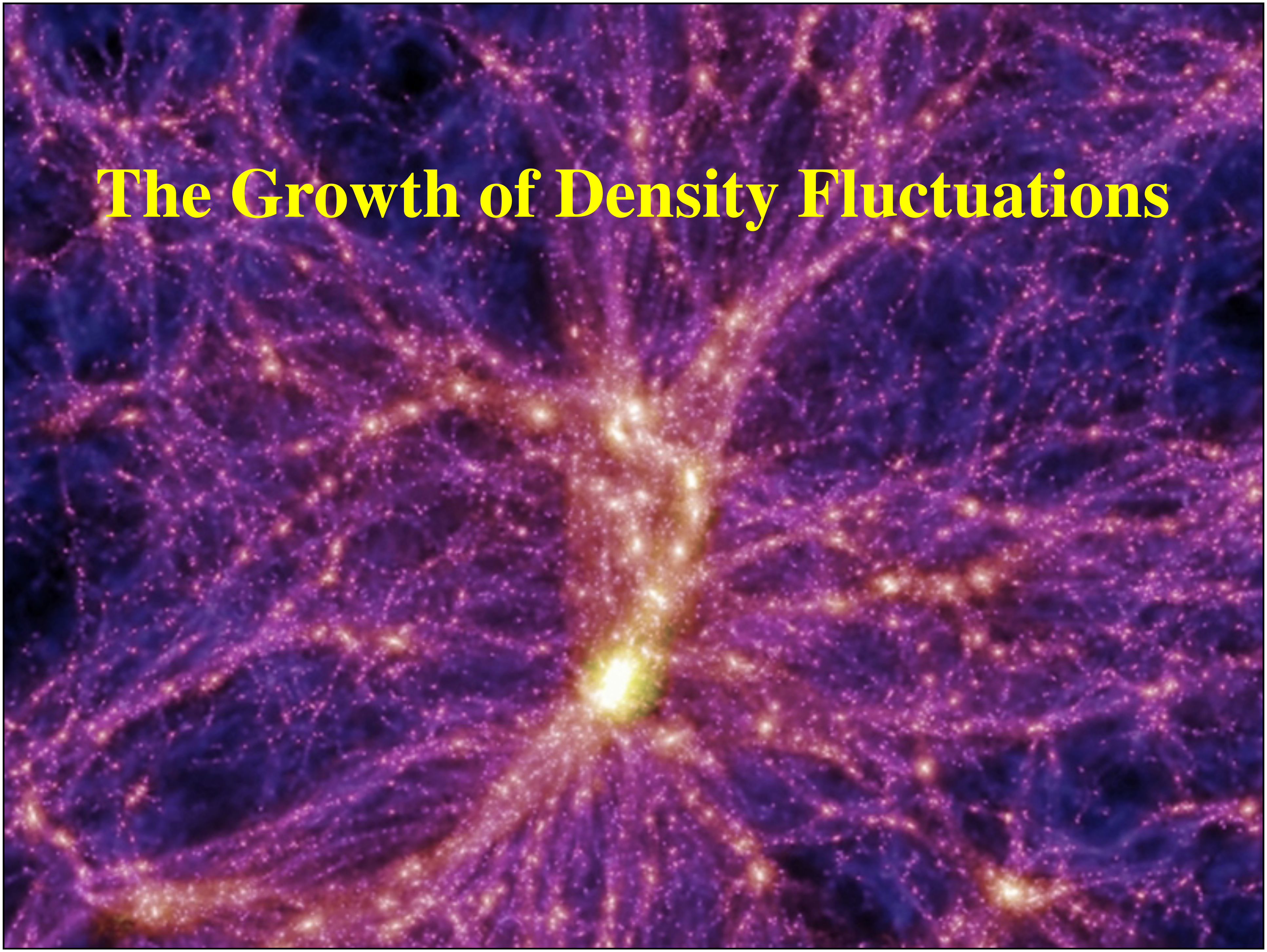
The Growth of Density Fluctuations



The Growth of Fluctuations

- Prior to matter radiation equality perturbations are prevented from growing due to radiation pressure
- Pressure opposes gravity effectively for all wavelengths below the Jeans Length:

