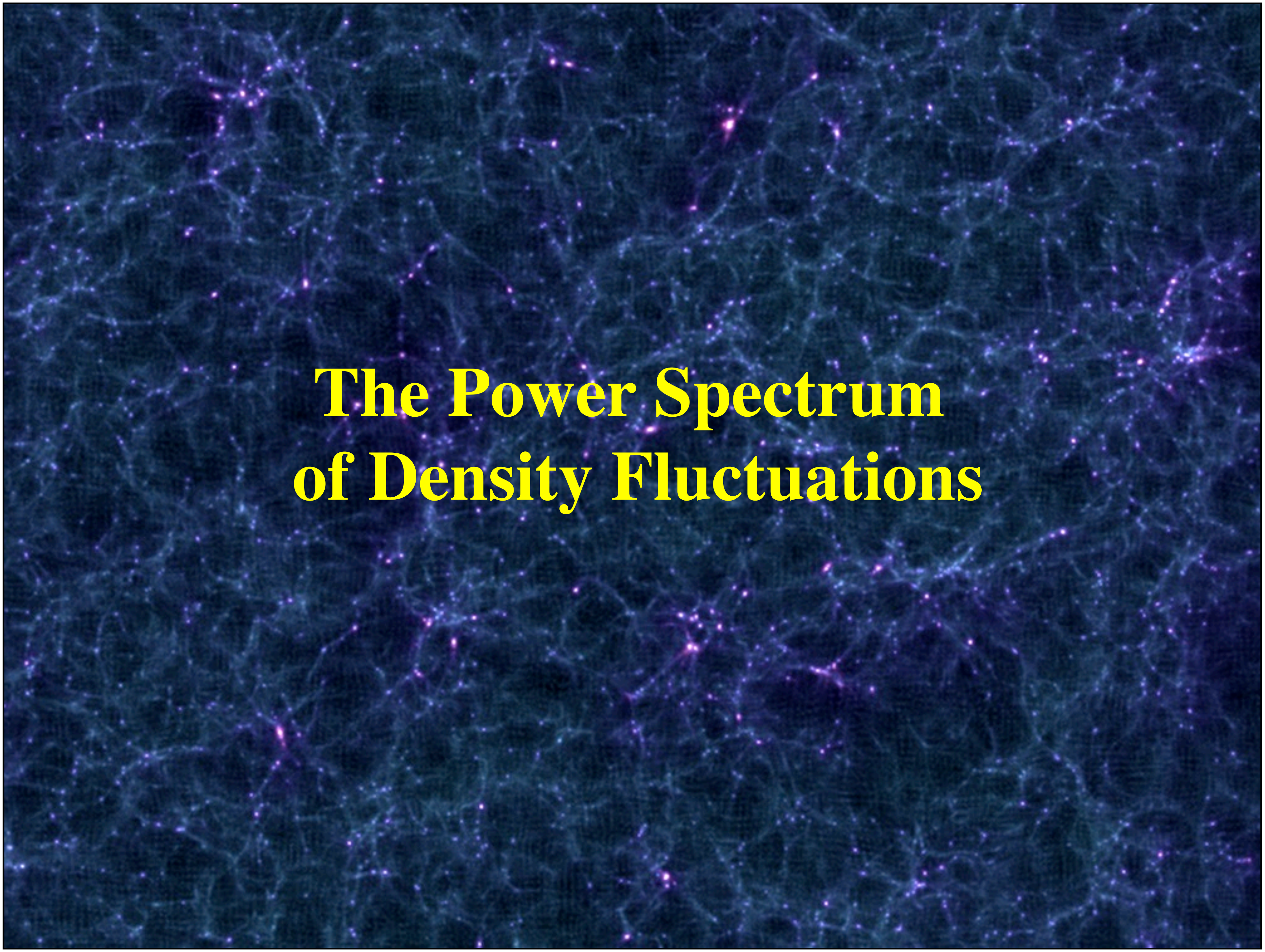
The Power Spectrum of Density Fluctuations



