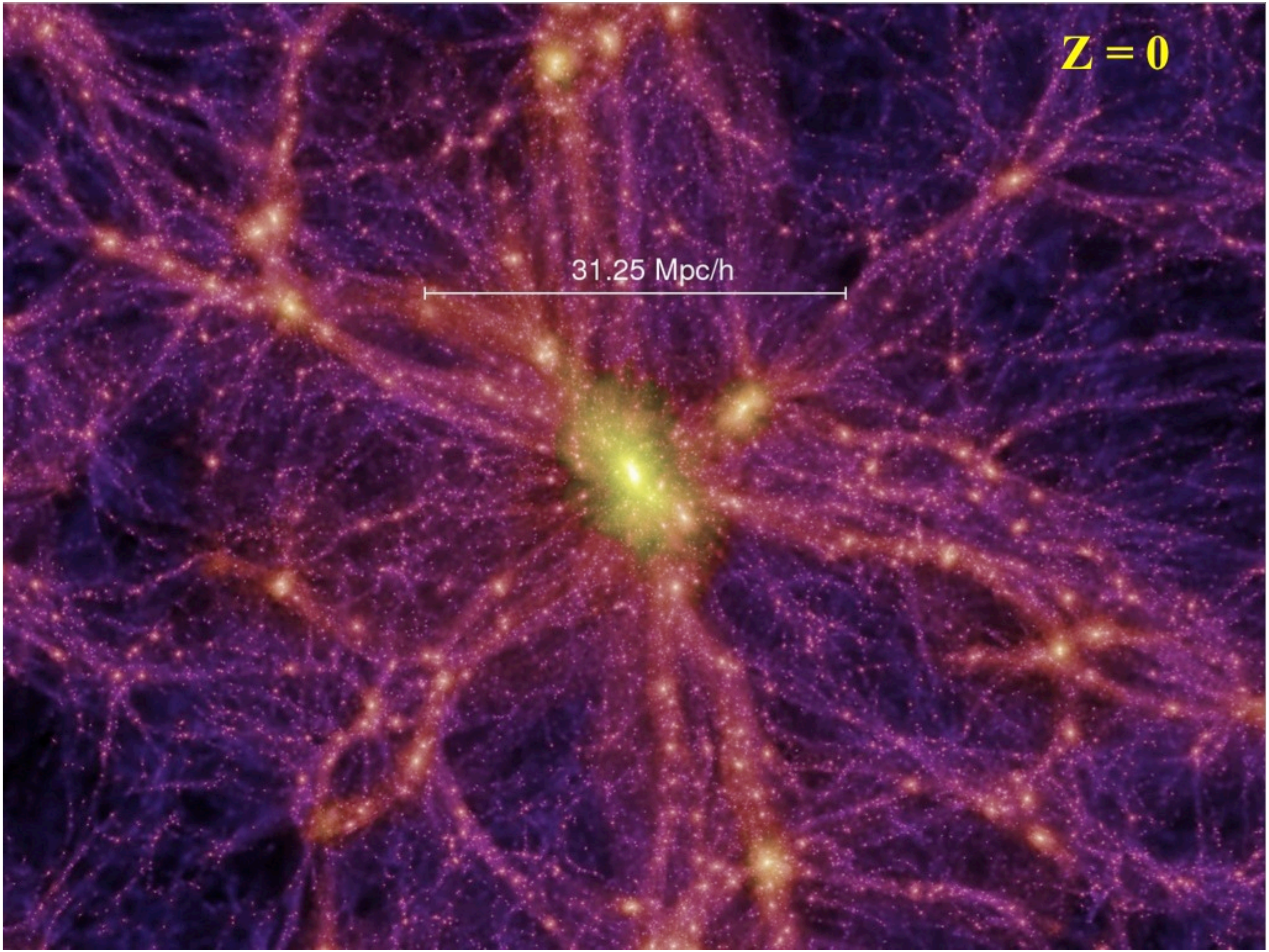
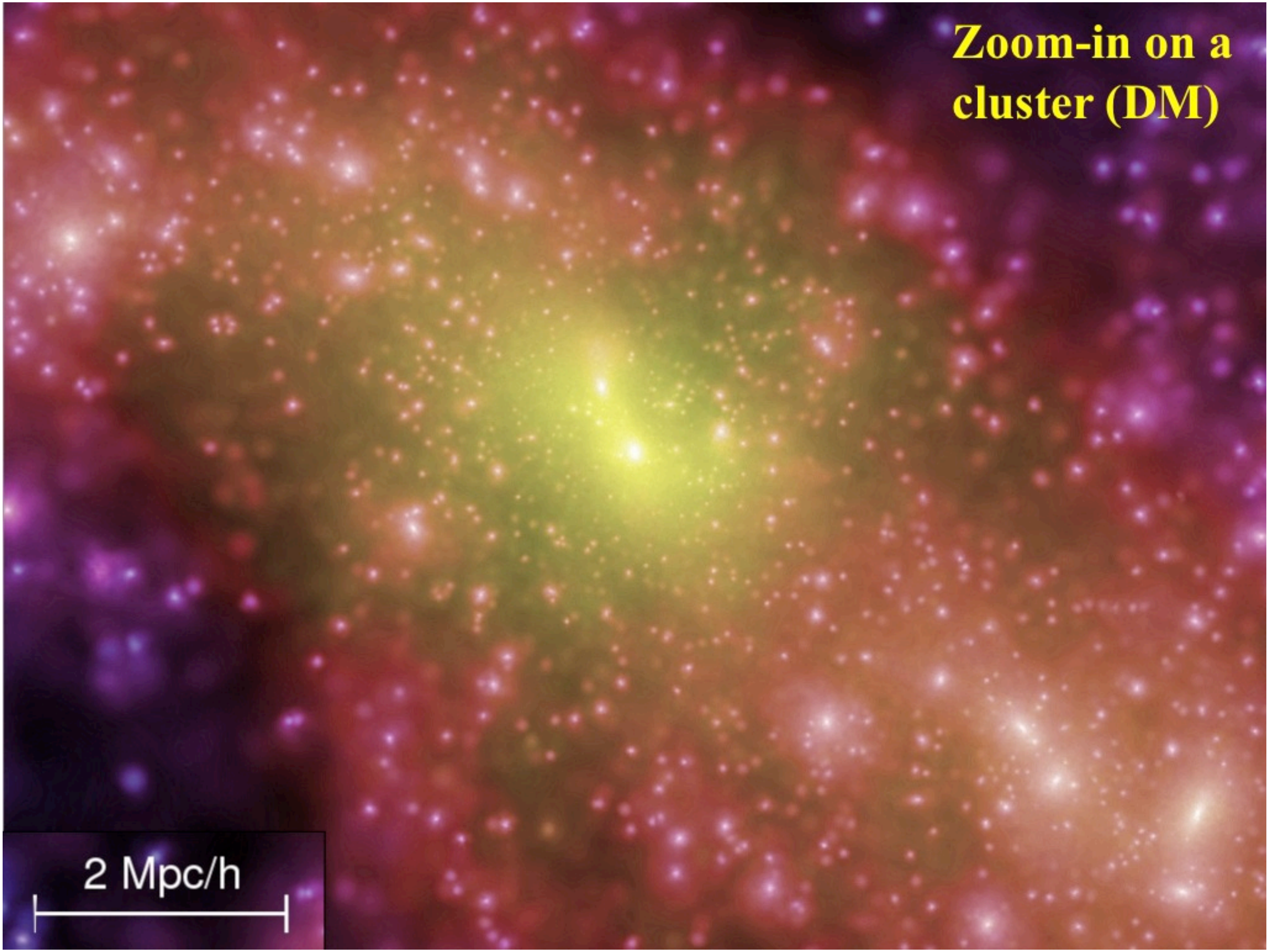