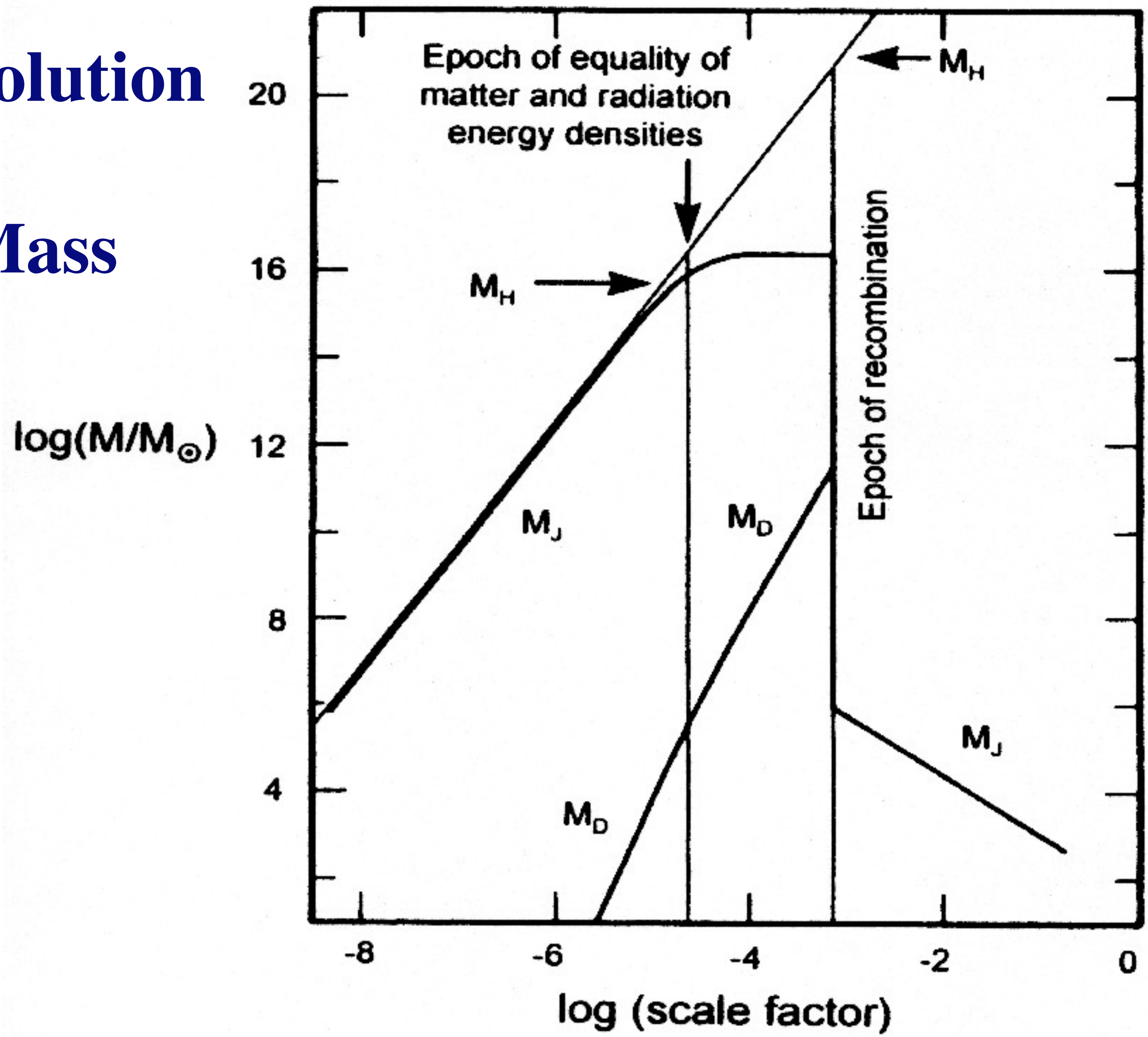
$$\boxed{\lambda_J = c_s \sqrt{\frac{\pi}{G\rho}}}$$

where $\boldsymbol{c_s}$ is the speed of sound and the equation of state is:

$$c^2{}_s = \frac{\partial P}{\partial \rho}$$

- Jeans Length is the scale at which sound waves can cross an object in about the time for gravitational collapse
- In a radiation dominated universe, Jeans Length is close to the horizon size
- At matter-radiation equality the sound speed starts to drop, and fluctuations can ***grow***

