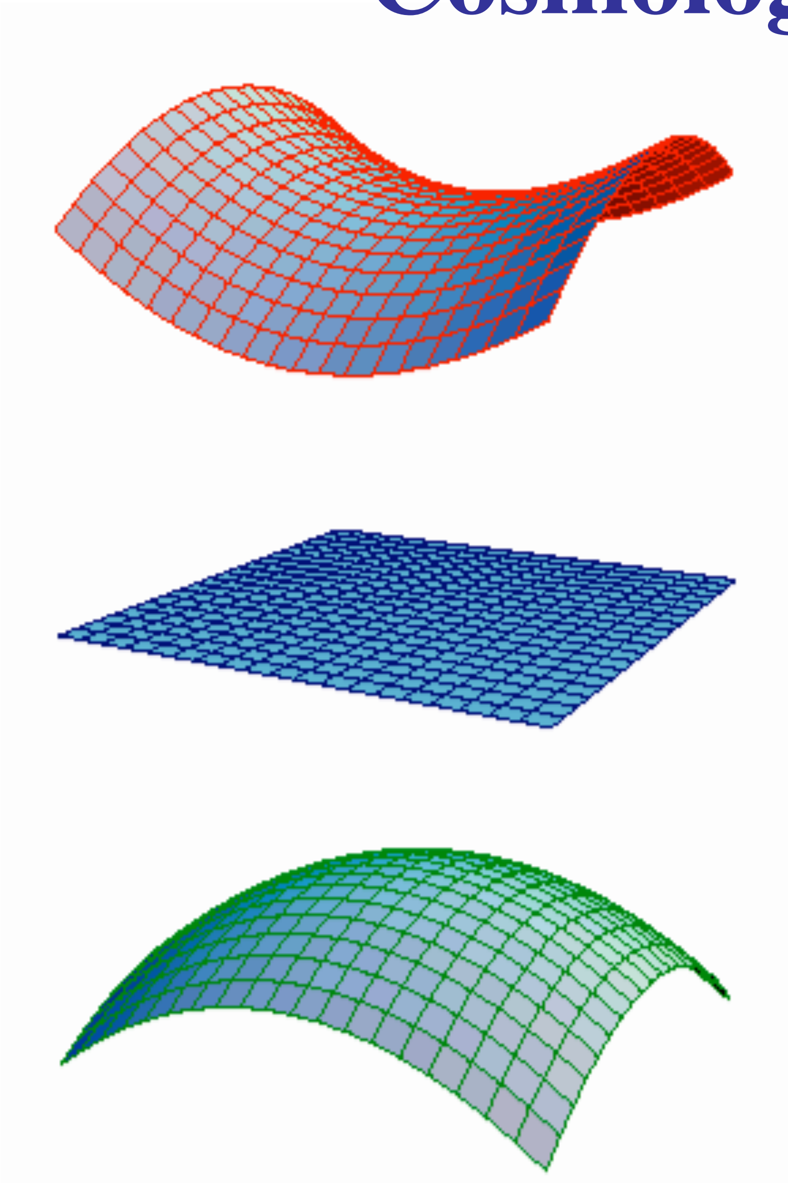