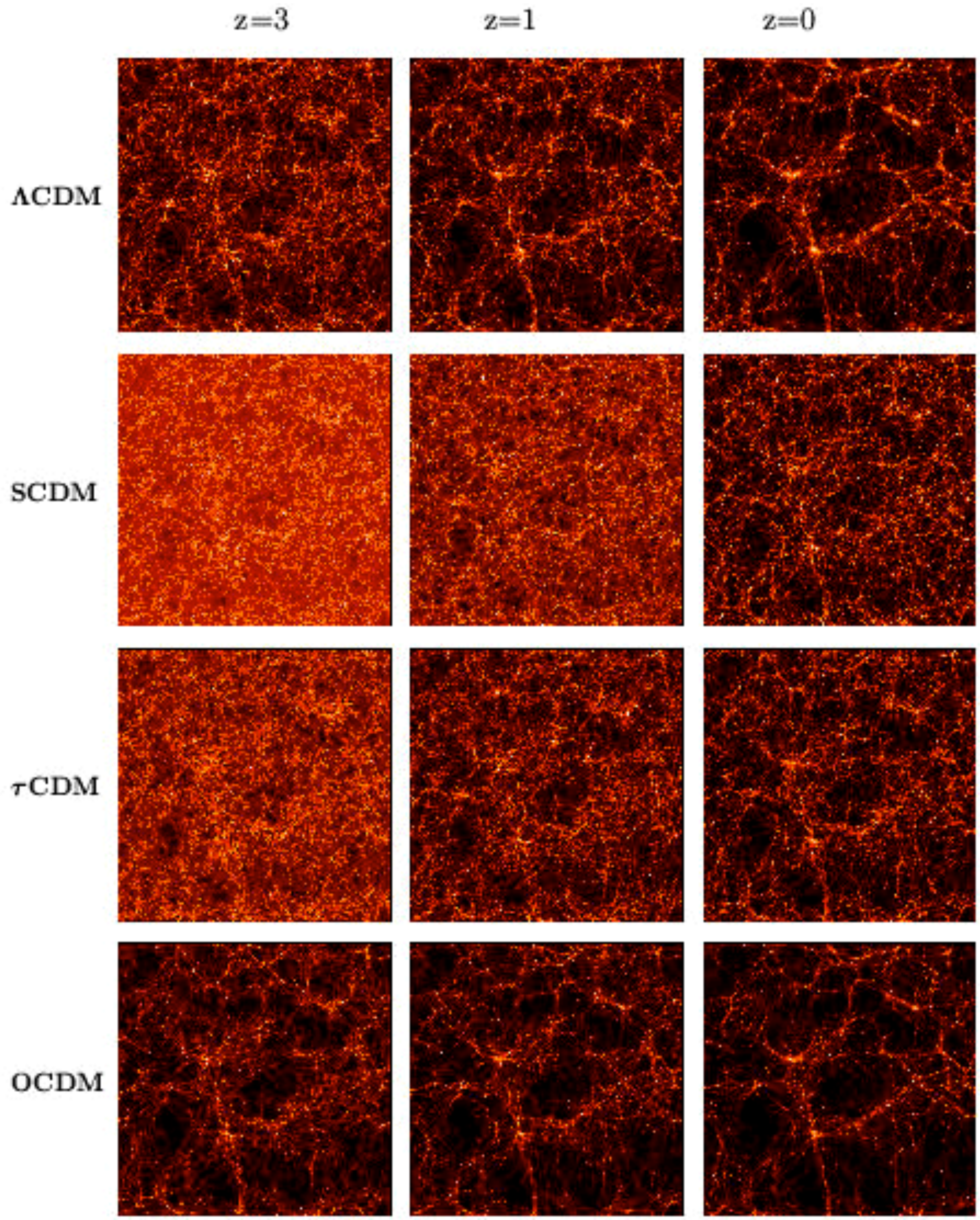
31.25 Mpc/h