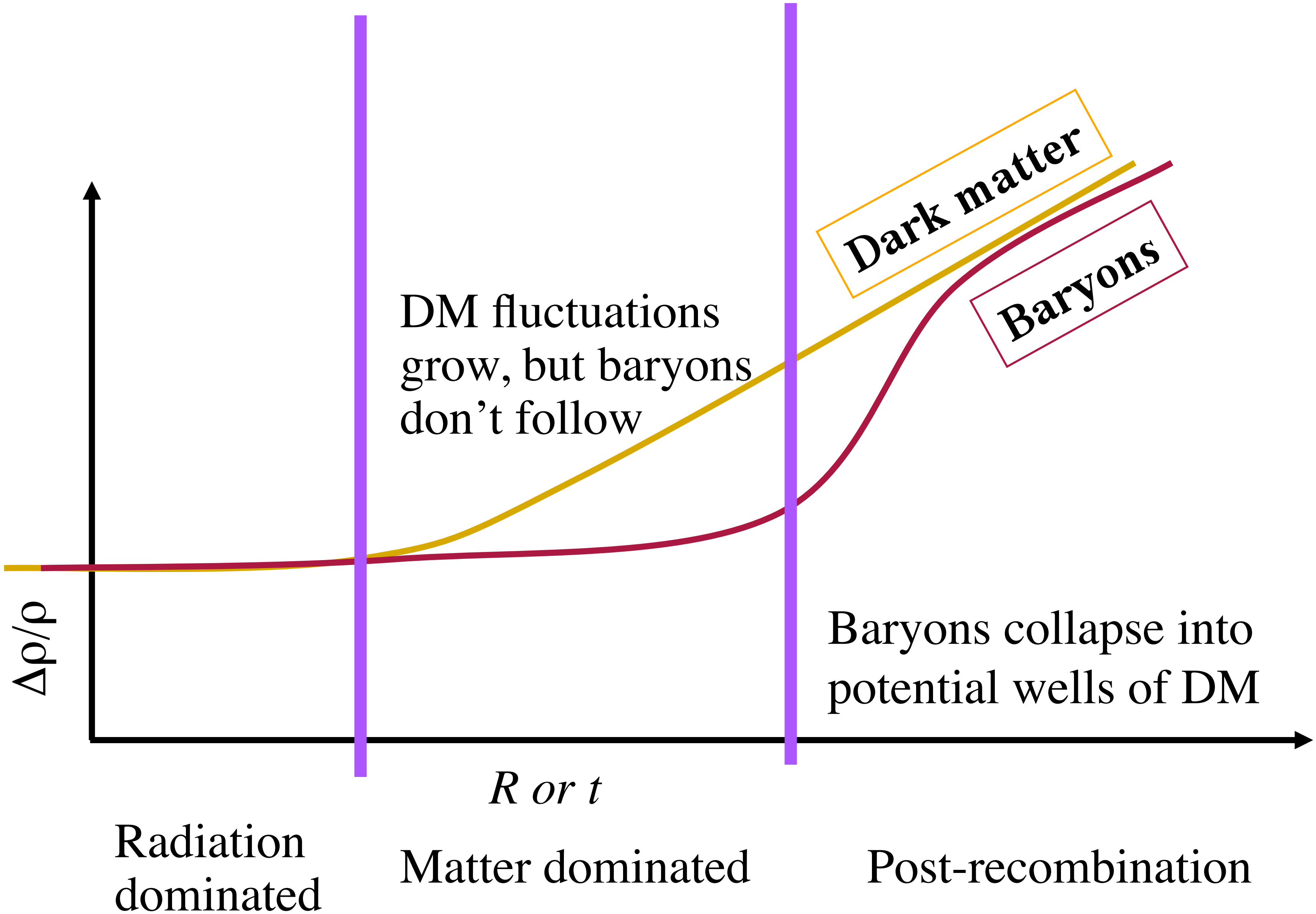
The Evolution of the Jeans Mass



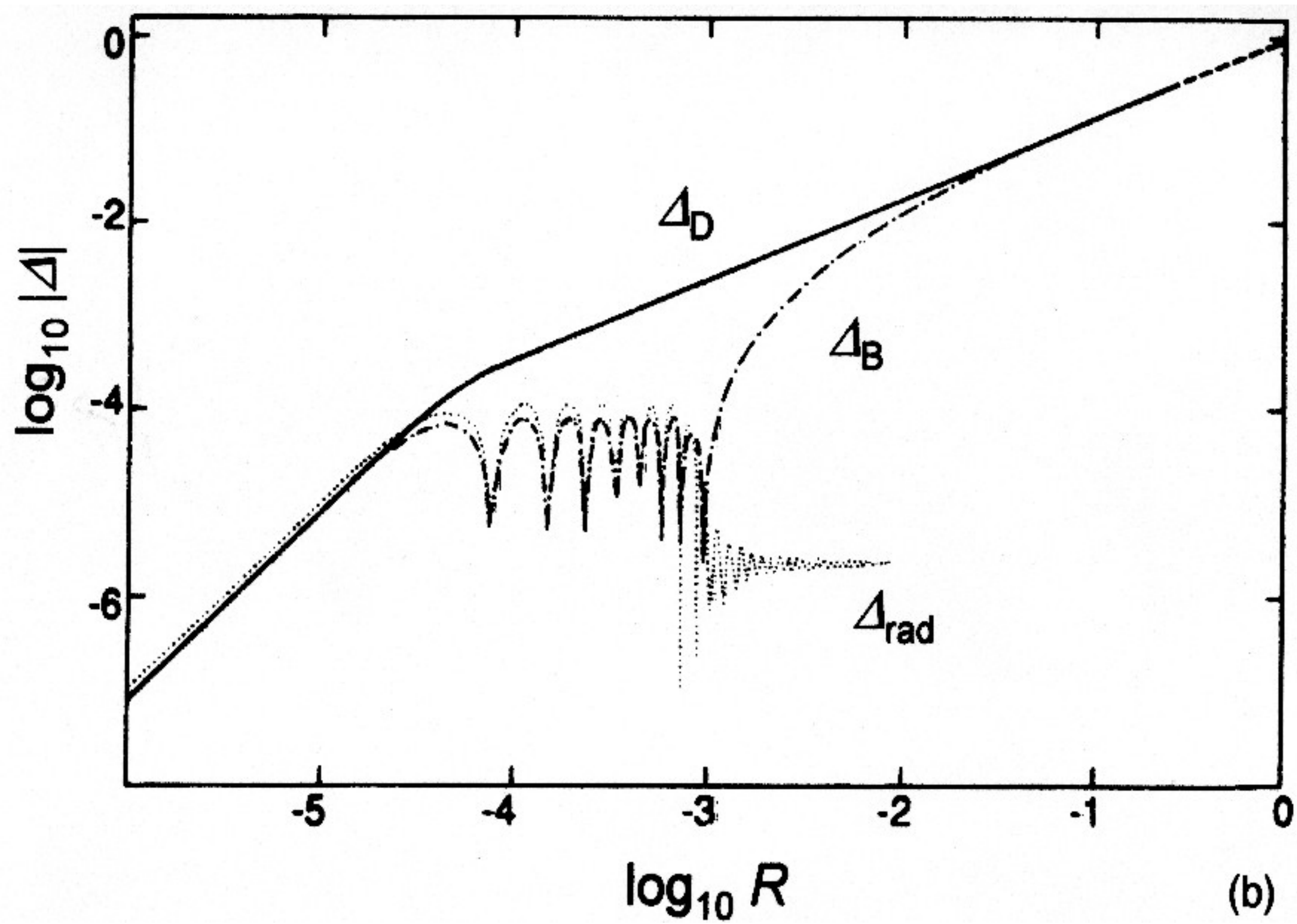
The Growth of Fluctuations

- ***Horizon scale at matter-radiation equality defines a particular scale of fluctuations***
- After matter-radiation equality baryons still affected by photon pressure due to Thomson scattering and perturbations oscillate as sound waves. These are responsible for the Doppler peaks we observe in the CMB
- After z_{rec} the fluctuations can grow, and any fluctuations or potential wells in the dark matter dominated density field will gravitationally attract baryons. So quickly the density fields will be similar again

