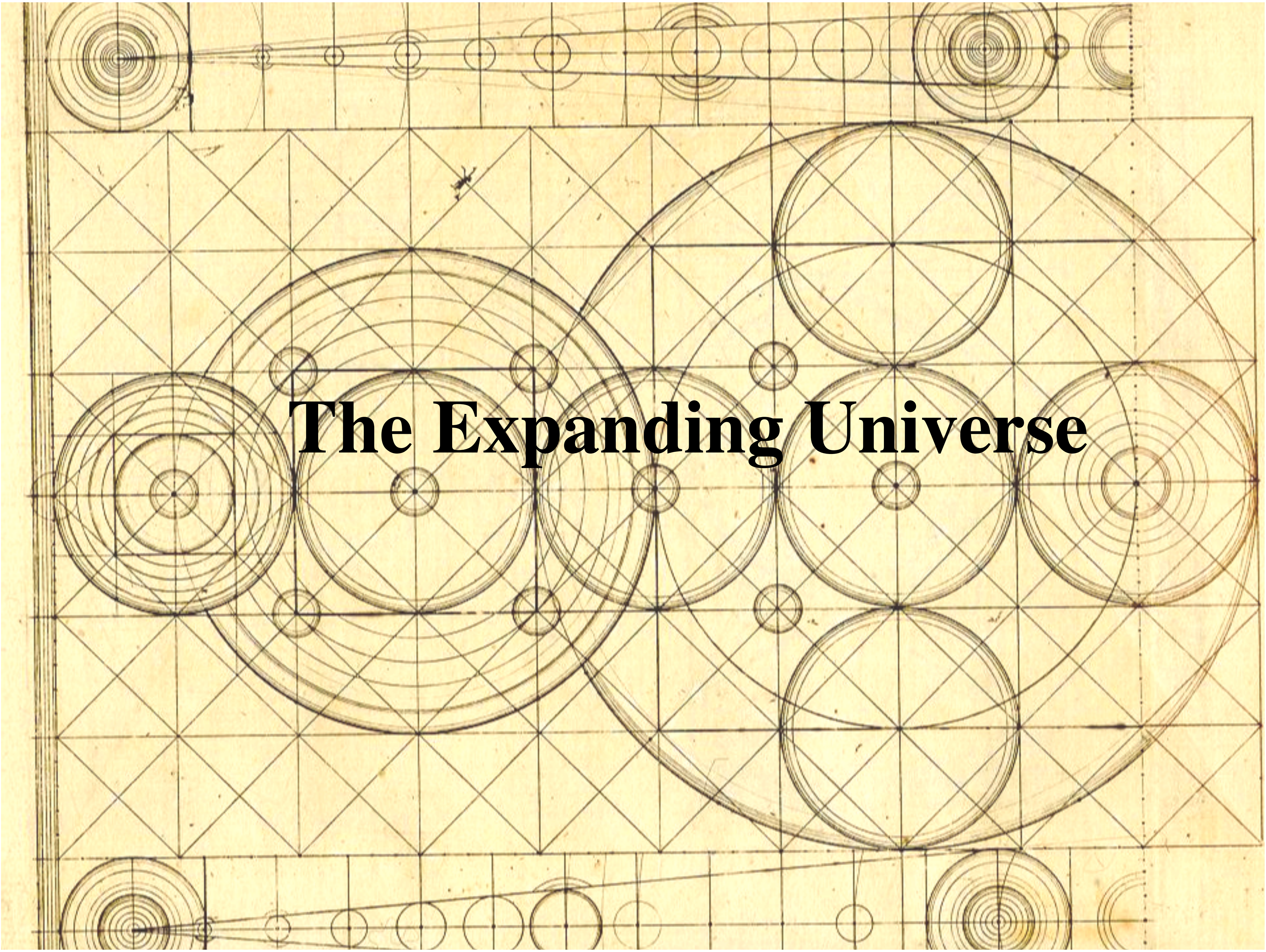
The Expanding Universe



Expansion Relative to What? Comoving and Proper Coordinates

There are fundamentally two kinds of coordinates in a GR cosmology:

- ***Comoving coordinates*** = expand with the universe

  Examples:

  - Unbound systems, e.g., any two distant galaxies
  - Wavelengths of massless quanta, e.g., photons

- ***Proper coordinates*** = stay fixed, space expands relative to them. Examples:

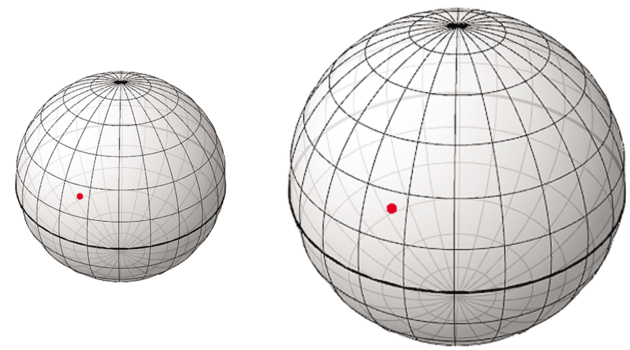
  - Sizes of atoms, molecules, solid bodies
  - Gravitationally bound systems, e.g., Solar system, stars, galaxies …

Expansion into What?