Z = 0
31.25 Mpc/h


Zoom-in on a cluster (DM)
2 Mpc/h

Simulations make qualitatively and quantitatively different predictions about the large-scale structure at different redshifts

This can be compared directly with the observations



Merger Tree

At small scales, galaxy merging is important. Each galaxy derives from a hierarchical sequence of mergers of smaller fragments

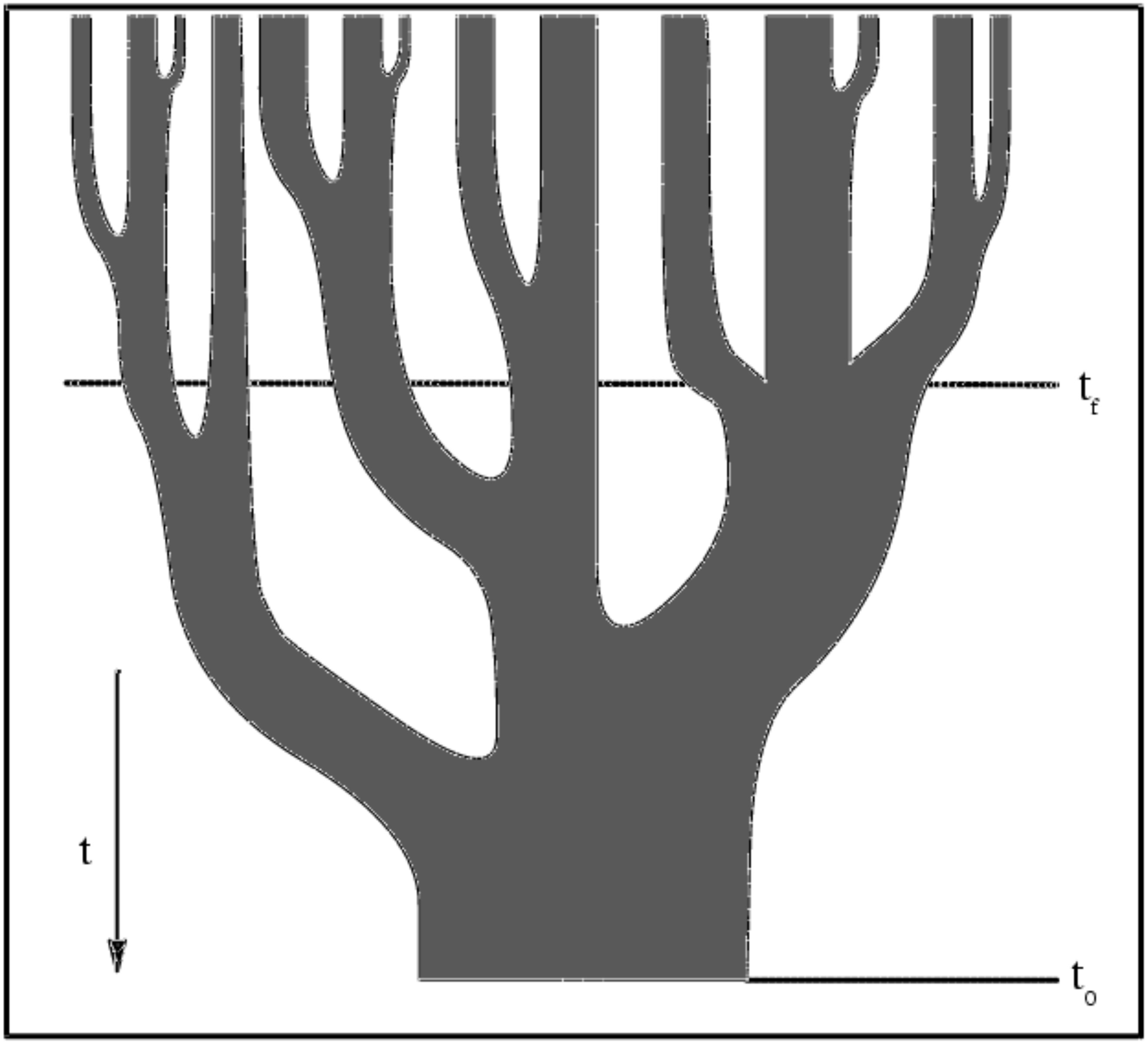
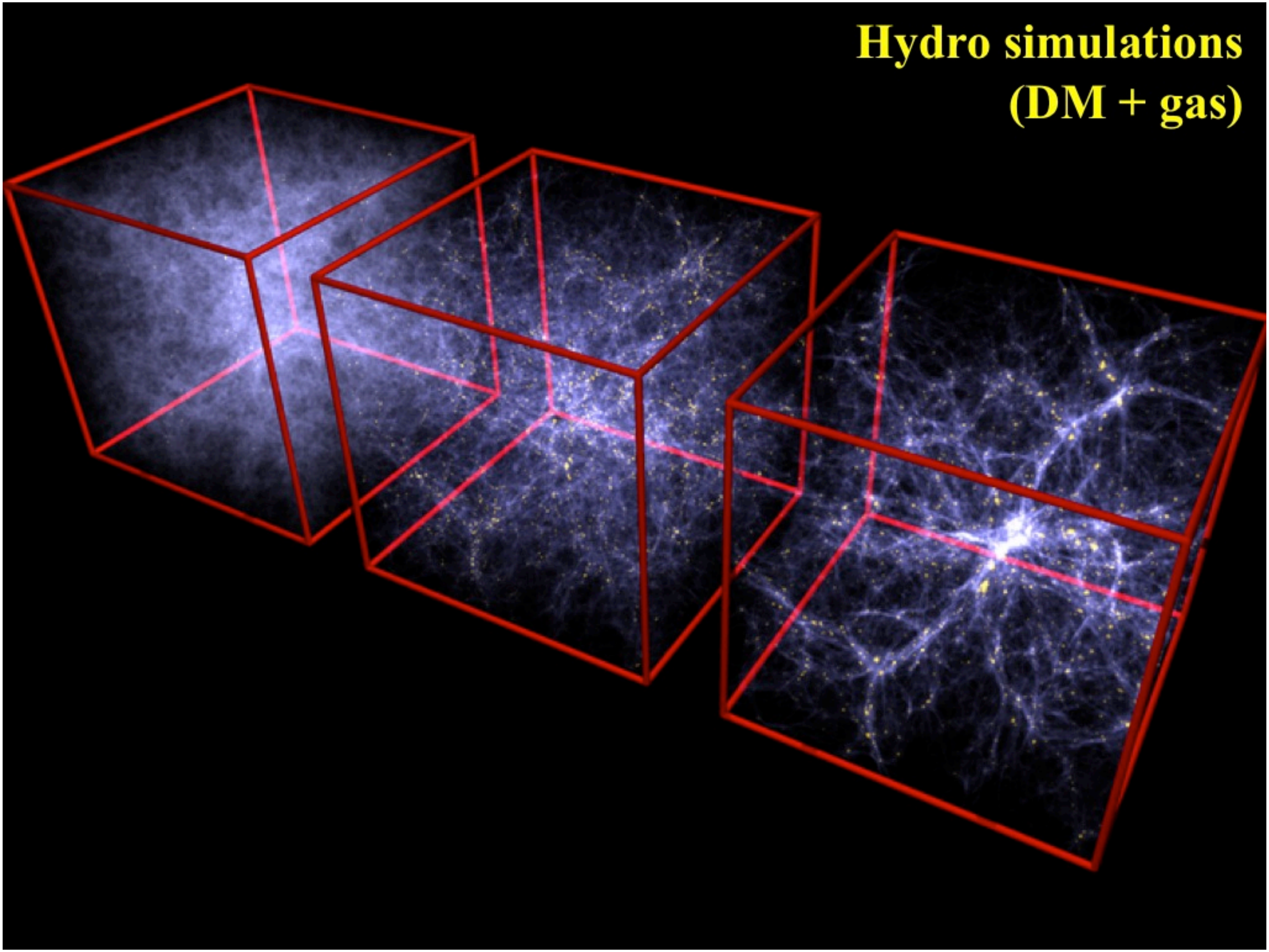