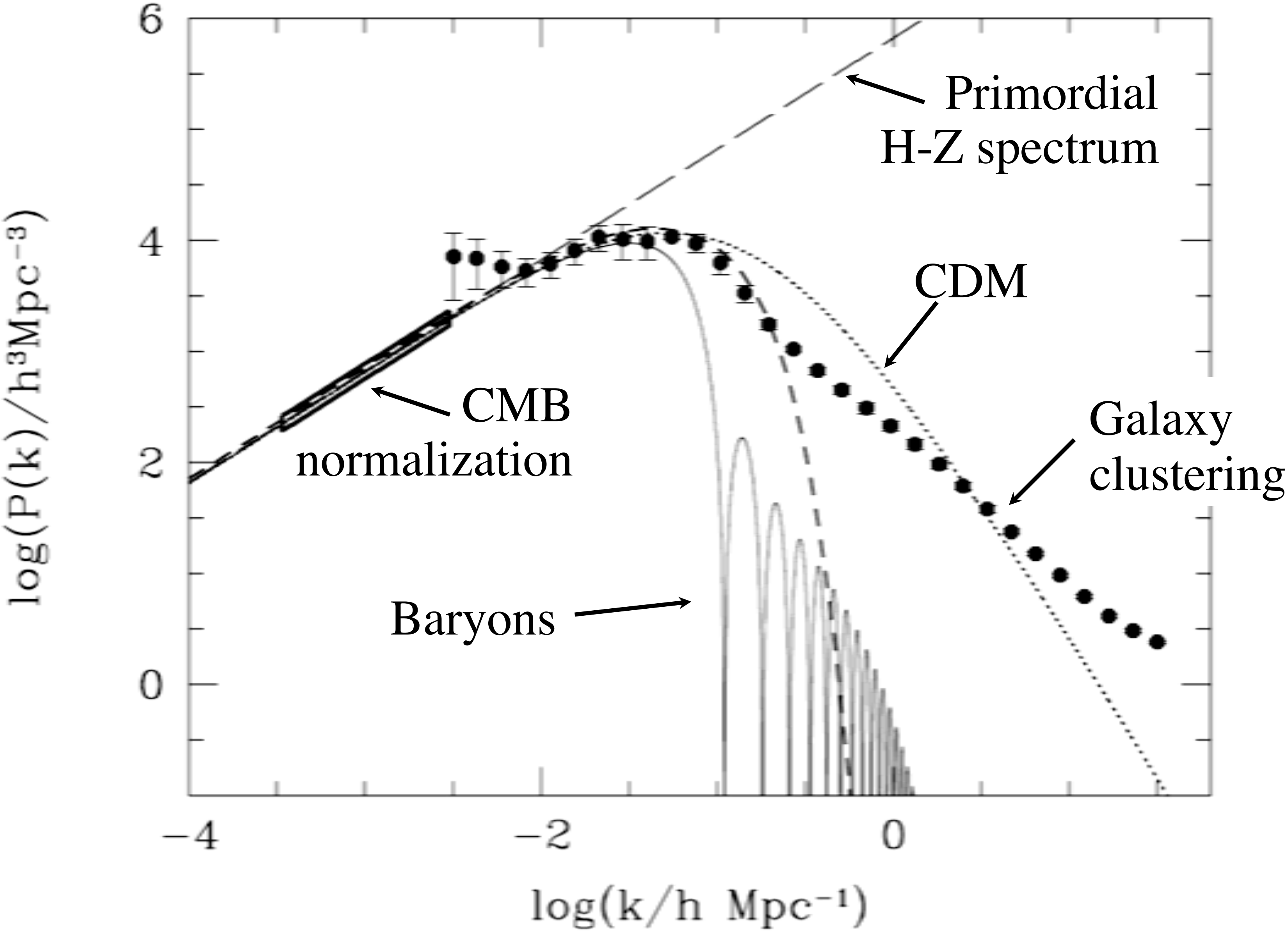
The Evolution of Fluctuations



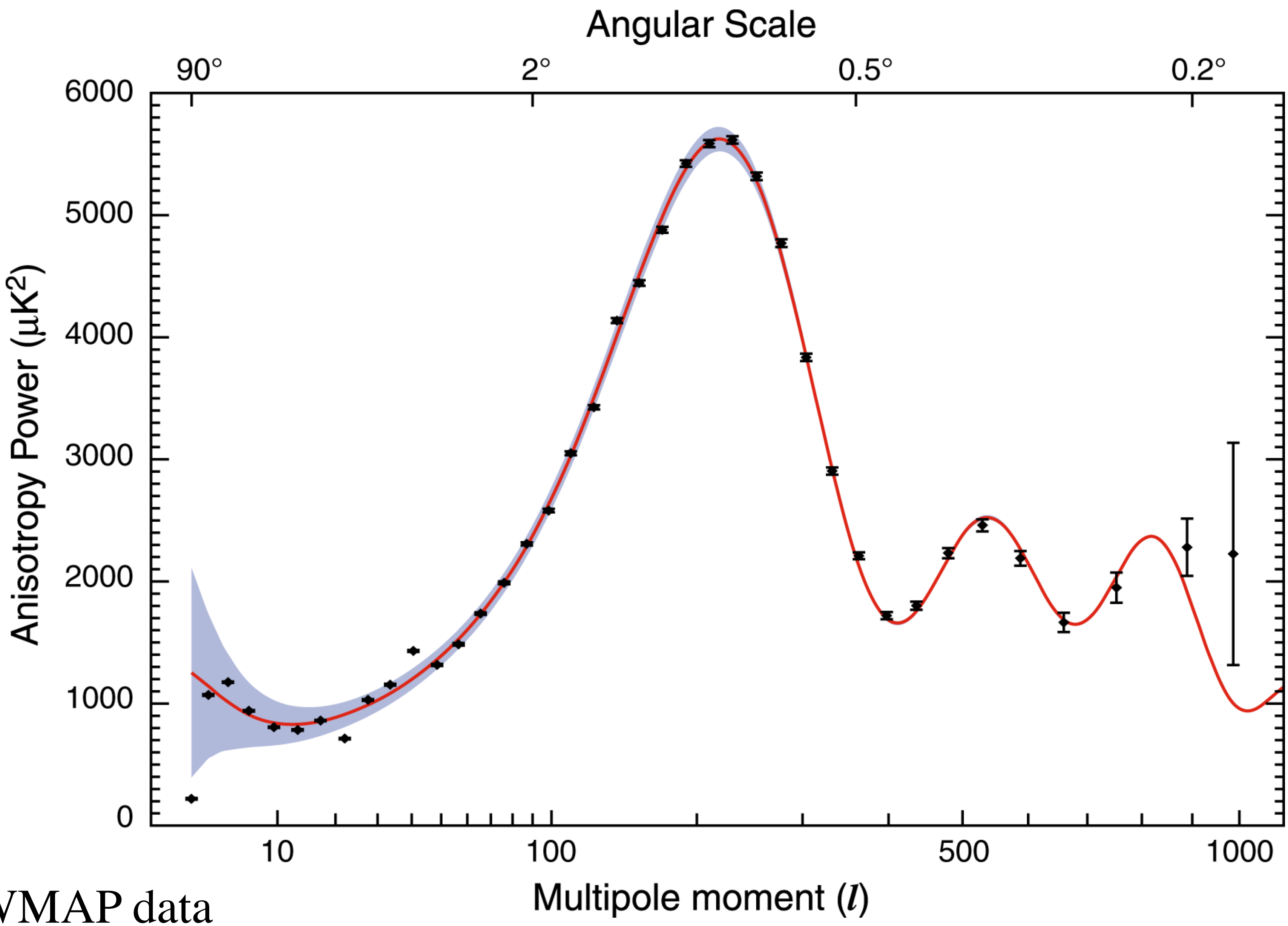
The Evolution of Fluctuations



The Power Spectra



And this is what we observe in CMBR:



WMAP data

Post-Recombination Universe

- Fluctuations continue to grow, and soon enter the non-linear regime, at progressively ever larger scales
- This evolution can only be followed numerically
- Once the gas infalling into the DM potential wells (dark halos) is compressed enough, ***dissipative effects*** begin to play a significant role (shocks, star formation, feedback). These are even difficult to simulate numerically!
- Energy dissipation accelerates collapse, and leads to higher densities which cannot be achieved by the dissipationless collapse (a factor of 8…). This is often called "***cooling***"