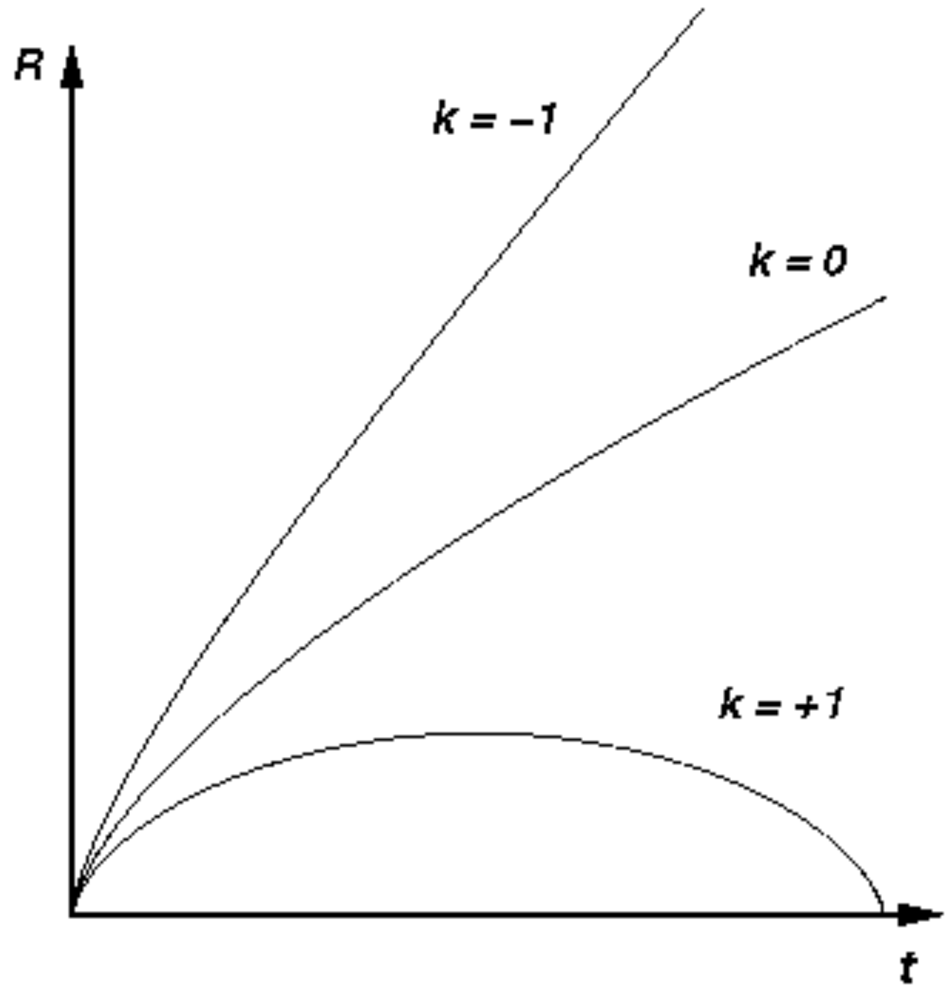
# Cosmological Parameters