Quantifying the Density Field

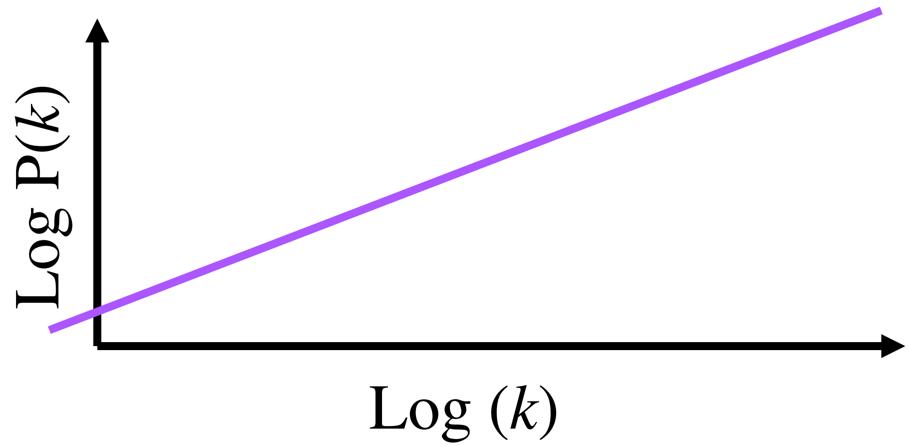
- Consider the overall fluctuating density field as a superposition of waves with different wavelengths, phases, and amplitudes
- Then we can take a Fourier transform and measure the **Power** on different scales, expressed either as wavelengths $\boldsymbol{\lambda}$ or frequencies or wave numbers $\boldsymbol{k = 1 / \lambda}$
- ***Density fluctuations field :*** $\delta = \dfrac{\rho - \bar{\rho}}{\bar{\rho}}$
- ***Fourier Transform of density field :*** $\delta_k = \sum \delta e^{-i\mathbf{k} \cdot \mathbf{r}}$
- ***Its Power Spectrum :*** $P(k) = \left\langle \left| \delta_k \right|^2 \right\rangle$

It measures the power of fluctuations on a given scale ***k***
Sometimes one uses the "mass scale"; $\boldsymbol{M \sim k^{-3(3+n)/4}}$

Density Fluctuation Spectrum

- A common assumption is that the fluctuations have the same amplitude $\delta \sim 10^{-4}$ when they enter the horizon
- This gives a ***scale-free*** or ***Harrison-Zeldovich*** spectrum
- Such spectrum is also predicted in the inflationary scenario
- More generally, it can be a power-law:

$$\boxed{P(k) = \left\langle \left| \delta_k \right|^2 \right\rangle \propto k^n}$$



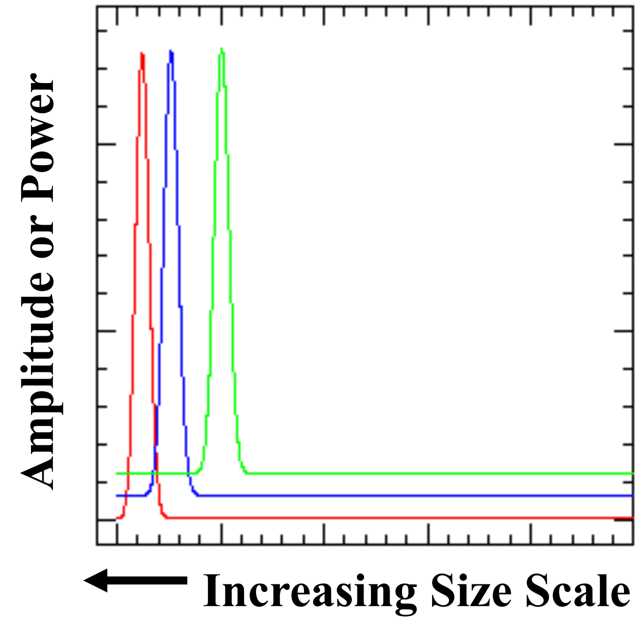
(for Harrison-Zeldovich, n = 1)

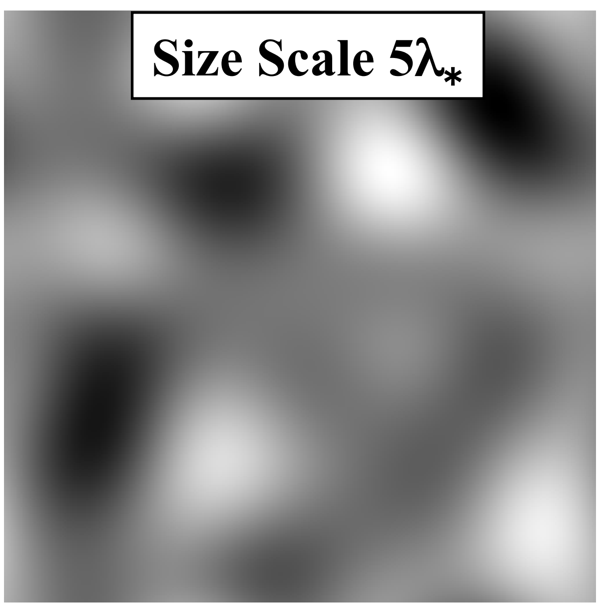
- The fluctuations grow under self-gravity, so the power increases (if all scales grow equally, the spectrum just shifts up in *log*), but they can be also erased or damped through various processes - the shape of the spectrum changes