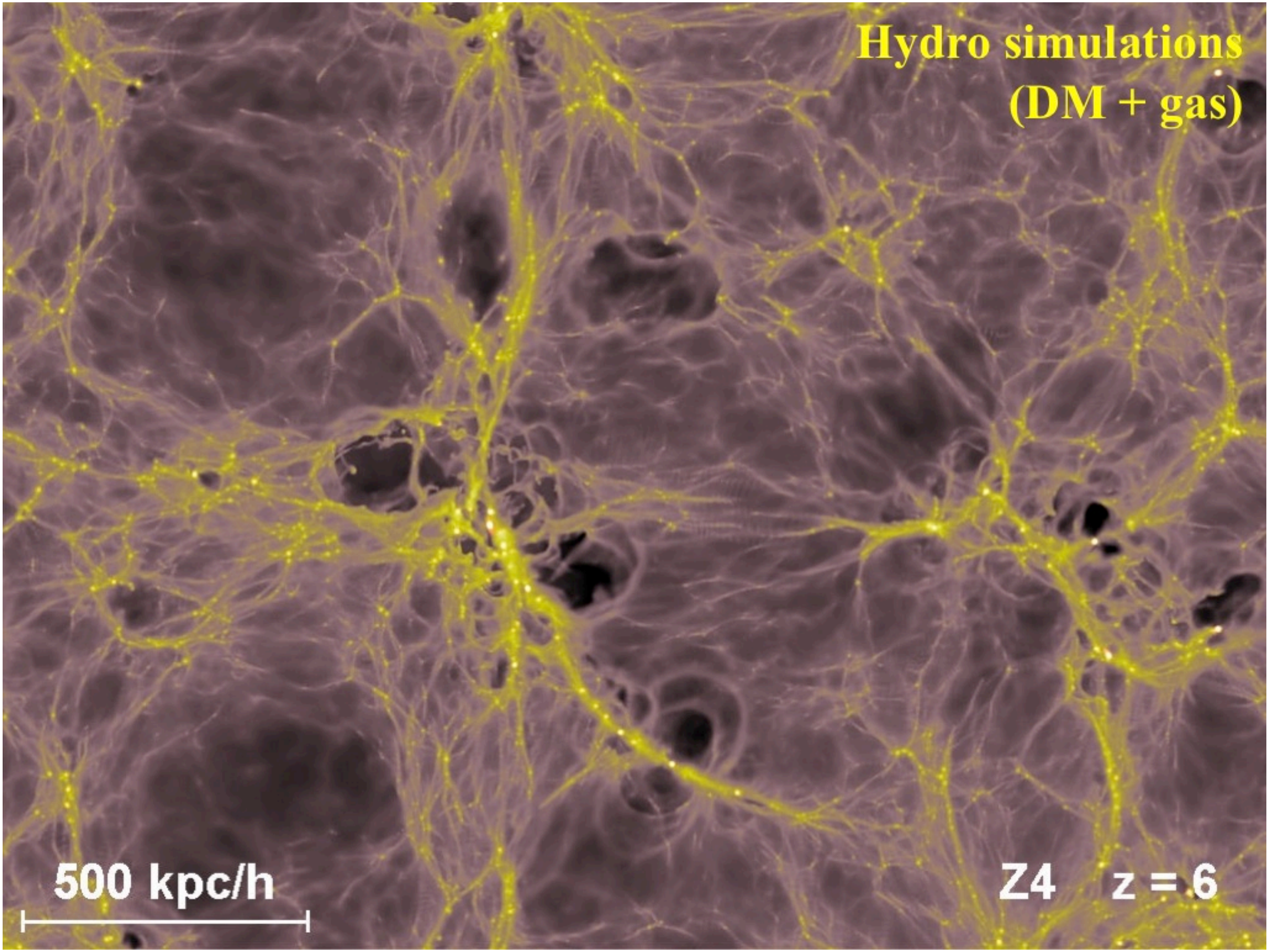
Figure 6. A schematic representation of a "merger tree" depicting the growth of a halo as the result of a series of mergers. Time increases from top to bottom in this figure and the widths of the branches of the tree represent the masses of the individual parent halos. Slicing through the tree horizontally gives the distribution of masses in the parent halos at a given time. The present time t_0 and the formation time t_f are marked by horizontal lines, where the formation time is defined as the time at which a parent halo containing in excess of half of the mass of the final halo was first created.