# Cosmological Parameters

Cosmological models are typically defined through several handy key parameters:

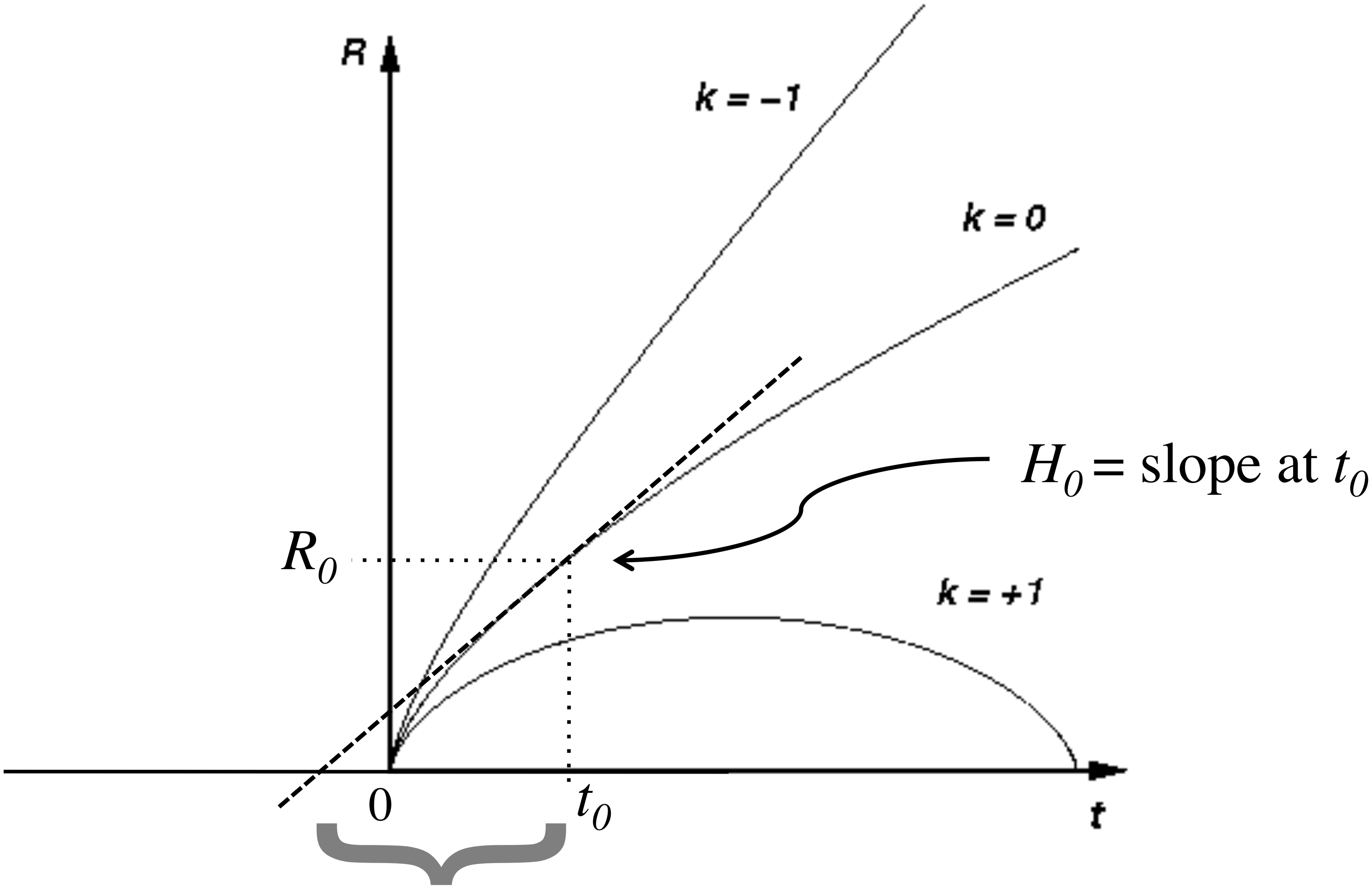
## 1. The Hubble Parameter

The **Hubble parameter** is the normalized rate of expansion:

$$\boxed{H \equiv \frac{\dot{R}}{R}}$$

Note that the Hubble parameter is not a constant! The **Hubble constant** is the Hubble parameter measured today -- we denote its value by $H_0$. Current estimates are in the range of $H_0$ = 65-75 km/s/Mpc -- we will discuss these efforts in more detail later.

# Hubble Constant Defines the Scale of the Universe




$1 / H_0$ = Hubble time

$c / H_0$ = Hubble length

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## 2. The Matter Density Parameter.

Rewriting the Friedmann Eqn. using the Hubble parameter, and for now set $\Lambda = 0$:

$$H^2 - \frac{8}{3}\pi G\rho = -\frac{kc^2}{R}$$

The Universe is flat if k=0, or if it has a critical density of

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

We define the **matter density parameter** as

$$\boxed{\Omega_M = \frac{\rho}{\rho_{crit}}}$$

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## 3. The "dark energy" density parameter

We can express a similar density parameter for lambda again by using the Friedmann equation and setting $\rho_M = 0$. We then get