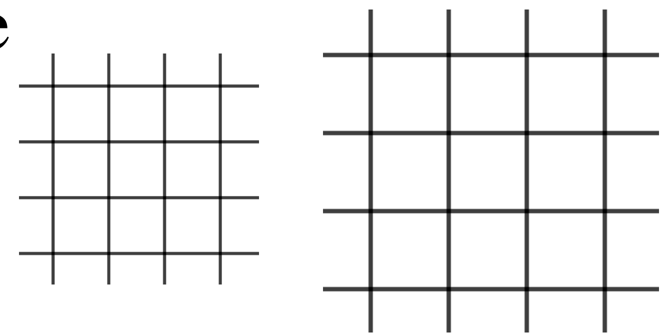
Into itself. There is nothing "outside" the universe

(Let's ignore the multiverse hypothesis for now)

A positive curvature universe is like the surface of a sphere, but in one extra dimension. Its volume is finite, but changes with the expansion of space.



A flat or a negative curvature universe is infinite in all directions; the comoving coordinate grid stretches relative to the proper coordinates

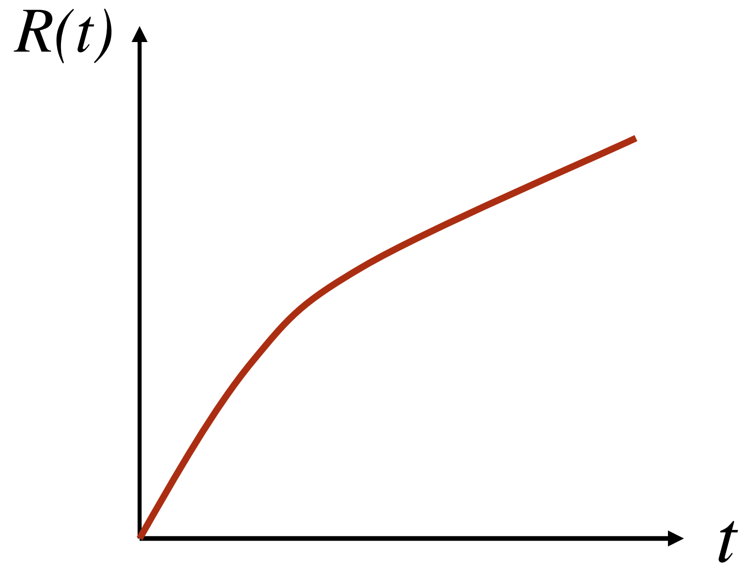


In either case, there is no "edge", and there is no center

(homogeneity and isotropy)

Quantifying the Kinematics of the Universe

We introduce a **scale factor**, commonly denoted as ***R(t)*** or ***a(t):*** a spatial distance between any two unaccelerated frames which move with their comoving coordinates