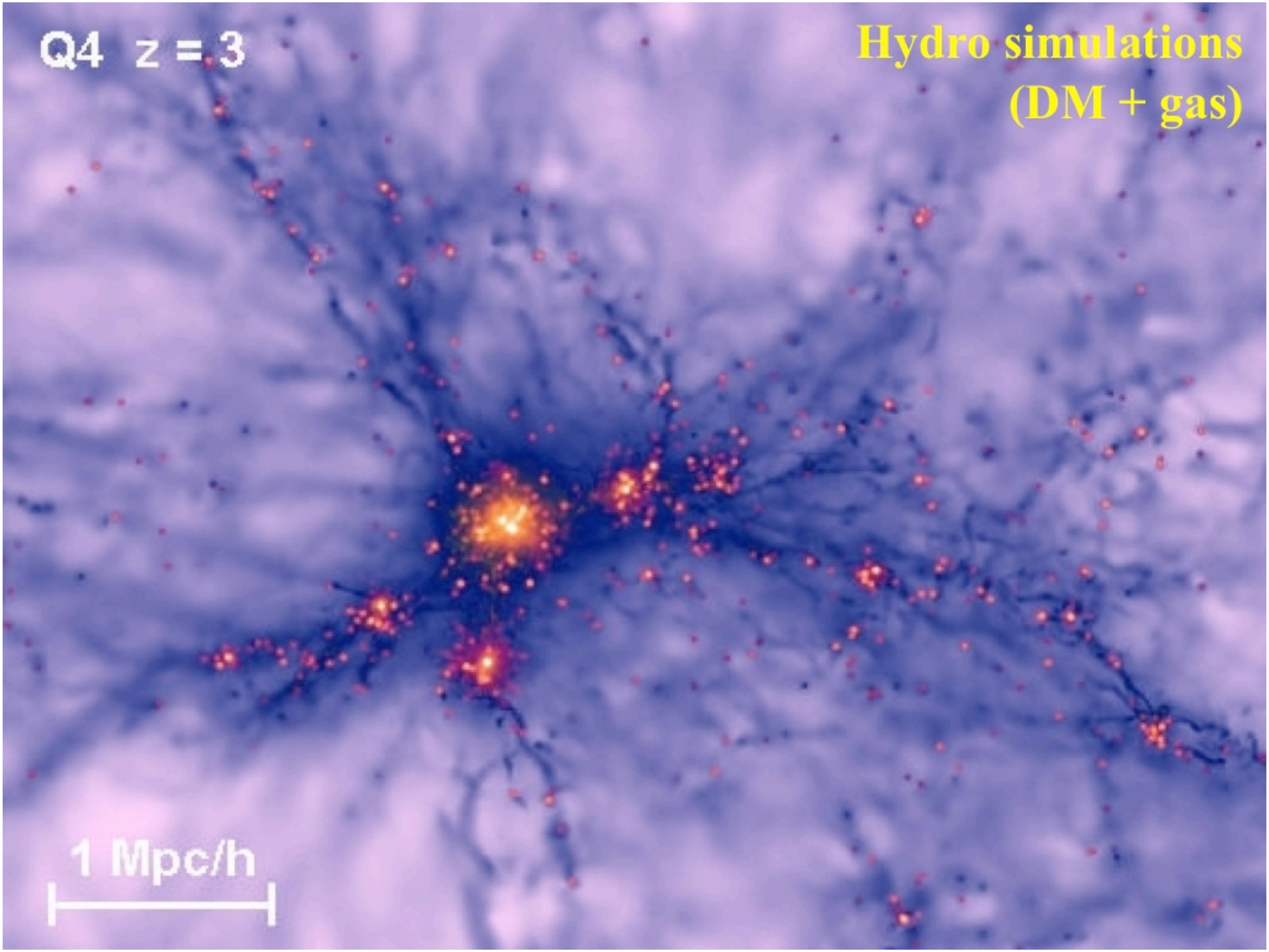
Hydro Simulation (Dissipative)