- A good mechanism is inverse Compton scattering of CMB photons on hot electrons; this process is effective only at $z < 100$ or so, since the CMB is too hot at higher z's

Cooling and Structure Formation

Define the ***cooling time*** as:

$$t_{cool} = E \,/\, |dE/dt| = 3\, n\, k\, T \,/\, [\, n^2\, \Lambda(T)\,]$$

where $\boldsymbol{n}$ is the particle number density, $\boldsymbol{k}$ is the Boltzmann constant, and $\Lambda(T)$ is a cooling rate function which depends on the chemical composition of the plasma

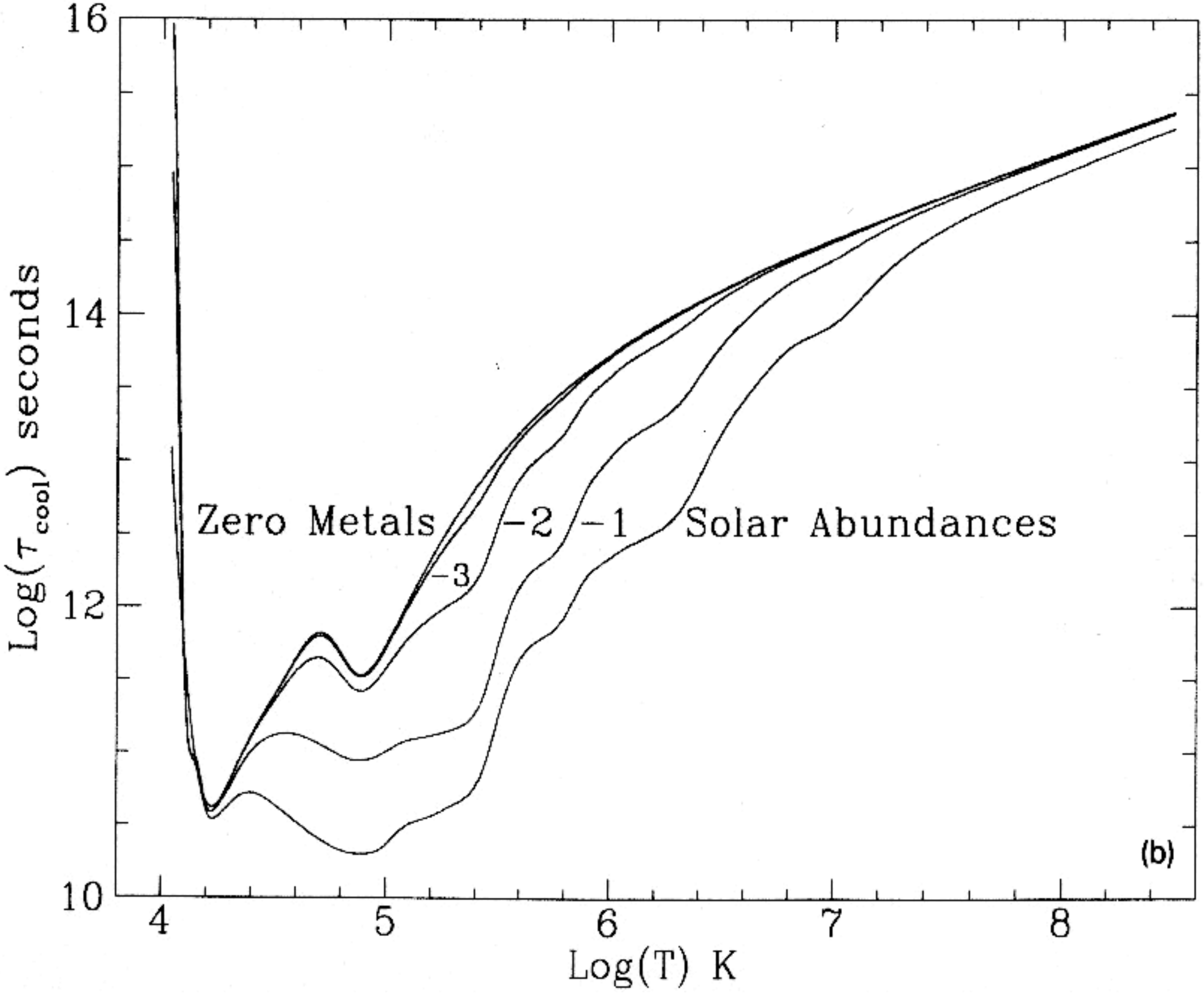
The key question is the relation between the free-fall time t_{ff}, cooling time, t_{cool}, and the Hubble time, t_H:

If $\boldsymbol{t_{cool} < t_{ff}}$ the cooling dominates the contraction, objects collapse faster and to smaller radii and higher densities; and vice versa

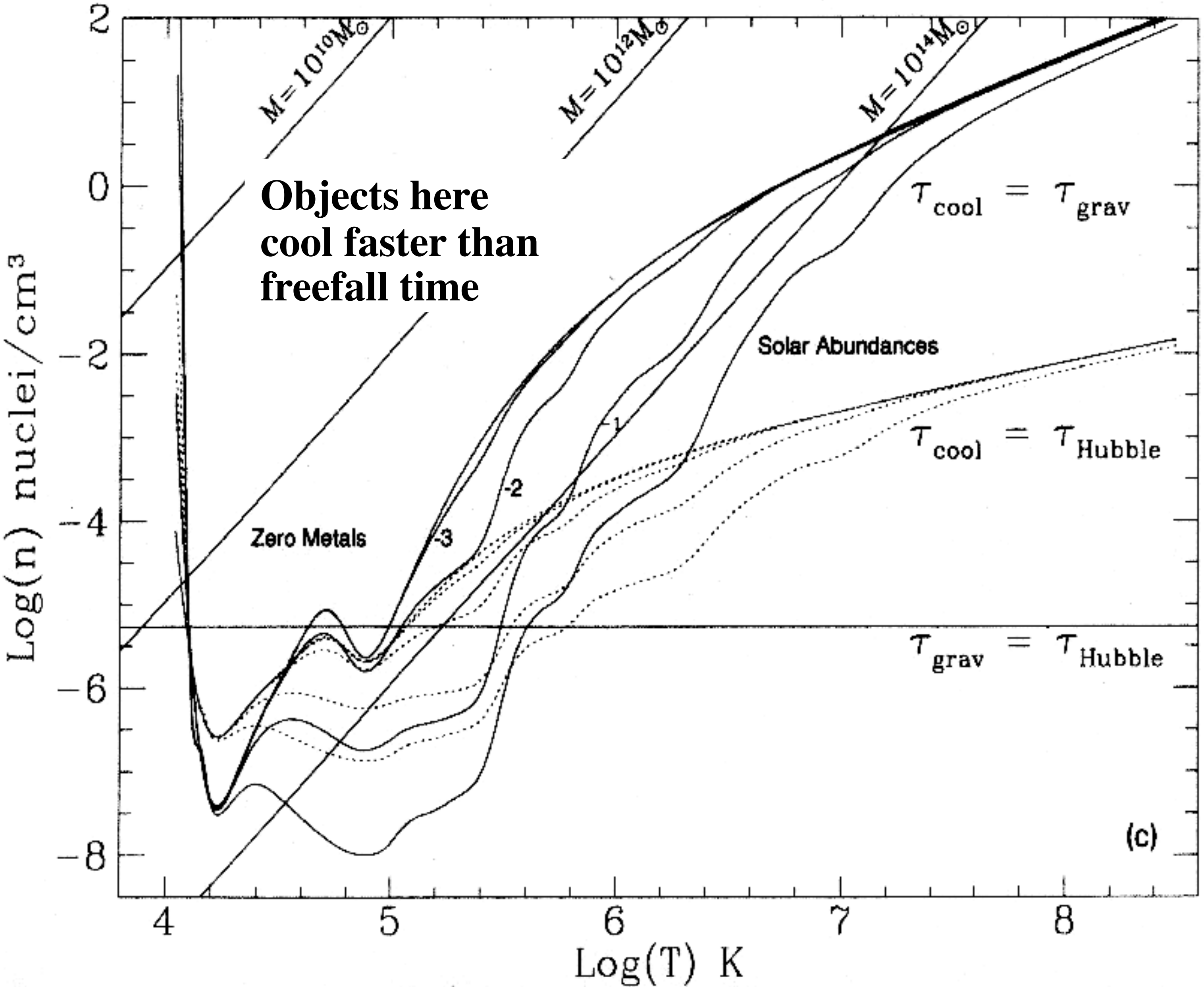
If $\boldsymbol{t_{cool} > t_H < t_{ff}}$ objects cannot form