$$\boxed{\Omega_\Lambda = \frac{\Lambda c^2}{3H^2}}$$

The total density parameter is then

$$\boxed{\Omega = \Omega_M + \Omega_L}$$

## 4. The deceleration parameter

$$\boxed{q = -R\ddot{R}/\dot{R} = \frac{\Omega_M}{2} - \Omega_L}$$

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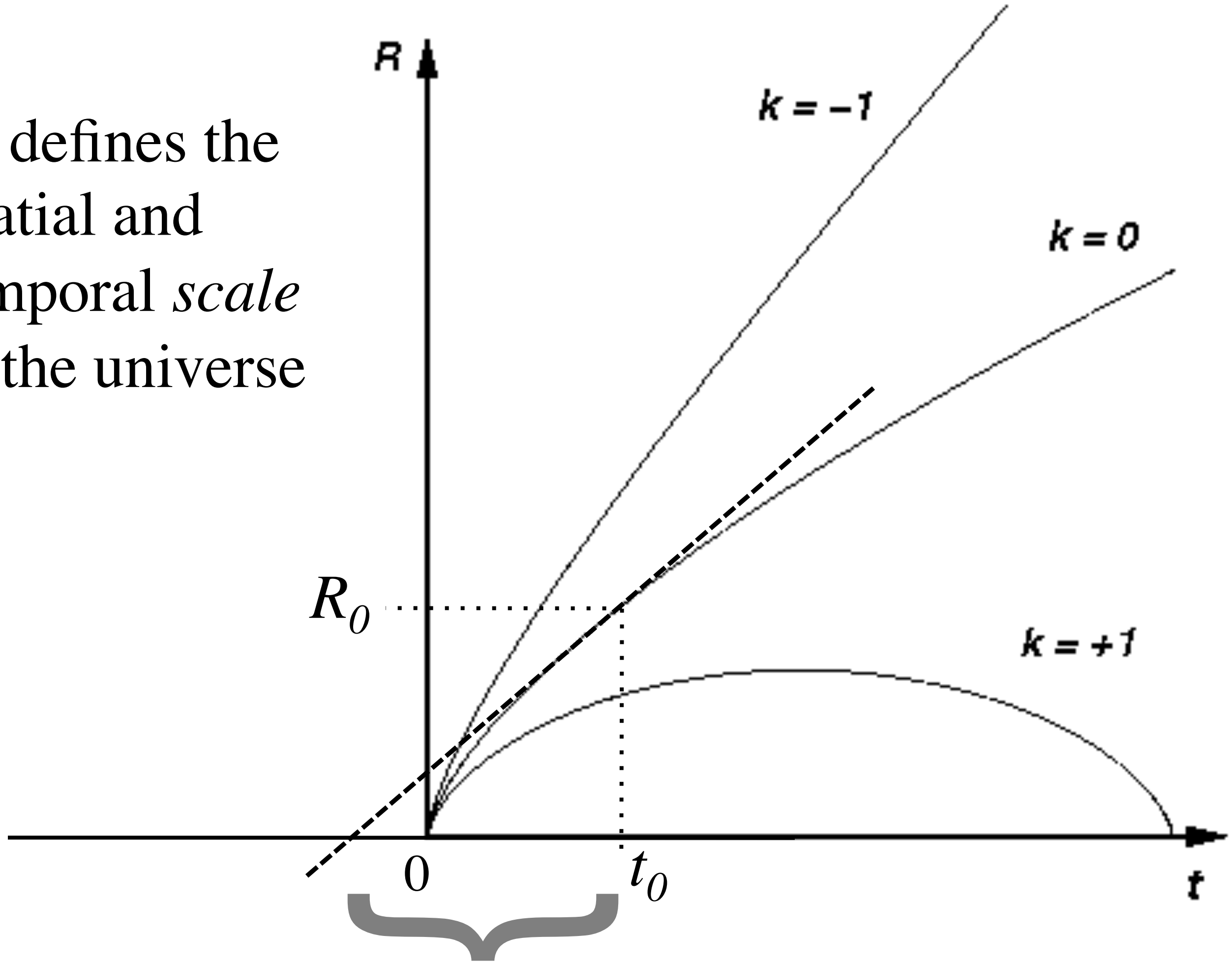
$H_0$ defines the spatial and temporal *scale* of the universe




The other parameters ($\Omega_x$) determine the *shape* of the *R(t)* curves

$1 / H_0$ = Hubble time

$c / H_0$ = Hubble length

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**A few notes:**

The Hubble parameter is usually called the Hubble constant (even though it changes in time!) and it is often written as:

$\boldsymbol{h} = H_0 / (100 \text{ km s}^{-1} \text{ Mpc}^{-1})$, or $\boldsymbol{h_{70}} = H_0 / (70 \text{ km s}^{-1} \text{ Mpc}^{-1})$

The current physical value of the critical density is

$$\rho_{0,\mathrm{crit}} = 0.921 \times 10^{-29}\, h_{70}^{2} \text{ g cm}^{-3}$$

The density parameter(s) can be written as:

$$\Omega_m + \Omega_k + \Omega_\Lambda = 1$$

where $\Omega_k$ is a fictitious "curvature density"

# Cosmological Parameters

Recall the definitions of the cosmological parameters:

$$\Omega_M = \rho_0/\rho_c = \frac{8\pi G}{3H_0}\rho_0$$

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2}$$

$$\Omega_M + \Omega_\Lambda + \Omega_k = 1$$

$$\Omega_k = -\frac{k}{R_0^2 H_0^2}$$

If $\Omega_k = 0$:

$$q_0 = \frac{1}{2}\Omega_M - \Omega_\Lambda$$

The Friedmann Eqn. is now:

$$\left(\frac{\dot{R}}{\dot{R}_0}\right)^2 = \Omega_M\left(\frac{R_0}{R}\right) + \Omega_k + \Omega_\Lambda\left(\frac{R}{R_0}\right)^2$$

# Next:

## Solving the Friedmann Equation, and the Equation of State