- Much harder than gravity alone
  - Yet crucial for the realistic astrophysical problems
- Resolution is a big problem: how many resolution elements of what size can you do?
  - We know that structure and physics in gas occur at all scales we can observe
- Physics is also often uncertain
  - For example, we do not really understand star formation
- Science examples:
  - Galaxy formation, including star formation, feedback by AGN and SNe, etc.
  - Supernova explosions
  - Star and planet formation

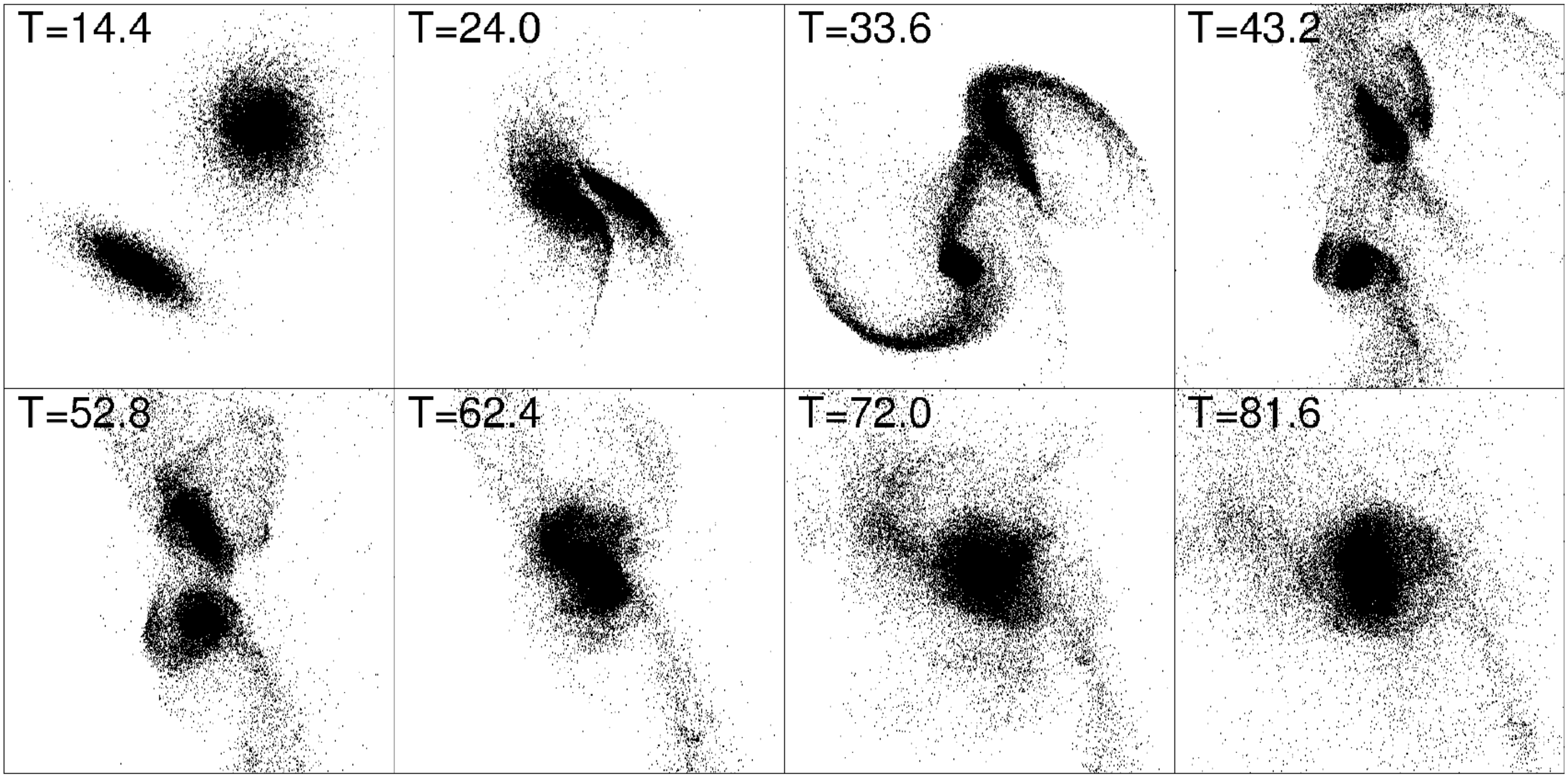
… etc. etc.