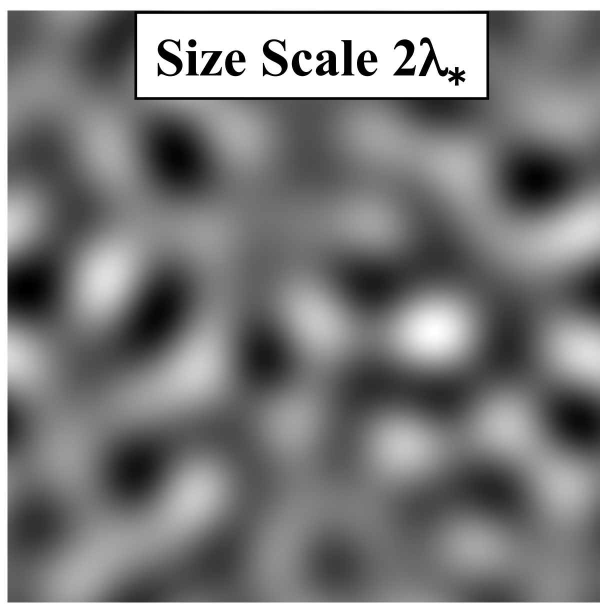
Images of Structure on Specific Scales

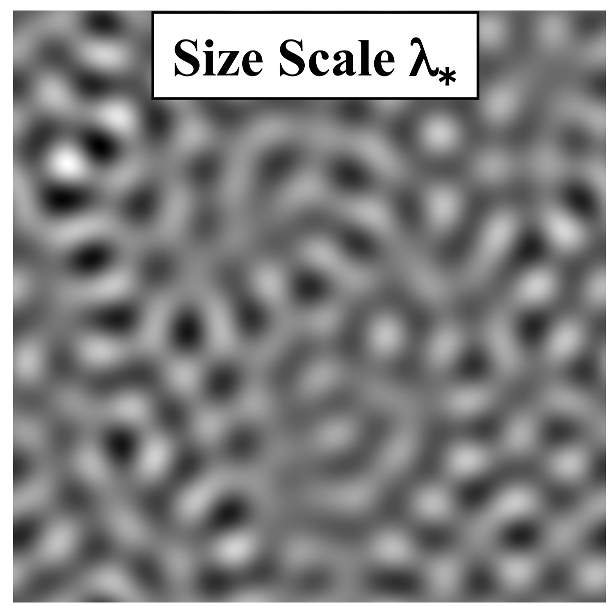
Consider a hypothetical case where the density field has fluctuations on some preferred scale, i.e., with a narrow range of frequencies

Power Spectra of fluctuations represented in images below:





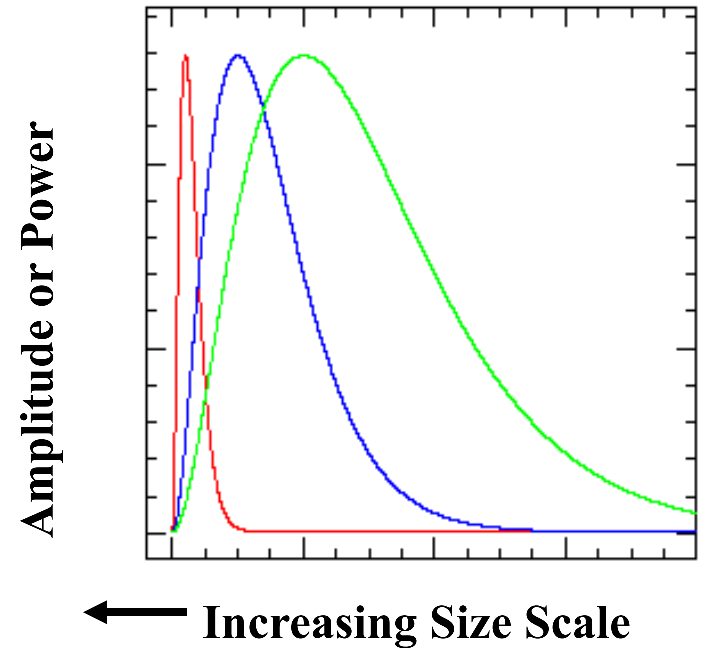


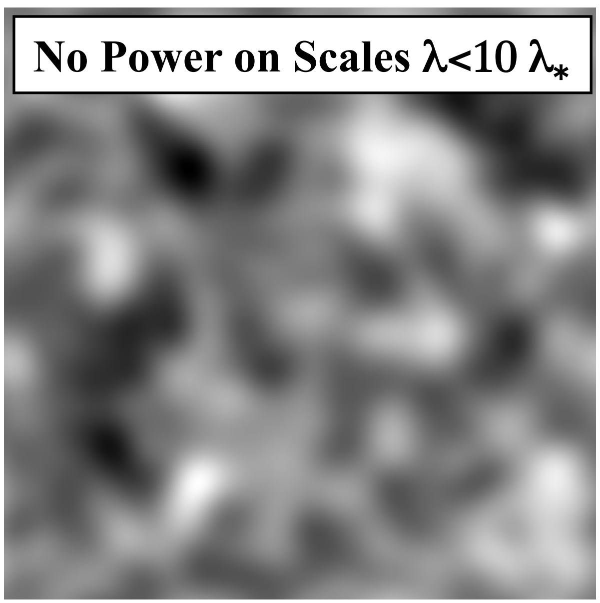


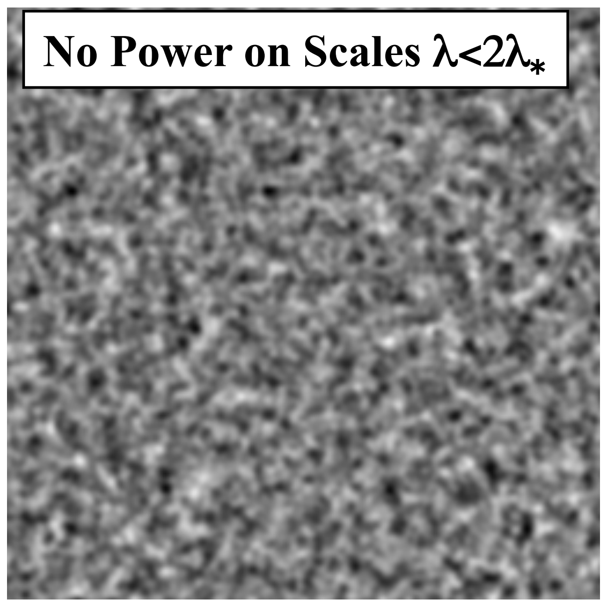
Images of Structures on a Broad Range of Scales

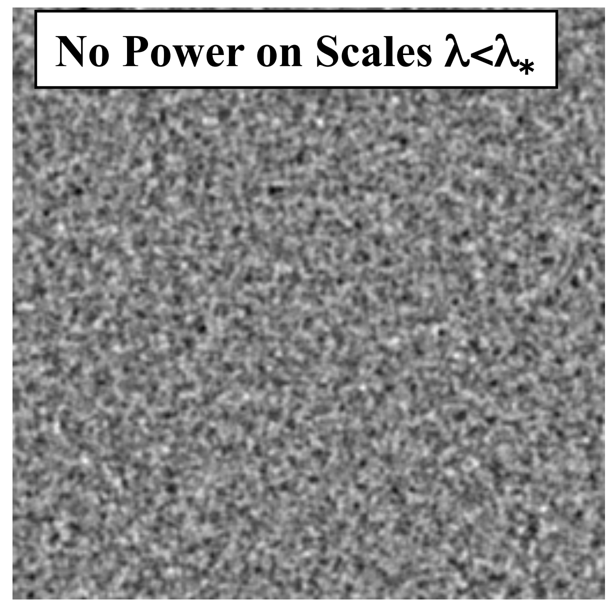
Consider a case where the range of frequencies is broader

Power Spectra of fluctuations represented in images below:









Types of Primordial Fluctuations

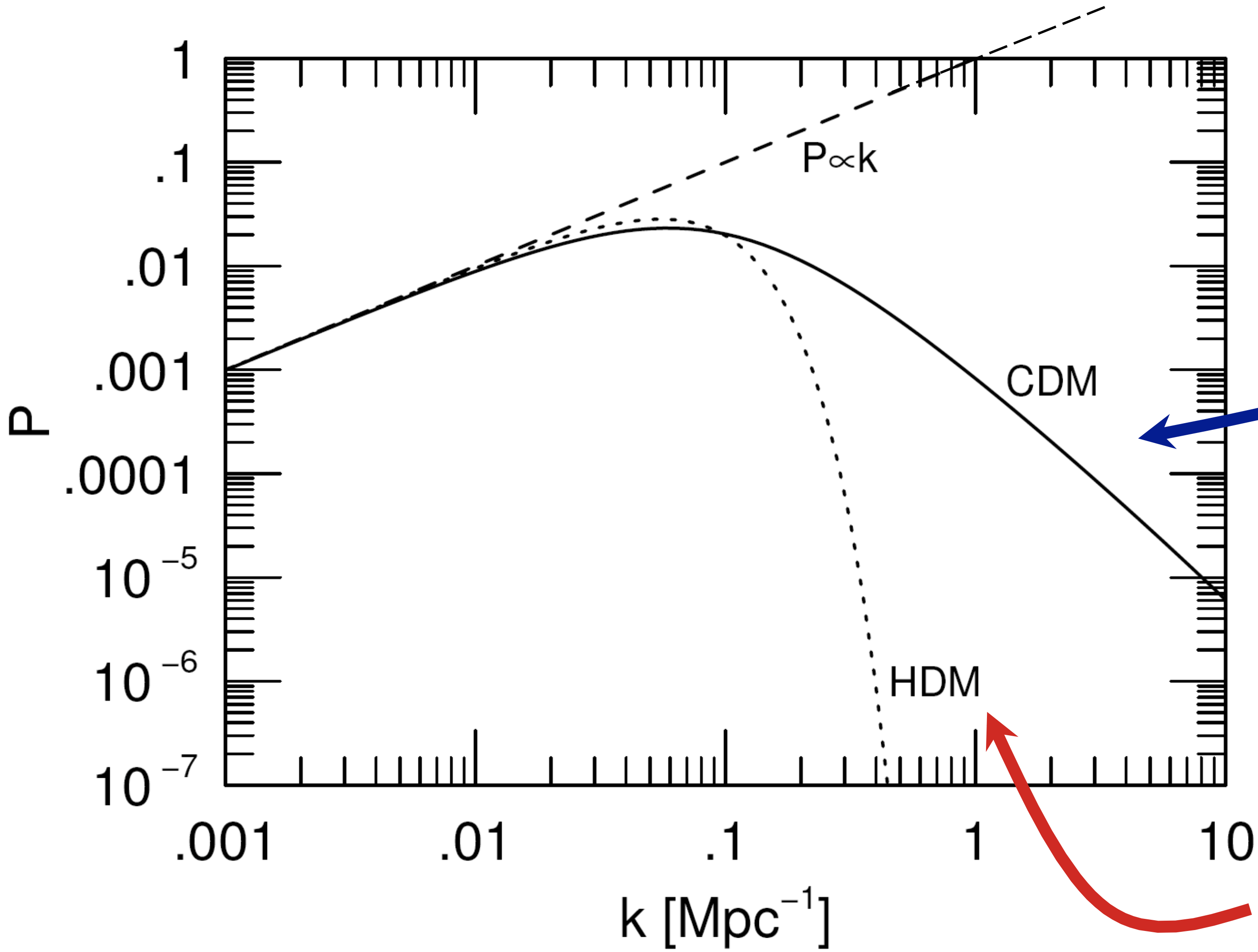
- **Adiabatic:** Corresponding to changes in volume in the early universe. Fluctuations in the number density of photons and matter particles are equal, but their mass densities change differently
  - Can be described as "1/*f* noise" ; an example is the Harrison-Zeldovich spectrum, with equal power on all mass scales (not frequencies). This is also what we see in the CMB
- **Isocurvature:** Start with no perturbations in the total mass/energy density field, but with fluctuations in the matter opposed to the radiation $\delta_\gamma = -\delta_m$
- **Isothermal:** Radiation field is unperturbed, fluctuations in matter only (rarely considered nowadays)
  - Can be described as "white noise" (equal power on all frequencies, so most power on small mass scales)

Dark Matter and Damping of Fluctuations

- Different types of dark matter form structure differently
- Baryonic dark matter is coupled to radiation, so it does not help in forming structure prior to the recombination
- Fluctuations can be erased or damped by sound waves (this is also called the Meszaros effect). This is important for slowly moving DM particles, i.e., cold dark matter (CDM)
- They can be erased by free streaming of relativistic particles, i.e., hot dark matter (HDM); diffusion of photons, which then "drag along" the baryons in the radiation-dominated era, does the same thing (this is also called the Silk damping)
- Thus HDM vs. CDM make very different predictions for the evolution of structure in the universe!
- In any case, the smaller fluctuations are always erased first