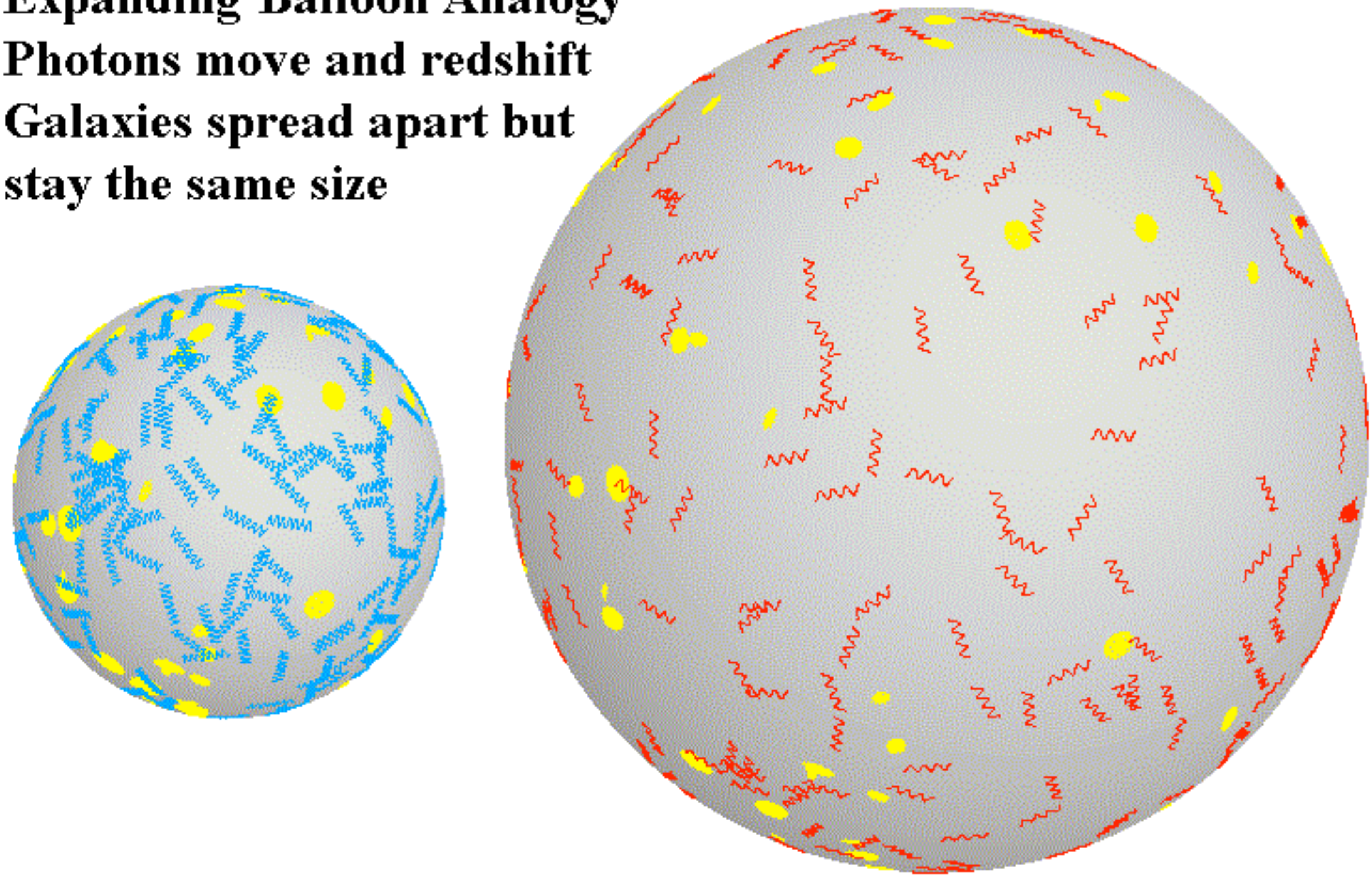


This fully describes the evolution of a homogeneous, isotropic universe

Computing ***R(t)*** and various derived quantities defines the **cosmological models**. This is accomplished by solving the **Friedmann Equation**

The Cosmological Redshift

Expanding Balloon Analogy
Photons move and redshift
Galaxies spread apart but stay the same size



Redshift as Doppler Shift

We define **doppler redshift** to be the shift in spectral lines due to motion:

$$z = \frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1 + v/c}{1 - v/c}} - 1$$

which, in the case of **v<<c** reduces to the familiar

$$z = \frac{v}{c}$$

The **cosmological redshift** is something different, although we are often sloppy and refer to it in the same terms of the doppler redshift. The cosmological redshift is actually due to the **expansion of space** itself.

Cosmological Redshift

A more correct approach is to note that the wavelengths of photons expand with the universe:

$$\frac{R(t_0)}{R(t_e)} = \frac{\lambda_0}{\lambda_e}$$

Or, by our definition of redshift:

$$z = \frac{\Delta\lambda}{\lambda}$$

We get:

$$\boxed{\frac{R(t_0)}{R(t_e)} = (1 + z)}$$

The two approaches are actually equivalent

Propagation of Light

Our view of the Universe depends upon the propagation of light through the curved space. To understand this, we need to consider the paths of null geodesics.

Suppose an observer sits at *r=0*. Consider radial light rays. Given that *ds=0*, then

$$\frac{dt}{R(t)} = \pm \frac{dr}{(1 - kr^2)^{1/2}}$$

Suppose a light ray is emitted at a time t_1 at a distance r_1 and is received today (r=0) at t_0

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = - \int_{r_1}^{0} \frac{dr}{(1 - kr^2)^{1/2}}$$