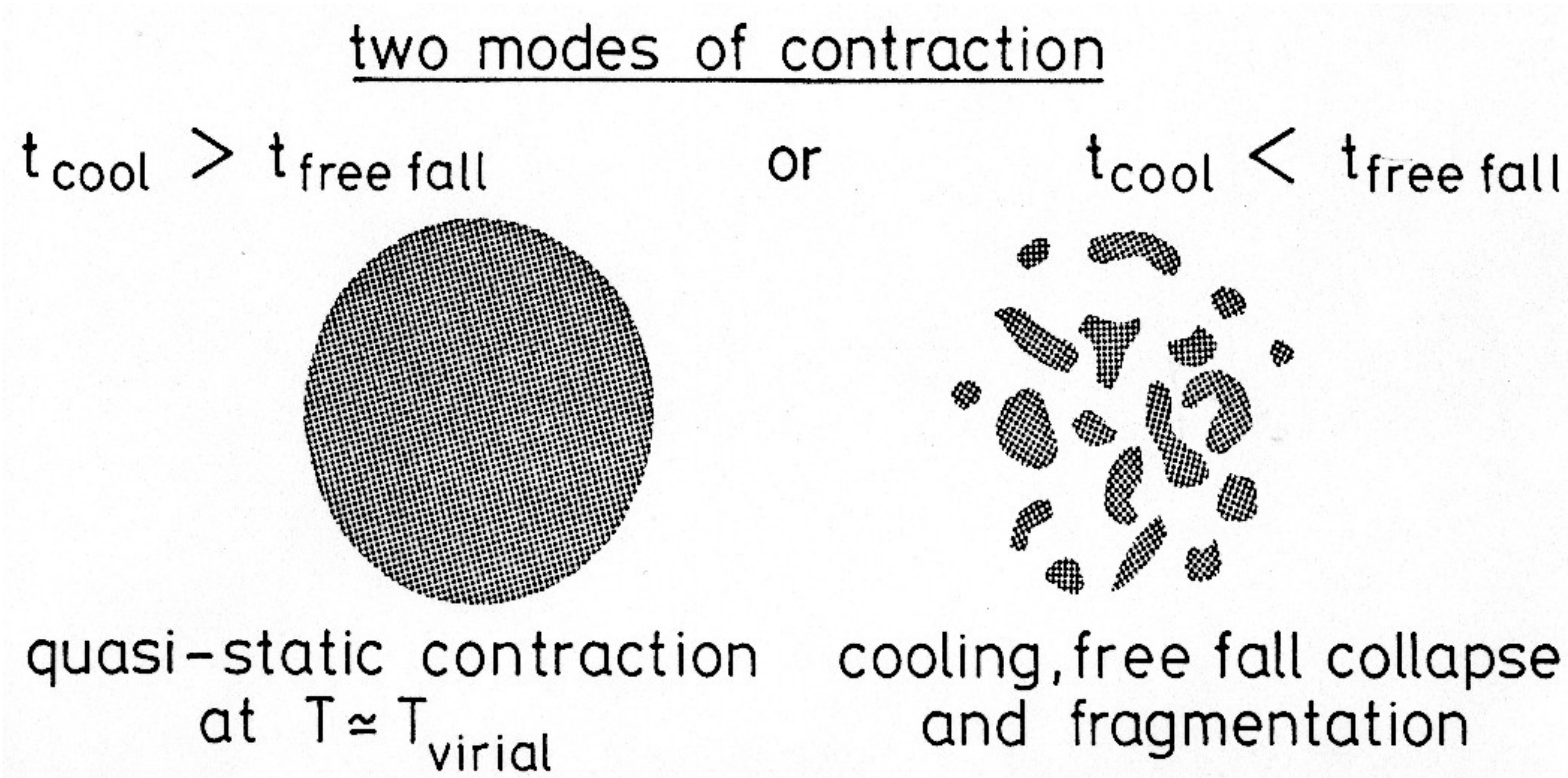
The position of objects in the "cooling diagram" plane $\{T,n\}$ thus determines their fate! Note also that $M_{Jeans} = f(T,n)$

The Cooling Curve



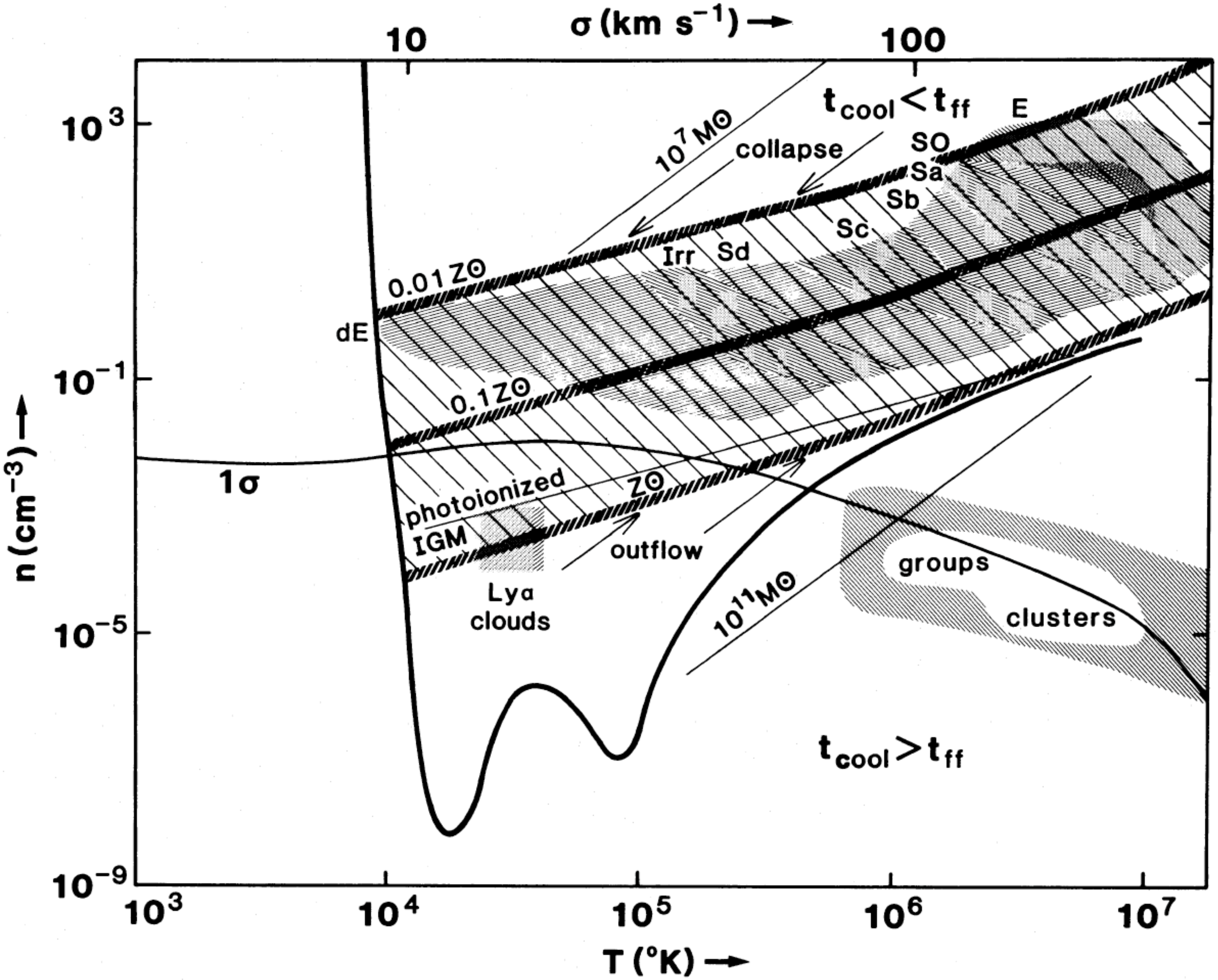
The Cooling Diagram





Fluctuations which cool faster than they fall together under gravity alone are subject to Jeans instability and fragmentation