Damping of Fluctuations

The primordial (H-Z) spectrum

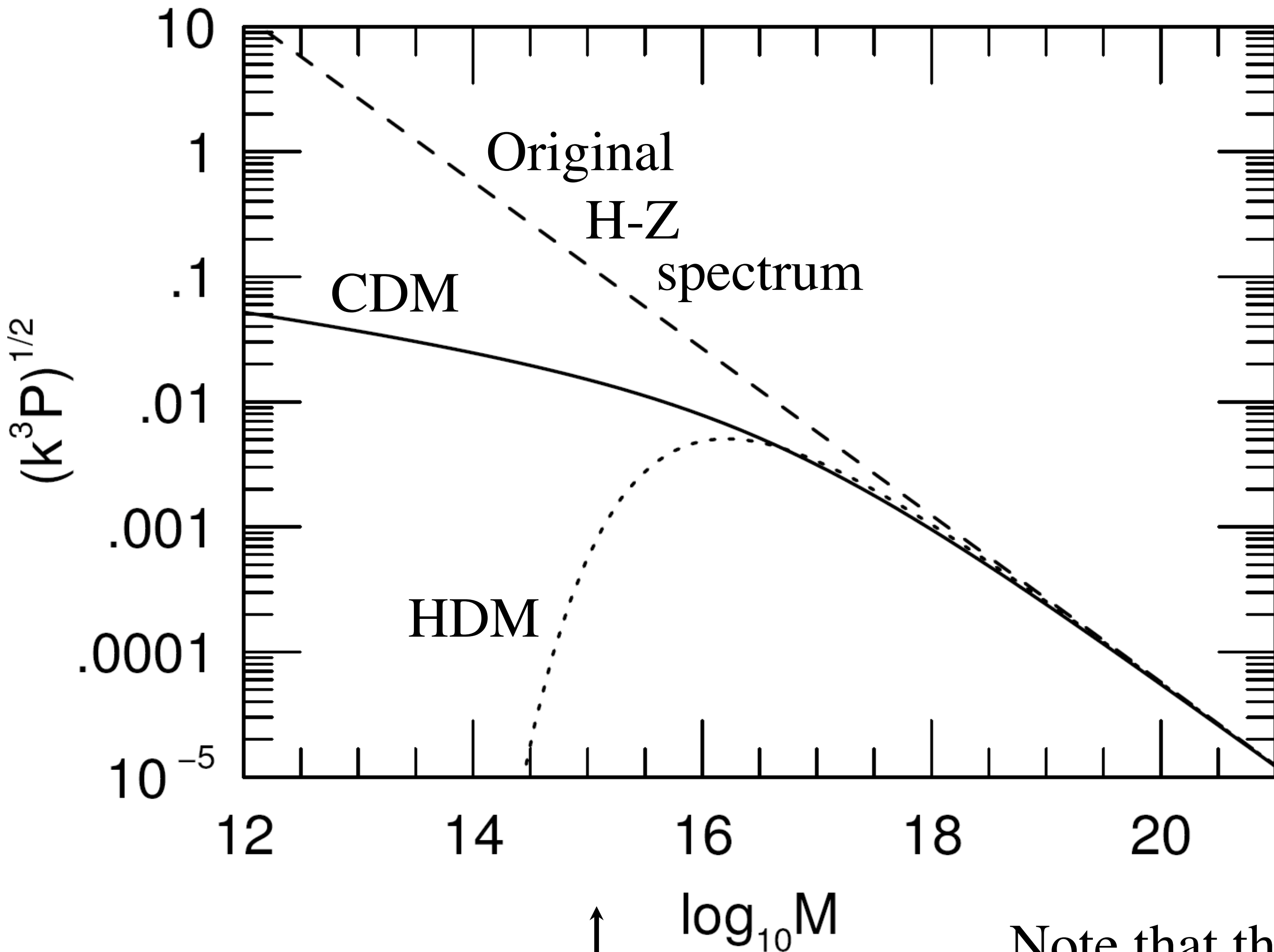


Sound waves diminish the strength of small scale fluc's for the CDM case

Relativistic streaming and photon diffusion erase them completely for the HDM case