Propagation of Light

Remembering that for a comoving source at distance *r*, the coordinate is fixed, then

$$\frac{dt_o}{R(t_o)} = \frac{dt_1}{R(t_1)}$$

Hence, in an expanding universe there will be a redshift

$$1 + z = \frac{\nu_1}{\nu_0} = \frac{dt_o}{dt_1} = \frac{R(t_o)}{R(t_1)}$$

Hubble's Law

If we consider a nearby source, then we can write

$$1 + z \approx 1 + \frac{\dot{R}(t_o)}{R(t_o)} dt$$

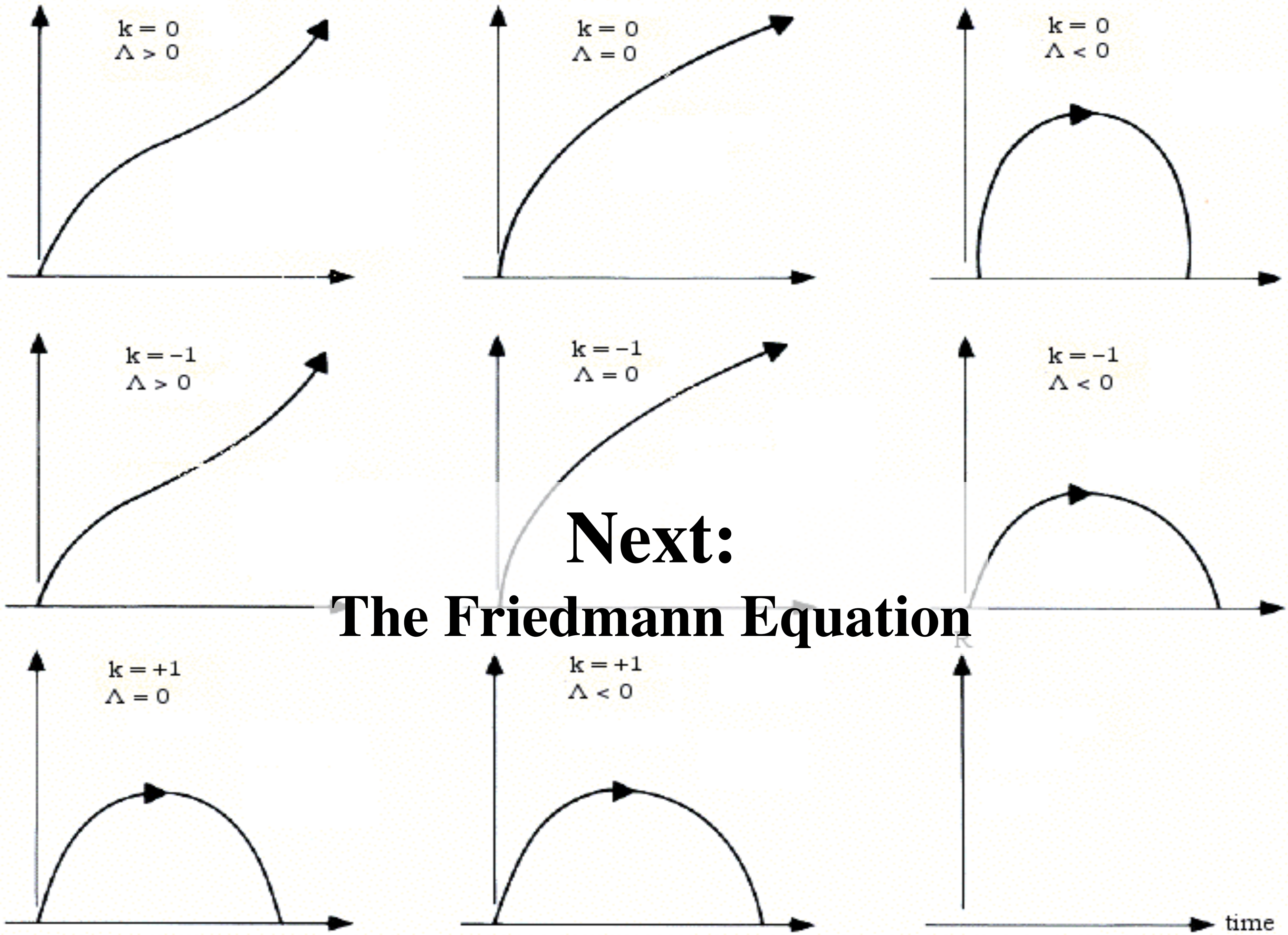
then

$$z \approx \dot{R}(t_o) r_1 = \frac{\dot{R}(t_o)}{R(t_o)} d = H(t) d$$

Hence, we can see we can derive Hubble's law from the Robertson-Walker metric. Hubble's "constant" *H(t)* gives the instantaneous expansion rate.

Is Energy Conserved in an Expanding (or Contracting) Universe?

- Consider energies of photons
- Consider potential energies of unbound systems



Next:

The Friedmann Equation