The Cooling Diagram



And indeed, we see that the cooling curve separates the dissipative structures (galaxies) from the dissipationless ones (groups and clusters)

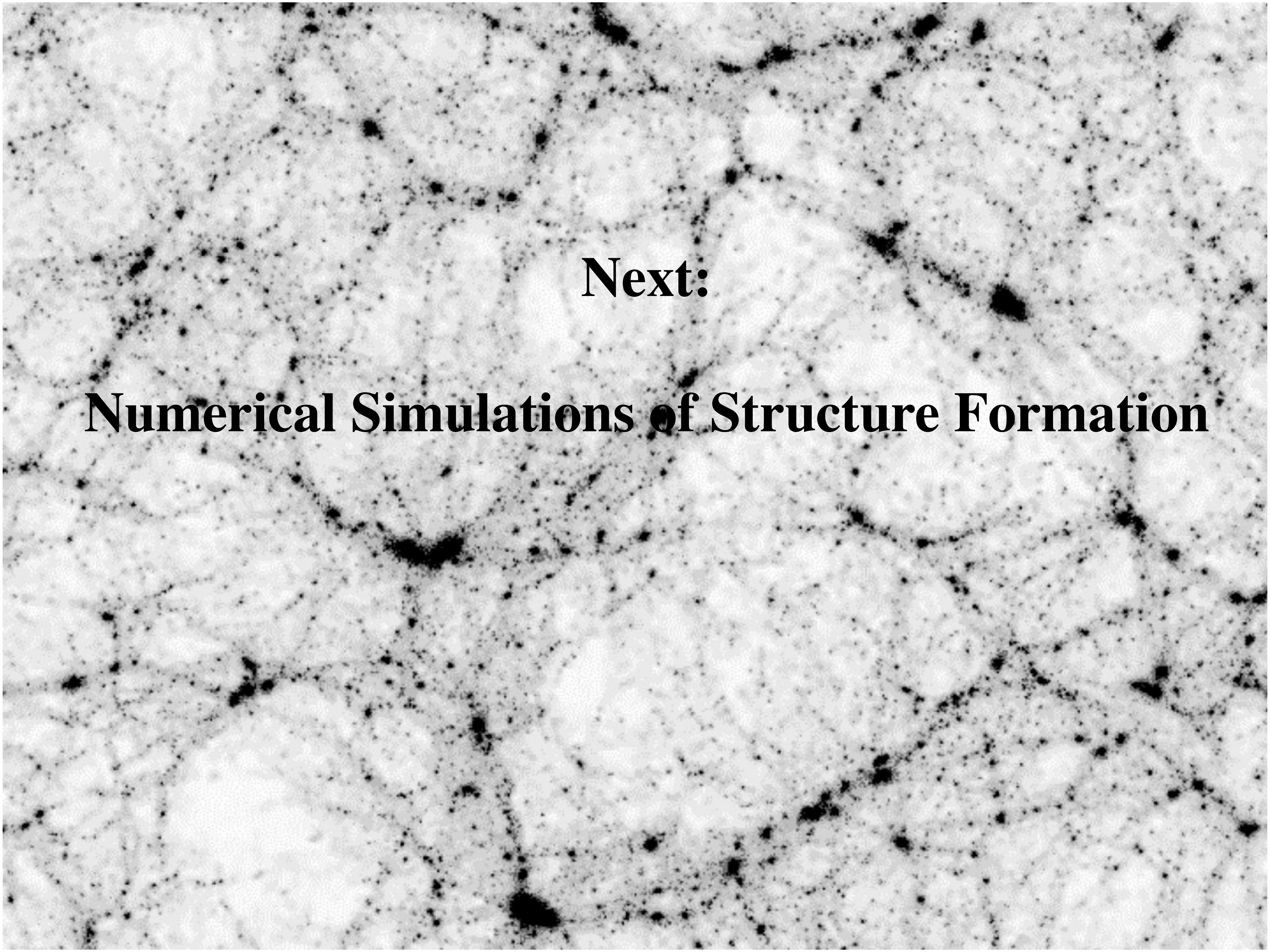
(from J. Silk)

Structure Formation Theory: A Summary of the Key Ideas (1)

- Structure grows from initial density perturbations in the early universe, via gravitational infall and hierarchical merging
- Initial conditions described by the primordial density (Fourier power) spectrum $P(k)$, often assumed to be a power-law, e.g., $P(k) \sim k^n$, $n = 1$ is called a Harrison-Zeldovich spectrum
- Dark matter (DM) plays a key role: fluctuations can grow prior to the recombination; after the recombination, baryons fall in the potential wells of DM fluc's (proto-halos)
- Damping mechanisms erase small-scale fluctuations; how much, depends on the nature of the DM: HDM erases too much of the high-freq. power, CDM fits all the data
- Collapse occurs as blobs ➙ sheets ➙ filaments ➙ clusters

Structure Formation Theory: A Summary of the Key Ideas (2)

- Pure gravitational infall leads to overdensities of ~ 200 when the virialization is complete
- Free-fall time scales imply galaxy formation early on (t_{ff} ~ a few $\times 10^8$ yrs), clusters are still forming (t_{ff} ~ a few $\times 10^9$ yrs)
- Characteristic mass for gravitational instability is the Jeans mass; it grows before the recombination, then drops precipitously, from ~ 10^{16} $M_\odot$, to ~ 10^5 $M_\odot$
- Cooling is a key concept:
  - Galaxies cool faster than the free-fall time: formation dominated by the dissipative processes, achieve high densities
  - Groups and clusters cool too slowly: formation dominated by self gravity, lower densities achieved
  - The cooling curve separates them



Next:

Numerical Simulations of Structure Formation