Damping of Fluctuations

The same, but represented in terms of the mass scales



Note that the cutoff mass is ~ galaxy cluster mass

Structure Formation in the HDM Scenario

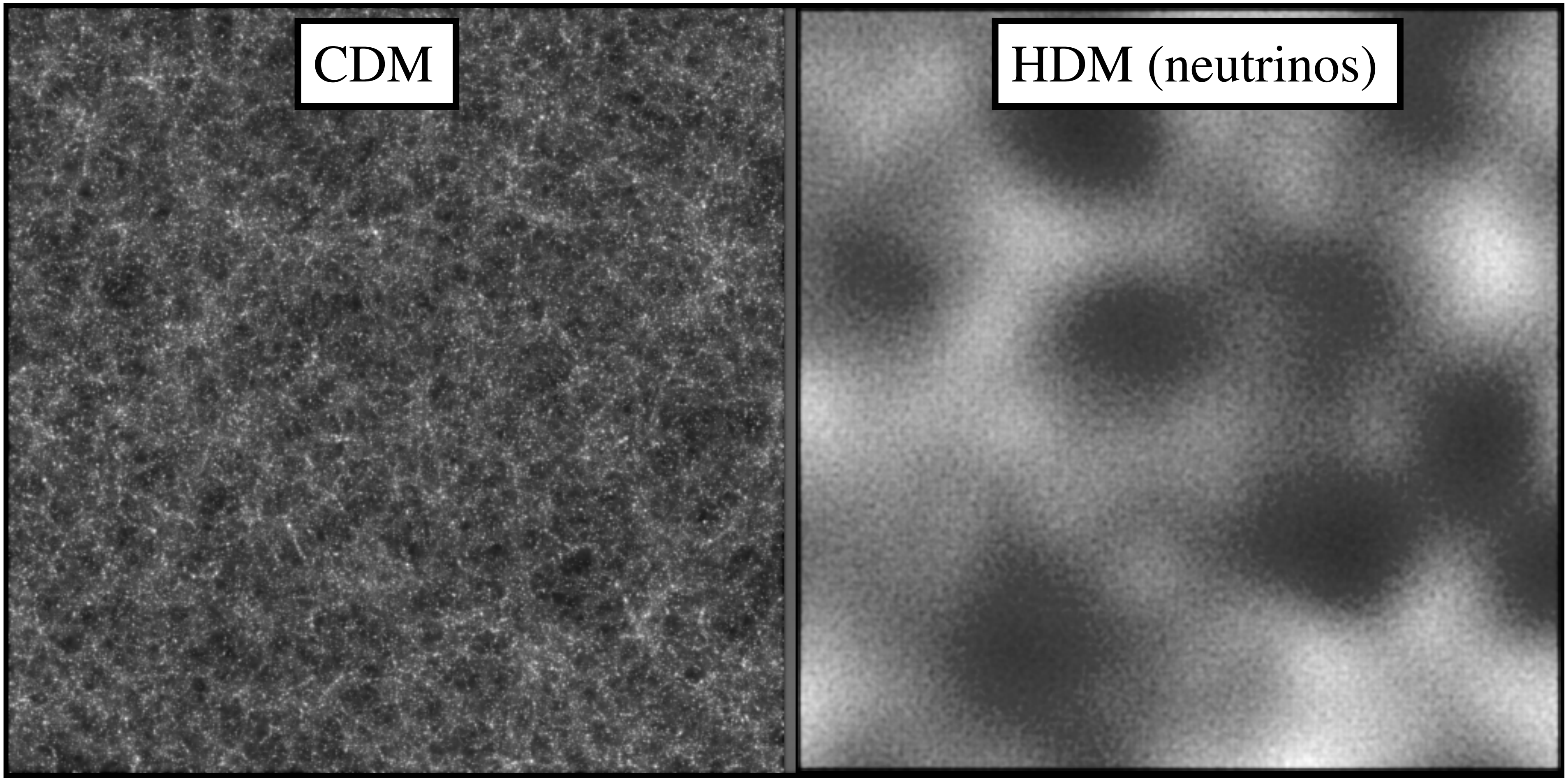
- HDM particles are relativistic, their speed means they can escape from small density fluctuations. This removes mass from the fluctuation and essentially smooths out any small fluctuations
- For example, large amounts of neutrinos will dissolve away mass fluctuations smaller than $10^{15}\ M_{\odot}$ before recombination. *Temperature fluc's in CMB should have large angular wavelengths and large amplitudes*
- Thus, only big lumps survive to collapse. These lumps are on the scale of clusters of galaxies, with relatively low overdensities, and thus collapse slowly. After the big structures have collapsed, fragmentation into smaller structures (like galaxies) can occur. Structure forms slowly, **"top-down".** *Galaxies form very late in the universe's history (~ now)*
- **This isn't what we see, so HDM doesn't work!**

Structure Formation in the CDM Scenario

- CDM particles don't diffuse out of small lumps. So lumps exist on all scales, both large and small
- Small lumps collapse first, big things collapse later. The larger overdensities will incorporate smaller things as they collapse, via merging
- Structure forms early with CDM, and it forms **"bottom-up"** Galaxies form early, before clusters, and clusters are still forming now
- This picture is known as **"hierarchical structure formation"**
- This closely matches what we observe
- It also produces the right kind of CMB fluctuations
- Thus, while we know that massive neutrinos (which would constitute HDM) do exist, most of the dark matter must be cold!