Hydro simulations
(DM + gas)


Hydro simulations
(DM + gas)
500 kpc/h
Z4
z = 6

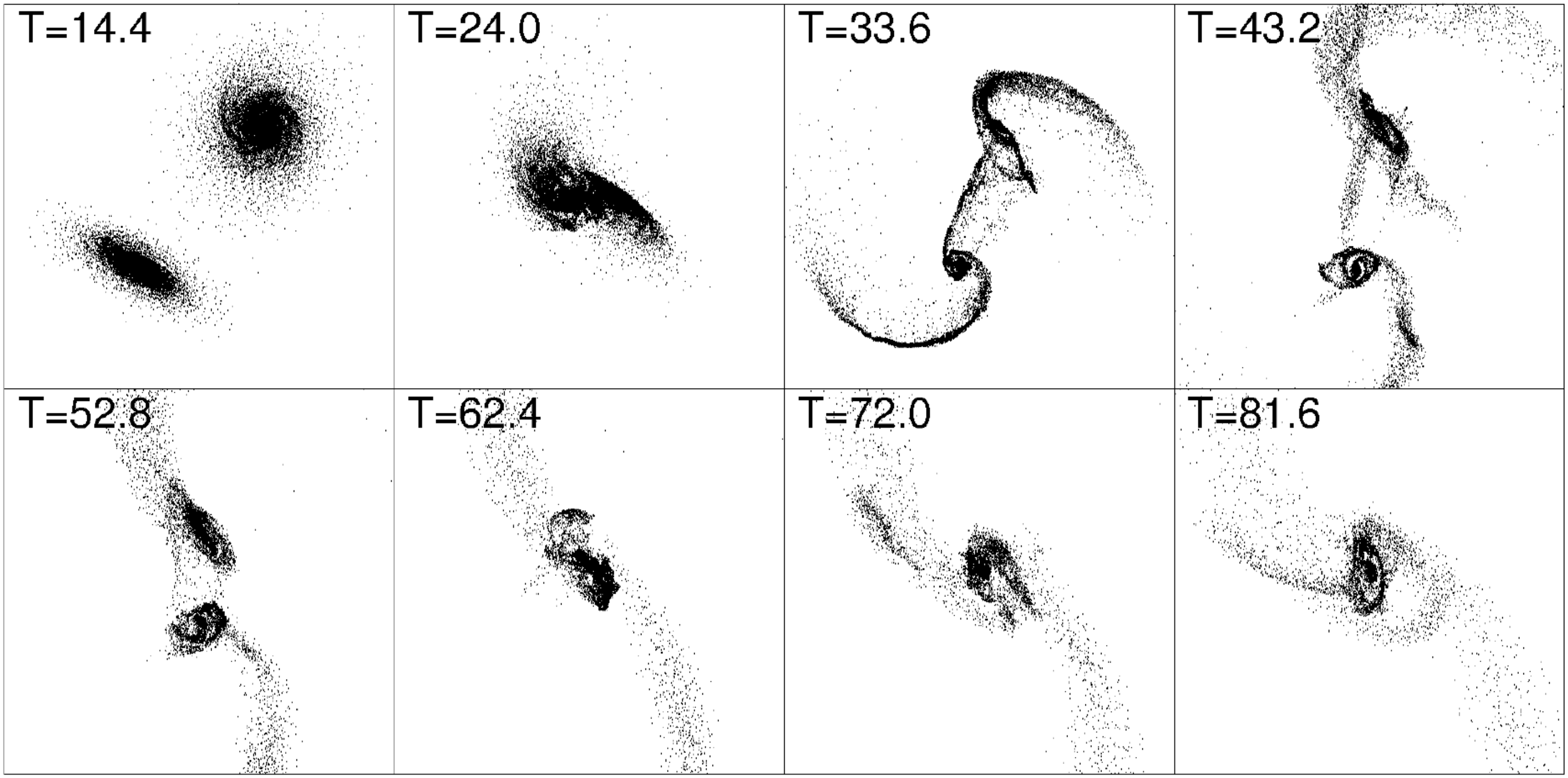

Q4 z = 3
Hydro simulations
(DM + gas)
1 Mpc/h

Galaxy Merger: Stars



Merge 2 nearly equal mass disk galaxies; in a few dynamical times, the remnant looks just like an elliptical galaxy

Galaxy Merger: Gas



In the same merger, gas quickly looses energy (since it is dissipative), and sinks towards the center of the remnant, where it can fuel a starburst, or an AGN if a massive black hole is present

Next:
Observations of Large Scale Structure