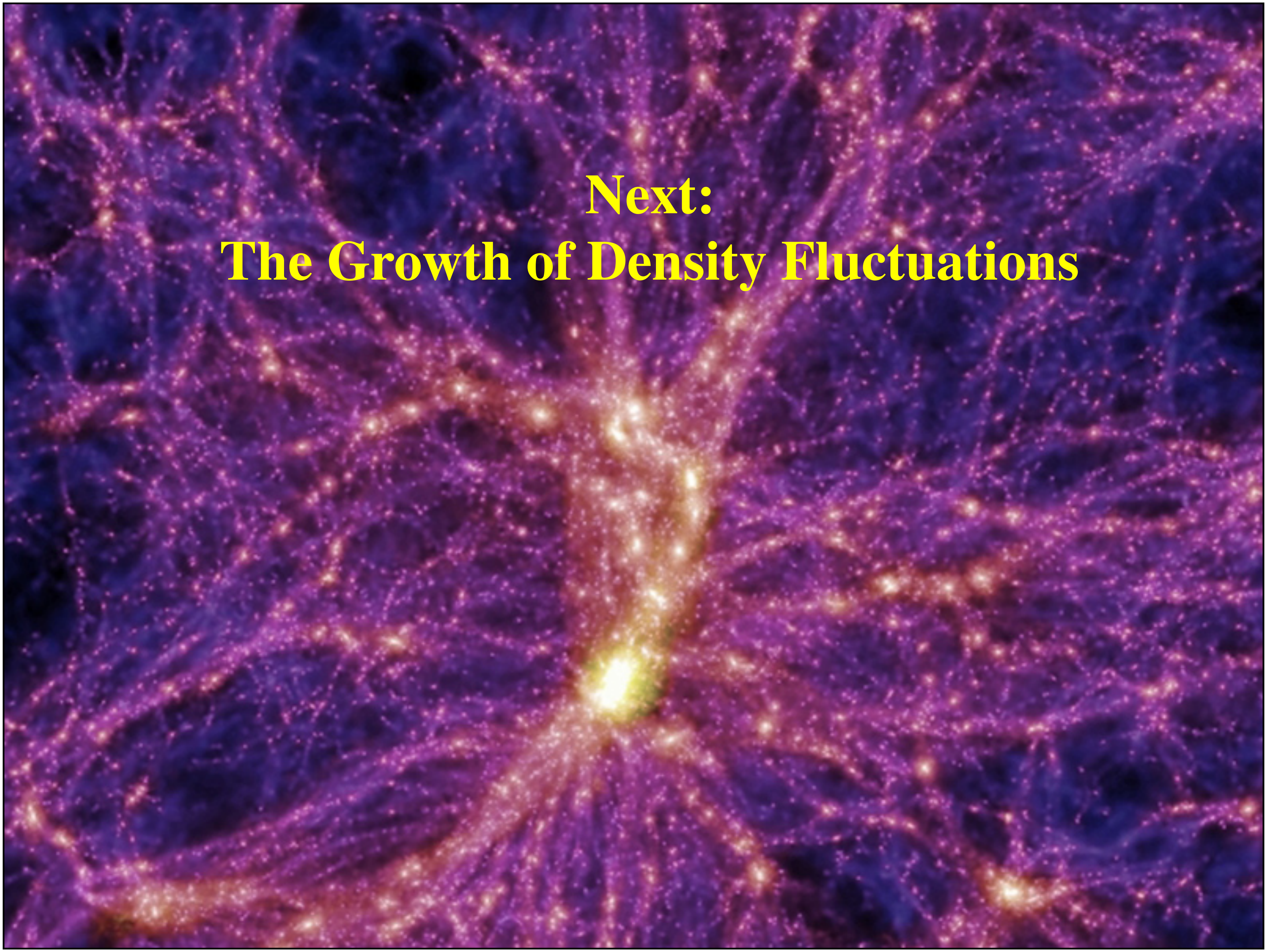
A Comparison of Structure Formation

Using Numerical Simulations



Box size = 100 h^{-1} Mpc

(Jenkins & Frenk 2004)


Next:
The Growth of Density Fluctuations